Two models for the simulation of multiphase flows in oil and gas pipelines

TACITE - TINA :

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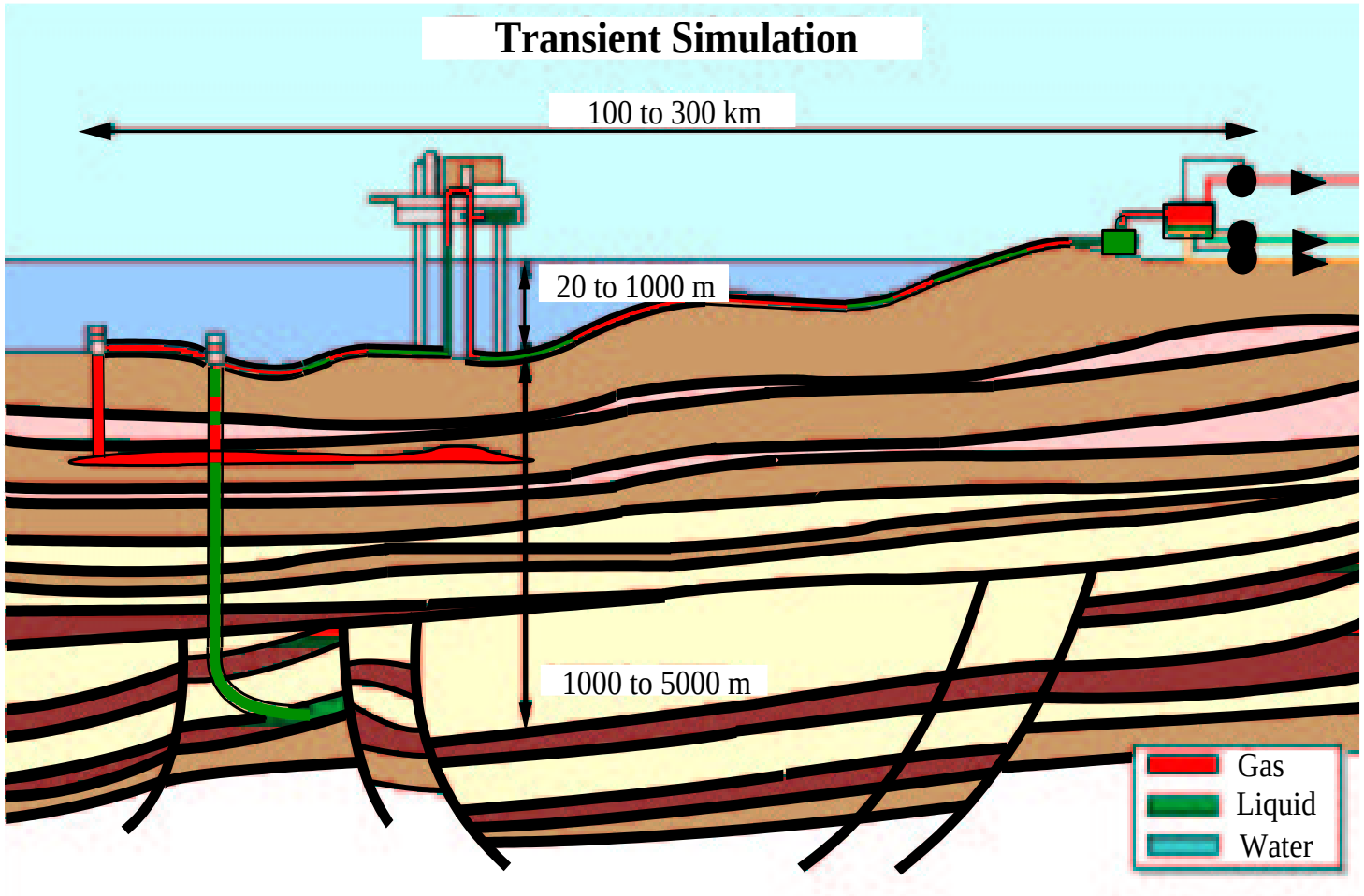
Multiphase production network

