Calculation of Low Mach Number Acoustics: A Comparison of MPV, EIF and linearized Euler equations

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Overview

- Why Aeroacoustics
- Asymptotic analysis for low Mach number flows
- The Multiple Pressure Variables (MPV) approach
- The Scaled Expansion about Incompressible Flow (EIF) scheme
- Direct simulation via heterogeneous domain decomposition
- Results and comparisons
- Summary

Why Aeroacoustics

- Aeroacoustics become more and more important
- Acoustic waves = weak pressure and density fluctuations
- Governing equations: Compressible Euler/Navier-Stokes equations
- Equations and schemes well known for years
- Still open problems:
  - Sound generation and propagation at once
  - Interaction between fluid and sound

Problems in Aeroacoustics

Large discrepancies between

- ⇨ flow speed and speed of sound
- ⇨ pressure wave amplitudes in flow and sound
- ⇨ length scales for vortex structures and acoustic waves
- ⇨ extent of the domain of interest and resolution in space/time.

Asymptotic analysis for low Mach number flows

$$f\left(\mathbf{x},t;M\right)=f^{(0)}\left(g(\mathbf{x},t,M)\right)+Mf^{(1)}\left(g(\mathbf{x},t,M)\right)+M^2f^{(2)}\left(g(\mathbf{x},t,M)\right)+o(M^2)$$

$$g(\mathbf{x},t,M)=(\mathbf{x},t) \longrightarrow \text{single scale asymptotic}$$

$$g(x,t;M)=(\eta=\mathbf{x},\xi=M\mathbf{x},t) \longrightarrow \text{multi scale asymptotic}$$

Asymptotic analysis for low Mach number flows

- Usually, asymptotics lead to a system of equations of leading order, 1st order, 2nd order etc.
- But: no closed system for leading order terms
- $p^{(0)}(t)$: thermodynamic pressure
- $p^{(1)}(\xi, t)$: acoustic pressure
- $p^{(2)}(\eta, \xi, t)$: hydrodynamic (incompressible) pressure
- $p = p^{(0)} + Mp^{(1)} + M^2 p^{(2)}$

Compressible Euler equations

$$
\begin{aligned}
\rho_t &+ \mathbf{U}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{U} &= 0\\
\mathbf{U}_t &+ (\mathbf{U}\cdot\nabla)\,\mathbf{U} + \frac{1}{M^2\rho}\nabla p &= 0\\
p_t &+ \mathbf{U}\cdot\nabla p + \gamma p\nabla\cdot\mathbf{U} &= 0
\end{aligned}
$$

