Modelling of Natural Convection Flows with Large Temperature Differences: A Benchmark Problem for Low Mach Number Solvers

H. Paillère‡ and P. Le Quéré§
‡CEA Saclay, DEN/DM2S/SFME, 91191 Gif-sur-Yvette Cedex, France
henri.paillere@cea.fr
§LIMSI, BP 133, 91403 Orsay Cedex, France
plq@limsi.fr

January 19, 2004

A natural convection test-case

The test-case we have selected is the differentially heated square cavity problem depicted in figure 1, which was the object of a well-known benchmark [1] for incompressible flow solvers and produced a set of reference solutions for different Rayleigh numbers Ra, ranging from 10^3 to 10^6. We recall that for a perfect gas, the Rayleigh number is defined as

$$Ra = Pr\ \frac{g\rho_o^2(T_h - T_c)L^3}{T_o\ \mu_o^2} \tag{1}$$

where Pr is the Prandtl number (0.71 for air), g is the gravity, L the height of the cavity, T_h and T_c the hot and cold temperatures applied to the vertical walls, T_o a reference temperature equal to $(T_h+T_c)/2$, ρ_o a reference density corresponding to T_o, and μ the coefficient of viscosity at T_o. The temperature differences may be defined by the non-dimensional parameter ϵ,

$$\epsilon = (T_h - T_c)/(T_h + T_c). \tag{2}$$

For small enough ϵ, compressibility effects may be neglected, and incompressible flow models with the Boussinesq approximation are valid and accurate enough to compute the flow and the heat transfer to the walls [2]. The latter is characterized by the local and average Nusselt numbers Nu and $\overline{Nu}$,

$$Nu(y) = \frac{L}{k_o(T_h - T_c)}\, k \frac{\partial T}{\partial x}\bigg|_w, \qquad \overline{Nu} = \frac{1}{L}\int_{y=0}^{y=L} Nu(y)\, dy \tag{3}$$

where $k(T)$ is the thermal conductivity and $k_o = k(T_o)$. For large temperature differences, the Boussinesq hypotheses break down and one needs to resort to a compressible flow model, or since the Mach numbers remain small, to a low Mach number approximation model. Chenoweth and Paolucci [3] have studied in detail the effects of Ra and ϵ on flow patterns and heat transfer, but used temperature-dependent properties as well as varying Prandtl number. Polezhaev [4] studied the $\epsilon = 0.2$ case with temperature-dependent properties and constant Prandtl number. Le Quéré et al. [5] have also studied the non-Boussinesq cases (up to $\epsilon = 0.6$) for constant Prandtl number and Sutherland's Law for the viscosity.

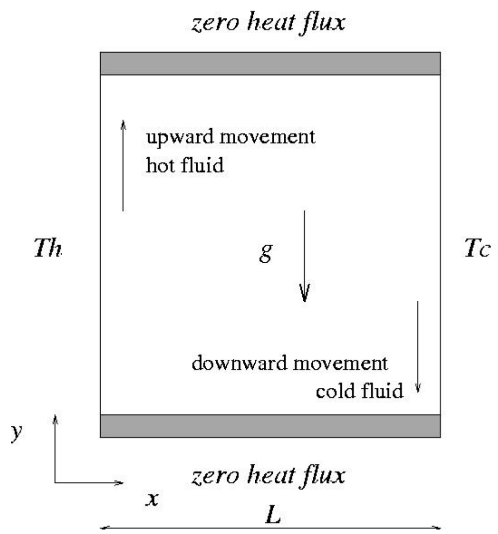

Figure 1: Differentially heated square cavity problem

Governing equations

We consider in this study the Navier-Stokes equations describing the flow of a compressible, calorically and thermally perfect gas, with constant Prandtl number. In conservative form, they read:

$$\begin{aligned}
&\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \underline{u}) = 0 \\
&\frac{\partial}{\partial t}(\rho u) + \nabla \cdot (\rho \underline{u} \otimes \underline{u} + p\underline{\underline{I}}) = \rho \underline{g} + \nabla \cdot \underline{\underline{\tau}} \\
&\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\rho \underline{u} H) = \nabla \cdot (k \nabla T) + \rho \underline{g} \cdot \underline{u} \\
&p = \rho R T = (\gamma - 1)\rho(E - \frac{1}{2}\|\underline{u}\|^2)
\end{aligned} \tag{4}$$