From the reservoir to the process installations



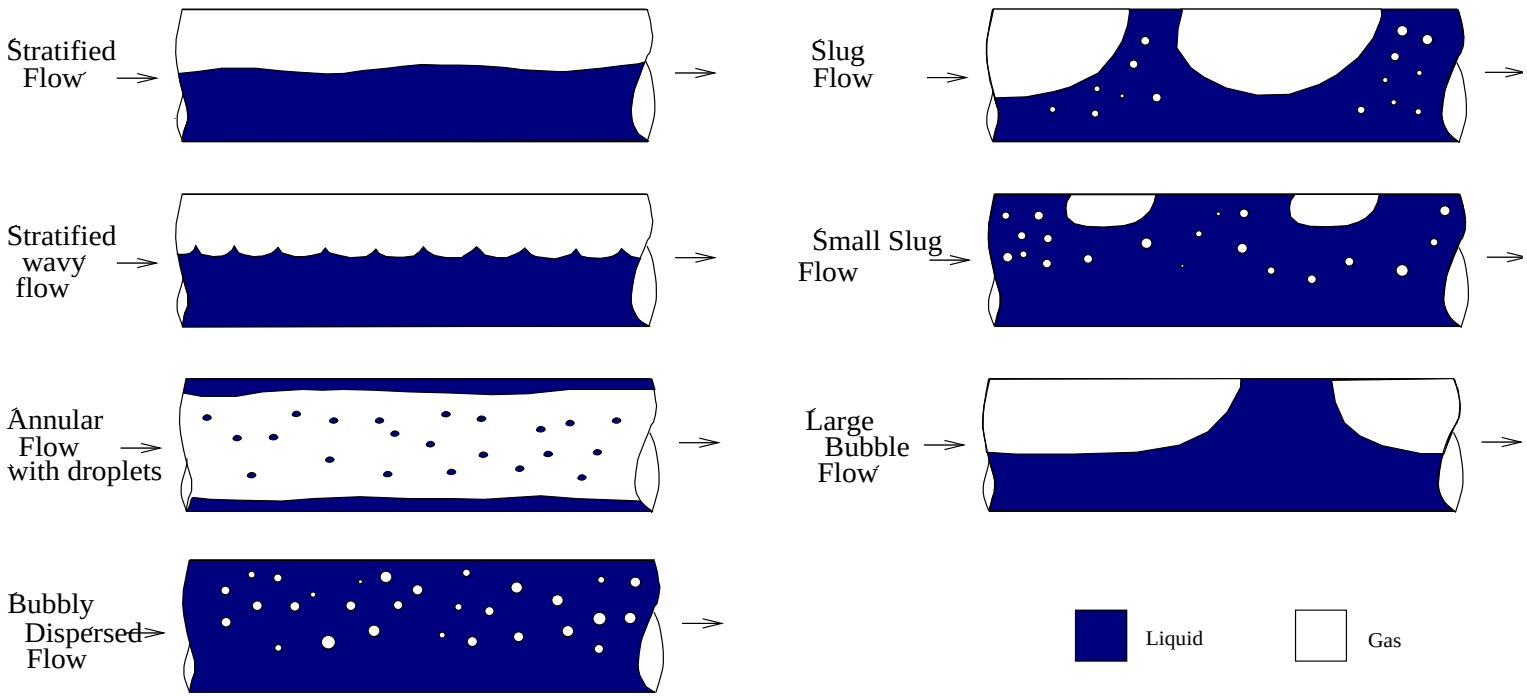
Offshore oilfields

Multiphase production

- Marginal fields near large decreasing reservoirs :
  - small accumulations of hydrocarbons
  - financially worthwhile if low cost developments
  - long subsea tiebacks to existing infrastructures
  - ex : North sea
- Deep water fields
  - great water depths ($\simeq 1000m$): no other choice
  - Gulf of Mexico, West Africa
- Water, oil and gas
  only two-phase flow in the following

Two-phase flow regimes

ex : Horizontal pipe



Depends on fluid velocities, gas fraction ...

Gravity

- Large differences in flow behavior between horizontal, inclined, and vertical pipe flow
- Gas density and gas/liquid disctribution change as a function of pressure (depth)
- Non uniform elevation of the pipeline

  - pipe lying on the seabed

  - riser reaching the platform

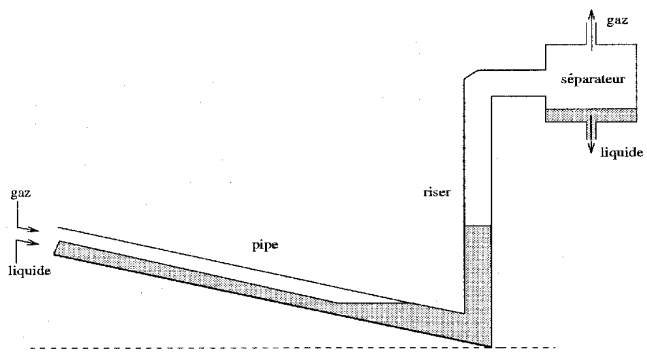
  - can induce large scale instabilities :