$$M = \frac{u_{ref}}{\sqrt{p_{ref}/\rho_{ref}}}$$

The Multiple Pressure Variables (MPV) approach

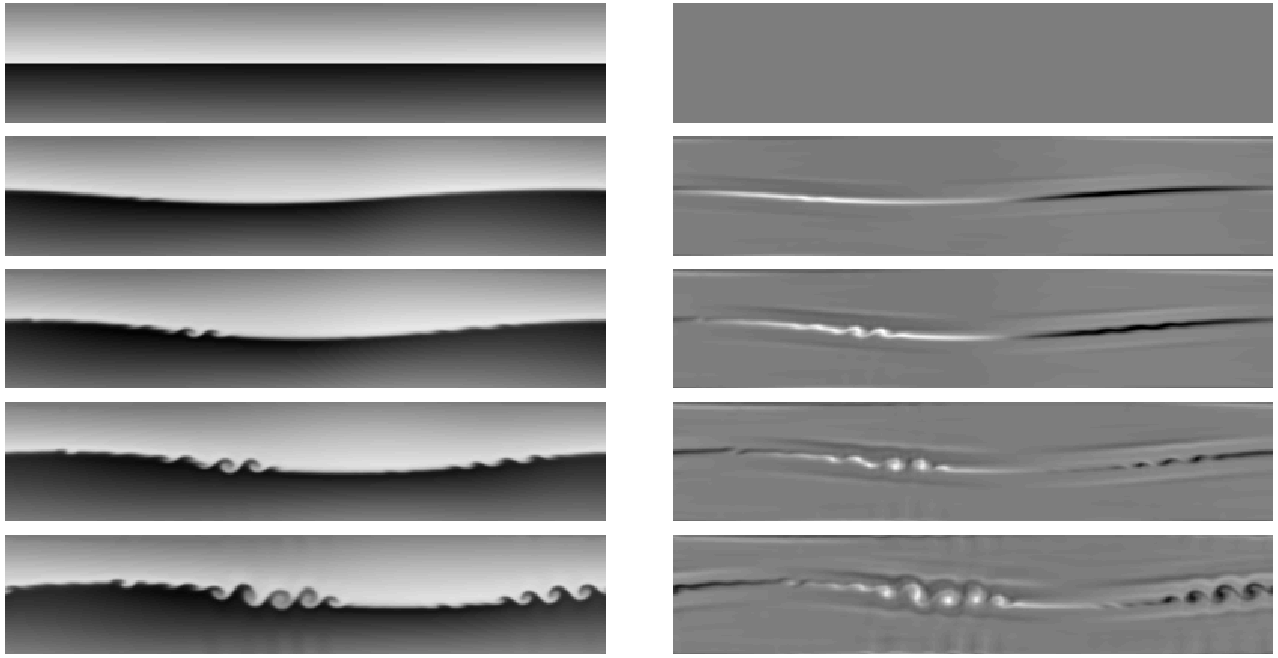
$$
\begin{aligned}
\rho_t \quad &+ \quad \mathbf{U}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{U} &&= 0 \\
\mathbf{U}_t \quad &+ \quad (\mathbf{U}\cdot\nabla)\,\mathbf{U} + \frac{1}{\rho}\nabla p^{(2)} &&= -\frac{1}{M\rho}\nabla p^{(1)} \\
M^2 p_t^{(2)} \quad &+ \quad M^2\mathbf{U}\cdot\nabla p^{(2)} + M^2\gamma p^{(2)}\nabla\cdot\mathbf{U} &&= -p_t^{(0)} - M p_t^{(1)} - M\mathbf{U}\cdot\nabla p^{(1)} \\
& && \quad -\gamma(p^{(0)} + M p^{(1)})\nabla\cdot\mathbf{U}
\end{aligned}
$$

$$
p^{(0)} := \frac{1}{|V|}\int_V p\,dx, \qquad p^{(1)} := \frac{1}{M|V_{ac}|}\int_{V_{ac}} p - p^{(0)}\,dx
$$

$$
p^{(2)} := \frac{1}{M^2}(p - p^{(0)} - M p^{(1)})
$$

The Multiple Pressure Variables (MPV) approach

Flow-Acoustics Interaction: Baroclinic vortex formation at M=1/20



Scaled Expansion about Incompressible Flow (EIF)

Assume

$$\begin{aligned}\rho &= \rho^{(0)} + M^2\rho^{(2)} + M^2\rho' \\ \mathbf{U} &= \mathbf{U}^{(0)} + M\mathbf{U}' \\ p &= p^{(0)} + M^2 p^{(2)} + M^2 p'\end{aligned}$$

with

$$\mathbf{U}_{0,t} + (\mathbf{U}_0 \circ \nabla)\,\mathbf{U}_0 + \frac{1}{\rho_0}\nabla p^{(2)} = 0$$

$$\nabla \cdot \mathbf{U}_0 = 0$$

$$\rho_0 = \frac{p_0}{c_0^2}$$

Scaled Expansion about Incompressible Flow (EIF)