where ρ, u, p, E and H are respectively the density, velocity, pressure, specific total energy and specific total enthalpy. $\underline{g}$ and $\underline{\underline{\tau}}$ represent respectively the gravity and the viscous stress tensor,

$$\underline{\underline{\tau}} = \mu \left(\nabla \underline{u} + \nabla^T \underline{u} - \frac{2}{3}(\nabla \cdot \underline{u})\underline{\underline{I}} \right)$$

μ and k are respectively the dynamic viscosity and the heat conduction coefficients. At low Mach numbers, the dissipation term $\nabla \cdot (\underline{\underline{\tau}} \cdot \underline{u})$ in the energy equations may be neglected, and was omitted in the equation.
For non-conservative solvers based on low Mach number asymptotic approximations of the Navier-Stokes equations, the energy equation may take different forms, and be written in terms of specific internal energy, enthalpy or temperature. For a perfect gas, and neglecting the energy dissipation term, the following form is advocated,

$$\rho c_p \left(\frac{\partial}{\partial t} T + \underline{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \frac{\mathrm{d}P}{\mathrm{d}t}$$

where $P(t)$ is the (spatially uniform) thermodynamic pressure in the domain, and c_p is the specific heat at constant pressure, which may be expressed in terms of the ratio of specific heats γ and the gas constant R, $c_p = \gamma R/(\gamma - 1)$. In this study, we shall assume either constant or temperature-dependent transport coefficients $\mu(T)$ and $k(T)$, given by Sutherland's Law:

$$\frac{\mu(T)}{\mu^*} = \left(\frac{T}{T^*} \right)^{\frac{3}{2}} \frac{T^* + S}{T + S}, \quad k(T) = \frac{\mu(T)\gamma R}{(\gamma - 1)Pr}$$

with $T^* = 273$ K, $S = 110.5$ K, $\mu^* = 1.68 \times 10^{-5}$ kg/m/s, $\gamma = 1.4$ and $R = 287$ J/kg/K, and $Pr = 0.71$.

We remark that at steady state, integration of the energy equation over the volume of the cavity represented in figure 1 implies that the average Nusselt numbers on the vertical walls denoted by L (left wall at T_h) and R (right wall at T_c) are equal:

$$\overline{Nu}_L = \overline{Nu}_R \tag{5}$$

Test Cases

Three test cases have been selected for which the flow is laminar and a steady-state solution exists,

- Test case T1: $Ra = 10^6$, constant properties, $\varepsilon = 0.6$
- Test case T2: $Ra = 10^6$, Sutherland's Law, $\varepsilon = 0.6$
- Test case T3: $Ra = 10^7$, Sutherland's Law, $\varepsilon = 0.6$

A fourth test case, T4, corresponding to an *unsteady* flow, will be specified soon.

Initial conditions

In each case, the problem is completely defined by the Rayleigh number, the value of ε and the following coefficients.

$$\begin{aligned} P_o &= 101325 \text{ Pa} \\ T_o &= 600 \text{ K} \\ R_{air} &= 287 \text{ J/kg/K} \\ \rho_o &= \frac{P_o}{RT_o} \\ Pr &= 0.71 \\ \gamma &= 1.4 \\ g &= 9.81 \ m/s^2 \end{aligned}$$

Spatially uniform initial conditions are imposed, $\forall(x, y) \in [0, L]^2$,

$$\begin{aligned} T(x,y) &= T_o \\ \rho(x,y) &= \rho_o \\ u(x,y) = v(x,y) &= 0 \end{aligned}$$

Boundary conditions

On the hot wall, a temperature of $T_h = T_o(1 + \varepsilon)$ is imposed, and on the cold wall, a temperature of $T_c = T_o(1 - \varepsilon)$ is imposed. On the horizontal walls, adiabatic conditions are applied. On all walls, the no-slip condition is imposed for the velocity.