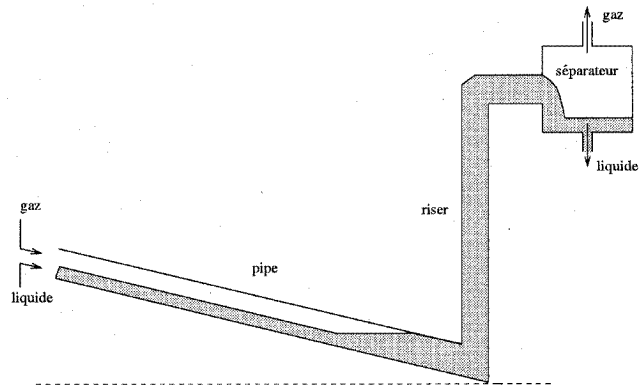
  terrain slugging, severe slugging

Severe-slugging phenomenon

It occurs for low velocity of gas and liquid phases.

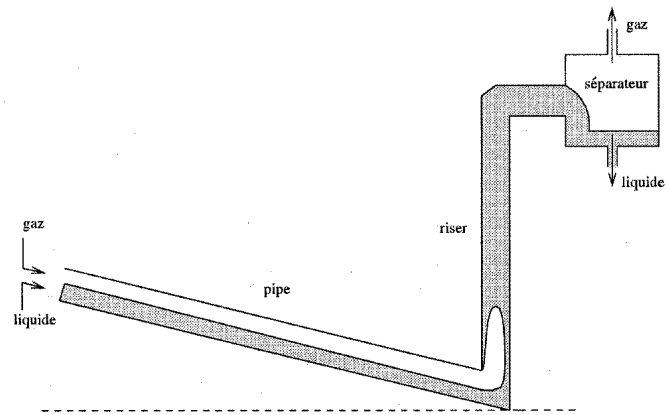
Cyclic phenomenon that can be broken down into 4 parts :



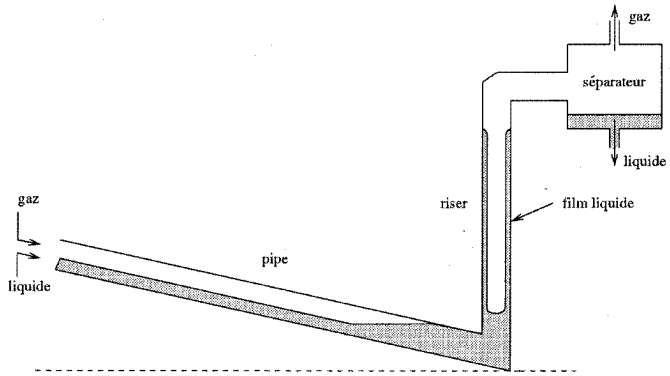


1. liquid accumulates at the low point

2. blockage until the pressure becomes sufficient to lift the liquid column



3. the liquid slug starts to go upward along the riser, the gas begins to flow



4. the gas arrives at the top of the riser and the pressure rapidly decreases causing liquid flow down

- Consequences :

  - Large surges in the liquid and gas production rates

  - Equipment trips and unplanned shutdowns if processing facilities are not adequatly sized

- Terrain slugging : low points in the topography of the pipe

Industrial objectives

Simulation tool for the design of multiphase production networks

- Maximise the production, minimize the risks and the costs

  - it is difficult to avoid severe-slugging (shut-downs ...)

  - over dimension the processing facilities

- Simulate transient phenomena induced either by

  - operating conditions : flowrates variations at the inlet, pressure variations at the outlet

  - the non uniform topography of the pipeline

- Characteristics of the flow

  - Long distance (10 to 100 kms), "low" velocities $\simeq$ m/s

  - Importance of gravity and compressibility: large variations in the gas density, gas fraction

- Accurate estimation of outlet flowrates

Two models for oil and gas pipelines

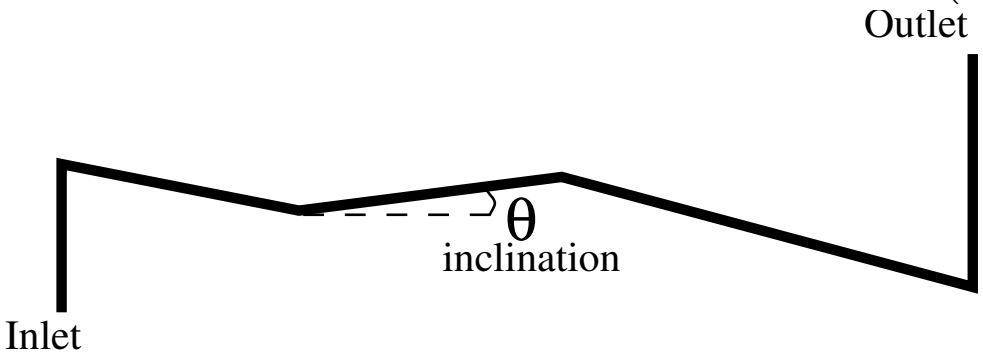
- I - Pipe and fluid representations
- II - Drift Flux model
- III - No pressure wave model