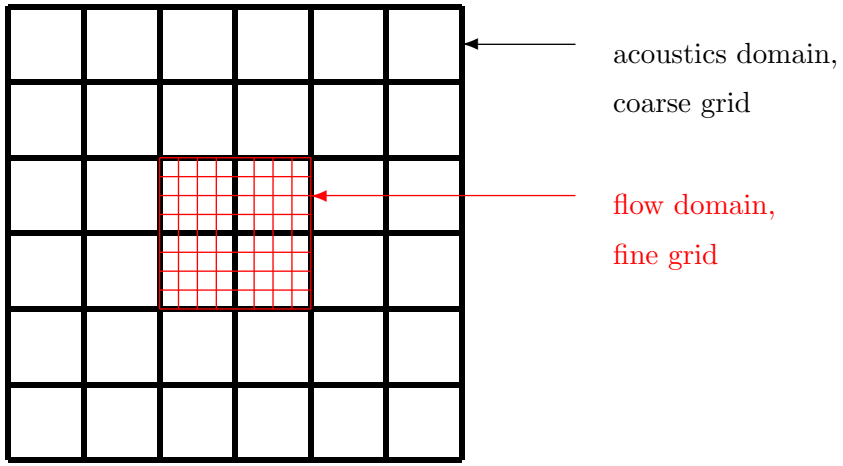
$$\rho'_t + \mathbf{U}^{(0)} \cdot \nabla \rho' + \frac{\rho^{(0)}}{M} \nabla \cdot \mathbf{U}' = -\rho_t^{(2)} - \mathbf{U}^{(0)} \cdot \nabla \rho^{(2)}$$

$$\mathbf{U}'_t + \left(\mathbf{U}^{(0)} \cdot \nabla\right) \mathbf{U}' + \left(\mathbf{U}' \cdot \nabla\right) \mathbf{U}^{(0)} + \frac{\nabla p'}{\rho^{(0)}\ M} = 0$$

$$p'_t + \mathbf{U}^{(0)} \cdot \nabla p' + \frac{\gamma p^{(0)}}{M} \nabla \cdot \mathbf{U}' = -p_t^{(2)} - \mathbf{U}^{(0)} \cdot \nabla p^{(2)}$$

Scaled Expansion about Incompressible Flow (EIF)

Coupling via embedded domains (volume coupling)



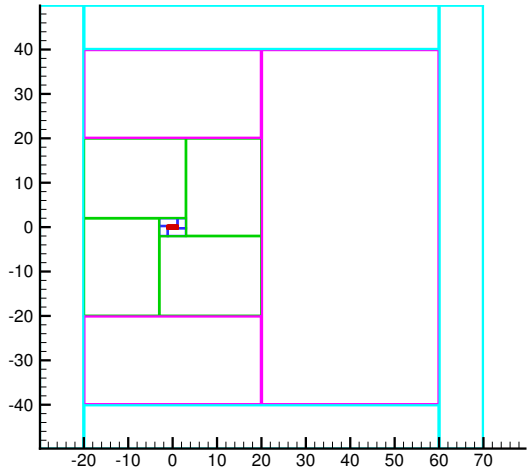
1. Advance incompressible flow on the fine grid to the next time level
2. Calculate the acoustic source terms on the fine grid
3. Put the acoustic source terms from the fine grid to the coarse grid
4. Advance the acoustic calculation on the coarse grid to the next time level