Required results

The participants are asked to produce "grid-converged" results, or to ensure that their numerical results are sufficiently accurate by refinining the mesh until the solutions vary no more (at least for the first four digits).

It is asked to submit results in the following format:

- Nusselt distributions on hot and cold walls, $\bar{y} \quad Nu(\bar{y})$, where $\bar{y} = y/L$.
  Name of file: $Tn - Nu - w - m$, where $n = 1, 2$ or 3 (test cases T1, T2, T3), $w = h$ (hot wall) or c (cold wall) and m is the participant letter which will be supplied.
  example: $T2 - Nu - h - B$ is the file containing the Nusselt distribution on the hot wall for participant B.

- Velocity distributions on axes $x = 0.5$ and $y = 0.5$, where the components u and v are non-dimensionalized with the reference velocity,

$$V_{ref} = \frac{\mu_o Ra^{0.5}}{\rho_o L}$$

  Name of files:
  $u - x0.5 - m$, containing two columns, $\bar{y}$ and $\bar{u}(\bar{x} = 0.5, \bar{y})$,
  $v - x0.5 - m$, containing two columns, $\bar{y}$ and $\bar{v}(\bar{x} = 0.5, \bar{y})$,
  $u - y0.5 - m$, containing two columns, $\bar{x}$ and $\bar{u}(\bar{x}, \bar{y} = 0.5)$,
  $v - y0.5 - m$, containing two columns, $\bar{x}$ and $\bar{u}(\bar{x}, \bar{y} = 0.5)$,
  where $\bar{x} = x/L$, $\bar{y} = y/L$ and $\bar{v} = v/V_{ref}$.

- a data file containing the following information

$$\bar{N}u_h$$
$$\bar{N}u_c$$
$$Nu(\bar{y} = 0.5)_h$$
$$Nu(\bar{y} = 0.5)_c$$
$$Nu_{\max h}$$
$$Nu_{\min h}$$
$$Nu_{\max c}$$
$$Nu_{\min c}$$
$$p_{\max}/Po$$
$$p_{\min}/Po$$
$$\frac{L}{V_{ref}} (\nabla \cdot \underline{u})_{\max}$$

$\frac{L}{V_{ref}}(\nabla \cdot \underline{u})_{\min}$

$\frac{L}{V_{ref}}(\nabla \cdot \rho\underline{u})_{\max}$

$\frac{L}{V_{ref}}(\nabla \cdot \rho\underline{u})_{\min}$

type of mesh, number of mesh points

CPU to reach steady state, and type of machine

name

The name of the file should be $Tn-dat-m$ where m is the participant letter.

Important dates

15th April, 2004, *Intention to participate*

Send to the organizers a one-page abstract describing the numerical method and code to be used to compute solution. In return, the organizers will send the participant his/her identification letter: m where $m = A, B, C\ \ldots$

28th May, 2004, *Numerical solutions*

Send to the organizers the required files

21 - 25 June, 2004, *Workshop*

The results of the natural convection test cases will be discussed during the workshop dedicated to Low Mach number flow benchmarks.

References

[1] G. De Vahl Davis and I.P. Jones. Natural Convection in a Square Cavity: a Comparison Exercise. *Int. J. Numer. Methods Fluids*, 3:227–248, 1983.

[2] D.D. Gray and A. Giorgini. The Validity of the Boussinesq Approximation for Liquids and Gases. *Int. J. Heat Mass Transfer*, 15:545–551, 1976.

[3] D.R. Chenoweth and S. Paolucci. Natural convection in an enclosed vertical air layer with large horizontal temperature differences. *J. Fluid Mech.*, 169:173–210, 1986.

[4] V.L. Polezhaev. Numerical solution of the system of two-dimensional unsteady Navier-Stokes equations for a compressible gas in a closed region. *Fluid Dyn.*, 2:70–74, 1967.

[5] P. Le Quéré, R. Masson, and P. Perrot. A Chebyshev Collocation Algorithm for 2D Non-Boussinesq Convection. *J. Comput. Phys.*, 103:320–335, 1992.