Pipe and Fluid

- Pipe

  - large length, small diameters (10 to 30 cm)

  - 1D model with variable inclination (here : constant diameter)



- Fluid description :

  - Two-phase Immiscible Flow : compressible gas and liquid, no mass transfer between phases

  - Multiphase compositionnal Flow :

    · mass tranfer between phases assuming thermodynamical equilibrium

    · lumping preprocessing procedure to reduce the number of components : 2 to 10 components

Drift flux model

- **Immiscible two-phase flow** :

  - Mass conservation of each phase

$$\frac{\partial}{\partial t}(\rho_\alpha R_\alpha) + \frac{\partial}{\partial x}(\rho_\alpha R_\alpha V_\alpha) = 0 \;\; \alpha = G, L$$

  - Thermo : Phase properties ρ_α.. as a function of (P,T)

- **Compositional two-phase flow** :

  - Mass conservation of each component k

$$\frac{\partial}{\partial t}(\sum_{\alpha=G,L} C_k^\alpha \rho_\alpha R_\alpha) + \frac{\partial}{\partial x}(\sum_{\alpha=G,L} C_k^\alpha \rho_\alpha R_\alpha V_\alpha) = 0 \;\; k = 1 \ldots N$$

R_α volumetric fraction, V_α velocity, C_k^α mass fraction of component k in phase α

  - Thermo :

Phase properties ρ_α, R_α, C_α^k as a function of $(P, T, C_k, k = 1..N)$

Drift flux model

- Momentum conservation equation for the mixture

$$\frac{\partial}{\partial t}(\sum_{\alpha} \rho_\alpha R_\alpha V_\alpha) + \frac{\partial}{\partial x}(\rho_\alpha R_\alpha V_\alpha^2 + P) = T_w - \rho g sin\theta$$

- Algebraic slip equation : $dV = V_G - V_L$

$$\Phi(V_M, x_G, \Gamma(P), dV, x) = 0$$

- Temperature

  T = cste or one mixture energy balance

Characteristics of the DFM Model

- **Non linear hyperbolic system of conservation laws**

$$\frac{\partial}{\partial t}W(x,t) + \frac{\partial}{\partial x}F(x,W(x,t)) = Q(x,W(x,t))$$

  - no algebraic expression of $F(x,W)$ and $Q(x,W)$
  - Jacobian $DF(x,W)$ computed numerically

- **Isothermal gas-liquid flow** : $\lambda_1 < \lambda_2 < \lambda_3$
  - under simplifying assumptions (S. Benzoni) :

$\lambda_1 = v_L - w$, $\lambda_3 = v_L + w$, $\lambda_2 = v_G$

$w \simeq$ sound velocity from about 50 m/s to several 100m/s

  - $\lambda_1 < 0$, $\lambda_3 > 0$: pressure pulses (sonic waves)
  - λ_2 : gas volume fraction waves (fluid transport)

λ_2 positive or negative, $\simeq 1$ m/s

  - $|\lambda_1| >> |\lambda_2|$, $|\lambda_3| >> |\lambda_2|$

Characteristics of the DFM Model

- **Isothermal compositional flow**:
  Hyperbolic system $(N+1) \times (N+1)$:
  Eigenvalues $\lambda_1 < ...\lambda_k... < \lambda_{N+1}$
  - $\lambda_1 < 0$, $\lambda_{N+1} > 0$: "pressure waves"
  - λ_k : composition waves : m/s
  - $|\lambda_1| >> |\lambda_k|$, $|\lambda_{N+1}| >> |\lambda_k|$
- **Low Mach Number Flows**
  Main interest : fluid transport "waves"
  responsible for the main dynamics in the pipeline

- **Boundary Conditions** (Two-phase immiscible flow)

  - Inlet Boundary $x = 0$: flow rates for each phase or for each component

$$\begin{aligned} \rho_G R_G V_G(0,t) &= Q_G(t)/Sect \\ \rho_L R_L V_L(0,t) &= Q_L(t)/Sect \end{aligned}$$