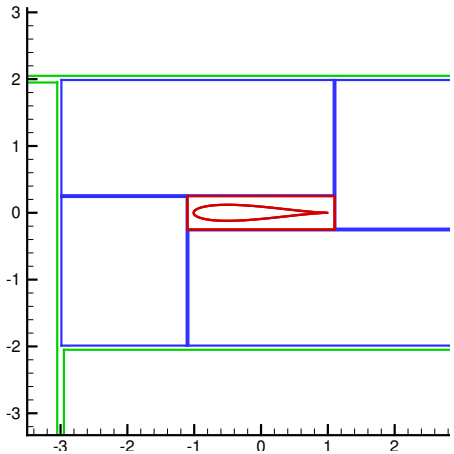
Direct simulation

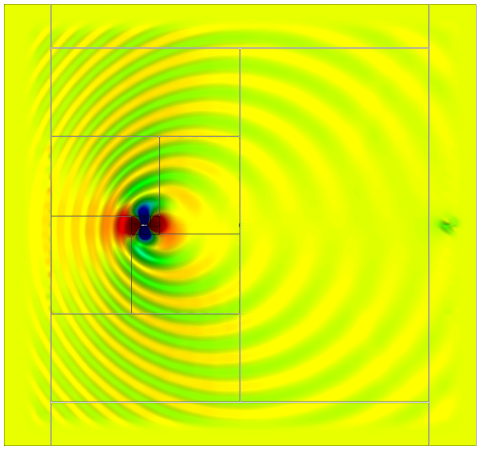
- Direct simulation offers valuable means of testing validity and applicability of aeroacoustic theories
- High order needed in space **and** time
- Very costly therefore only simple problems
- Domain Decomposition to reduce computational costs

Heterogeneous Domain Decomposition

Coupling on the surface only







Heterogeneous Domain Decomposition

Heterogeneous in

- the mesh (different cell sizes, structured-unstructured meshes)
- the mathematical modeling (Navier–Stokes, Euler, linearized Euler)
- different numerical schemes (Finite-Volume, Finite-Difference, Discontinuous-Galerkin)
- different time-steps (optimal time-step w.r.t. CFL condition)
- discontinuous fluxes at the interfaces to avoid reflections

Computations in time domain only

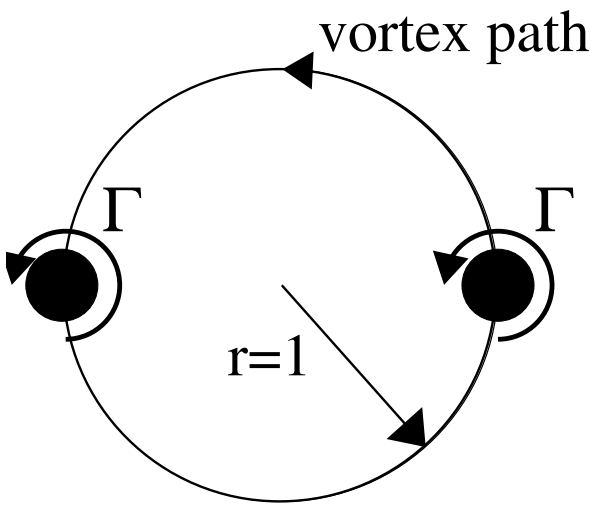
Direct simulation

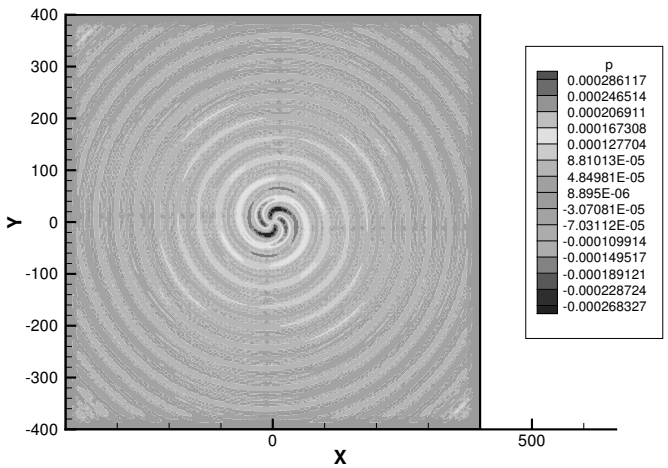
ADER

- arbitrary order in space **and** time
- Finite Volume method (conservative)
- time-integrated fluxes at the cell boundaries: solution expanded in Taylor series in time, time-derivatives replaced by successive use of original PDE, sequence of Generalized Riemann Problems
- very fast on cartesian grids with constant Δx, Δy
- unstructured meshes: ADER-DG faster

Results and Comparisons

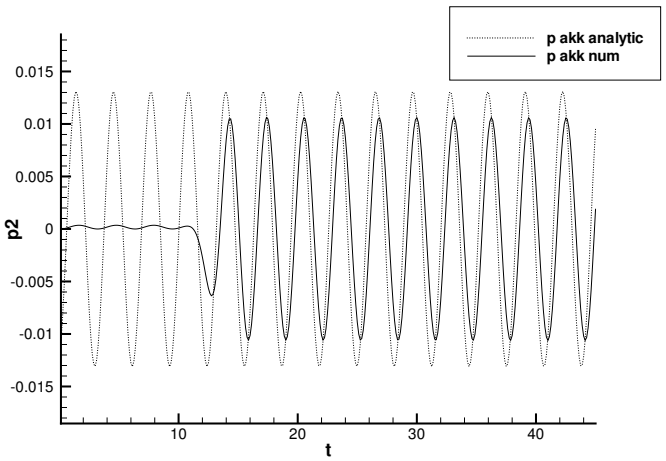
Test case: Pair of co-rotating vorteces at M=0.094657

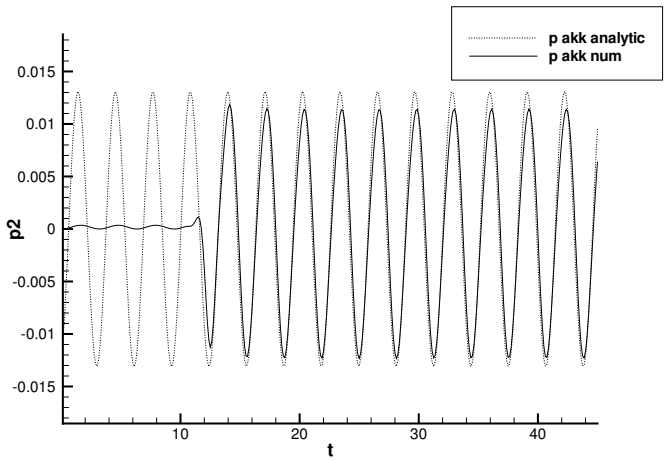




Results and Comparisons

Results of the EIF scheme at point (150,0)
2nd order scheme vs. 4th order scheme

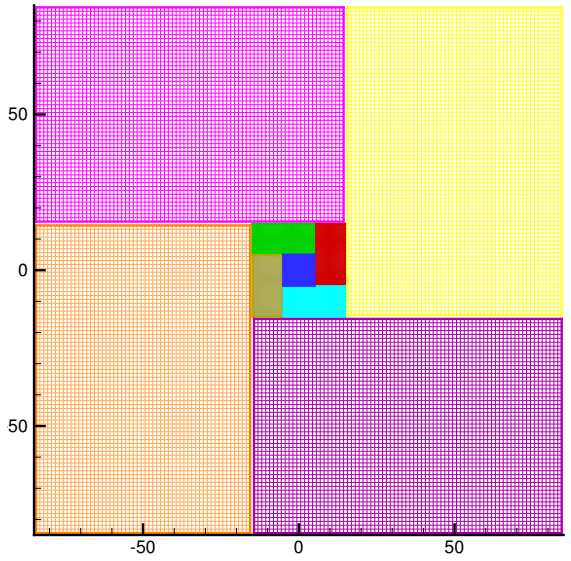


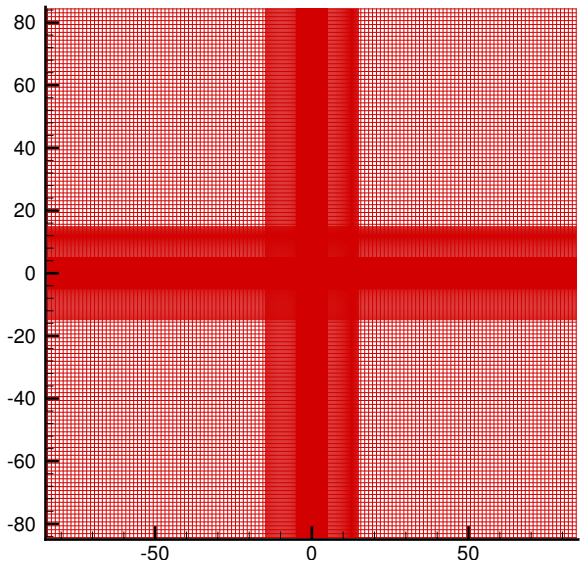


Results and Comparisons

Results of direct simulation: ADER vs. MPV

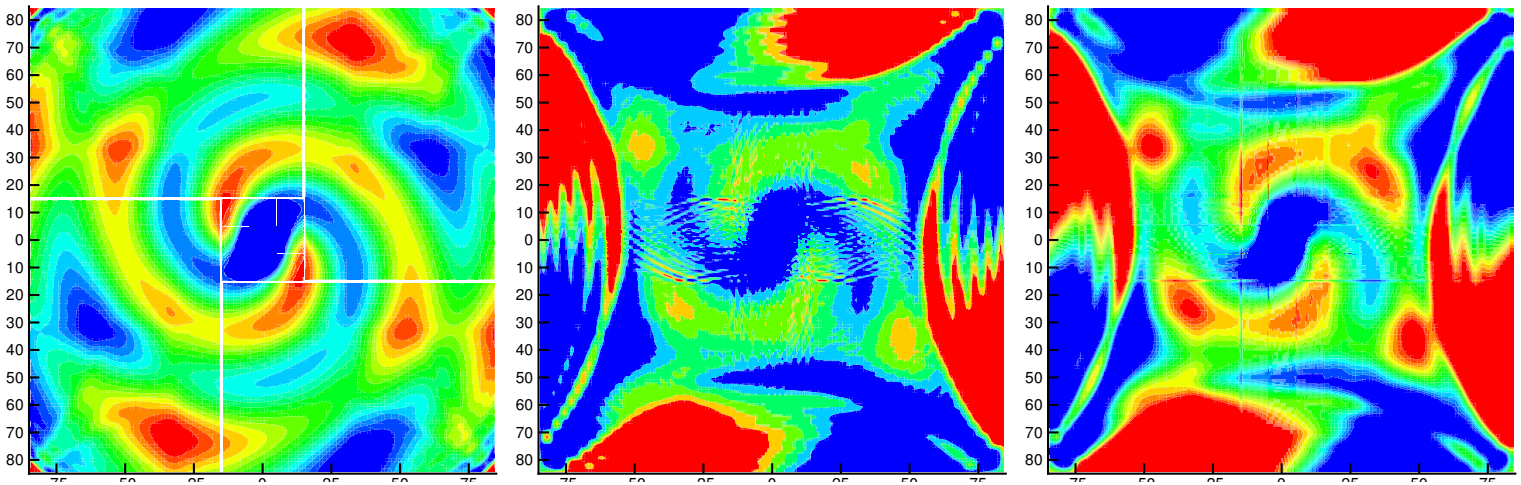
The grids $\Delta x = 0.05, 0.3125, 1.25$





Results and Comparisons

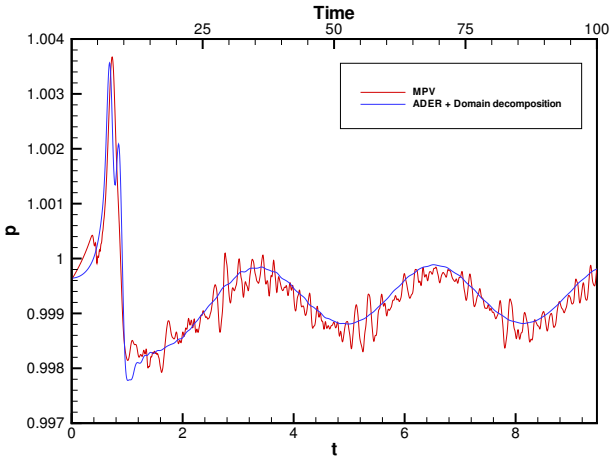
Results of direct simulation: ADER vs. MPV
Results at time=100

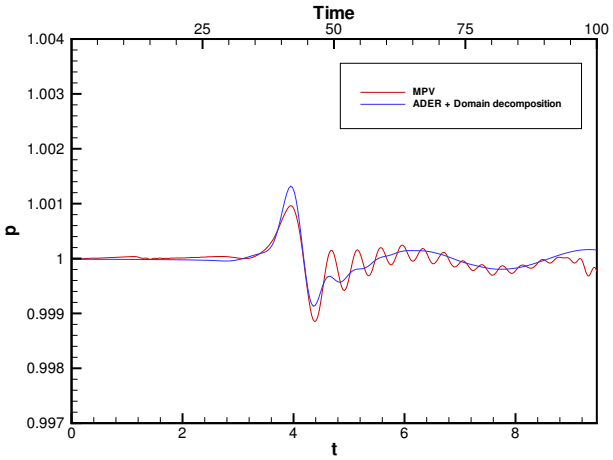


ADER pressure p | MPV pressure p | MPV pressure $p^{(1)}$

Results and Comparisons

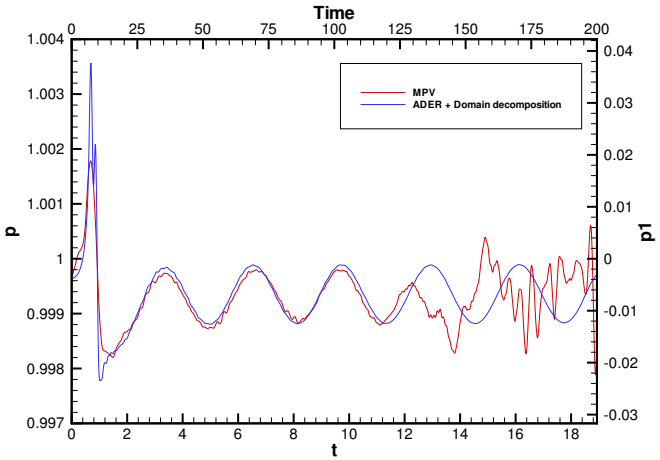
Results of direct simulation: ADER vs. MPV - total pressure
Observation points (x = 10.15625,0) and (x = 50.625,0)

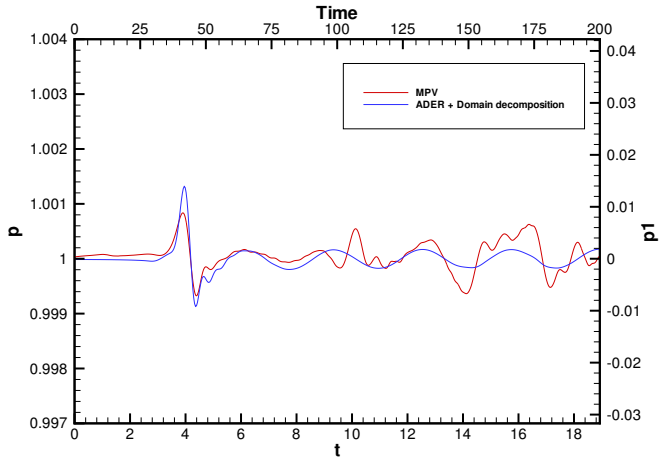




Results and Comparisons

Results of direct simulation: ADER vs. MPV - total pressure/$p^{(1)}$
Observation points (x = 10.15625,0) and (x = 50.625,0)





Results and Comparisons

CPU-time comparison

- EIF-scheme: about 5 min + incompressible solution
- ADER: 589 min
  (using 200x200 + 4 * 32x64 + 4 * 56x112 = 73 280 grid cells, much larger time steps in the outer regions)
- MPV: 737 min
  (using 376x376 = 141 376 grid cells, global small cell time step)

MEMORY resources comparable

Summary

- 3 different schemes shown
- EIF fastest scheme, but no recoupling of acoustics to the flow
- ADER arbitrary high order schemes give the correct solution in the low Mach regime when only on scale has to be resolved in every region. Reduction of computational costs by heterogeneous domain decomposition
- MPV is able to approximate acoustic/flow interaction also. Has to be included in the coupling code
- Parallelization of the coupling is non-trivial