  - Outlet boundary $x = L$: pressure, liquid can not go back into the pipe

$$\begin{aligned} P(L,t) &= P_{outlet}(t) \\ R_L(L,t) &= 0 \text{ si } V_L < 0 \end{aligned}$$

- **Initial condition**
  Steady flow with BC at $t = 0$ $Q_G(0)$, $Q_L(0)$, $P_{outlet}(0)$

- **Transient Flow** induced by BC variations, instabilities (severe-slugging)

Numerical scheme

- **Cell centered Finite Volume scheme**

  cell $]x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}[$, unknown $W_i \simeq W(x_i)$

$$\Delta x_i \frac{d}{dt} W_i + H_{i+\frac{1}{2}} - H_{i-\frac{1}{2}} = \Delta x_i Q_i$$

- **Numerical flux**

  - difficult to use algebraically constructed approximate Riemann solver like Roe scheme

  - Rusanov : uniform dissipation on all the waves, too dissipative on void fraction waves

  - Idea : introduce enough numerical dissipation but preserves the different orders of magnitude

$$H(U,V) = \frac{1}{2}(F(U) + F(V)) + \frac{1}{2} D(U,V)(U - V)$$

$$\boldsymbol{D(U,V) = |DF(U)| + |DF(V)|}$$

Time discretisation

- **Explicit scheme**

  - CFL based on sonic waves

  - time steps too small / time of fluid transport

- **Linearly implicit scheme**

  Linear implicitation of the source term and of the flux

$$\begin{aligned} H(U^{n+1}, V^{n+1}) \quad \simeq \quad & H(U^n, V^n) + approx\left(\frac{\partial}{\partial U} H(U^n, V^n)\right)(U^{n+1} - U^n) \\ & + approx\left(\frac{\partial}{\partial V} H(U^n, V^n)\right)(V^{n+1} - V^n) \end{aligned}$$

$$\frac{\partial}{\partial U} H(U, V) \quad \simeq \quad \frac{1}{2}(DF(U) + D(U, V))$$

Does not account for the different wave velocities

Semi-implicit scheme

- explicit on the slow waves and linearly implicit on the fast waves
- CFL based on fluid transport waves, time steps in agreement with the main phenomena
- Splitting of the flux into a "slow waves" part and a "fast waves" one

  - cf. G. Fernandez PHD thesis for the Euler equations

  - The small eigenvalues are cancelled in the flux derivatives, similar to the "diagonal" approach of Fernandez
  ex : $DF(U)$ replaced by $\overline{DF}(U) = T\bar{\Lambda}T^{-1}$,
  $\bar{\Lambda}_k = \lambda_k(U)$ for $k = 1, N+1$, $\bar{\Lambda}_k = 0$ for $k = 2, ..., N$
- CFL :
  $\Delta t < min(CFL_v \frac{\Delta x}{2max\lambda_k, k=2,...,N+1}, CFL_p \frac{\Delta x}{2max\lambda_k, k=1,N+1})$,
  $CFL_v = 0.8$, $CFL_p = 20$

Second order scheme

Wall friction and gravitational terms induce large $\frac{\partial}{\partial x}(P)$: second order scheme necessary

- **MUSCL approach** :
  - linear reconstruction on "physical" variables : P, c_i, V_M
  - minmod limiter
- **RK2 type scheme** to enable CFL 0.8 on slow waves
  - second order time scheme on the slow waves
  - first order on the fast waves

Boundary conditions (immiscible flow)

- **Inlet BC** imposed on the numerical fluxes

  - Fictitious cell : W_0

$$\begin{cases} H_G(W_0, W_1) & = & Q_G(t) \\ H_L(W_0, W_1) & = & Q_L(t) \\ L_1^t W_0 & = & L_1^t W_1 \end{cases}$$

  - Non linear system, sometimes very difficult to solve

- **Outlet BC**

  - Pressure BC : fictitious $I+1$ cell, W_{I+1}

$$\begin{cases} P(W_{I+1}) & = & P_{outlet}(t) \\ L_2^t W_{I+1} & = & L_2^t W_I \\ L_3^t W_{I+1} & = & L_3^t W_I \end{cases}$$

- Change in the flow direction at the oulet : $v_L < 0$

  - failure of any numerical treatment based on the sign of the eigenvalues or on a adequat fictitious state ($R_{I+1} = 0$)

  - Empirical and simple approach :
  if $H_L(W_I, W_{I+1}) < 0$ then $(H_L)_{I+\frac{1}{2}} = 0$

  - approach justified theoritically on a scalar model (T. Gallouët) : leads to a monotone continuous numerical flux and satisfies the BC.

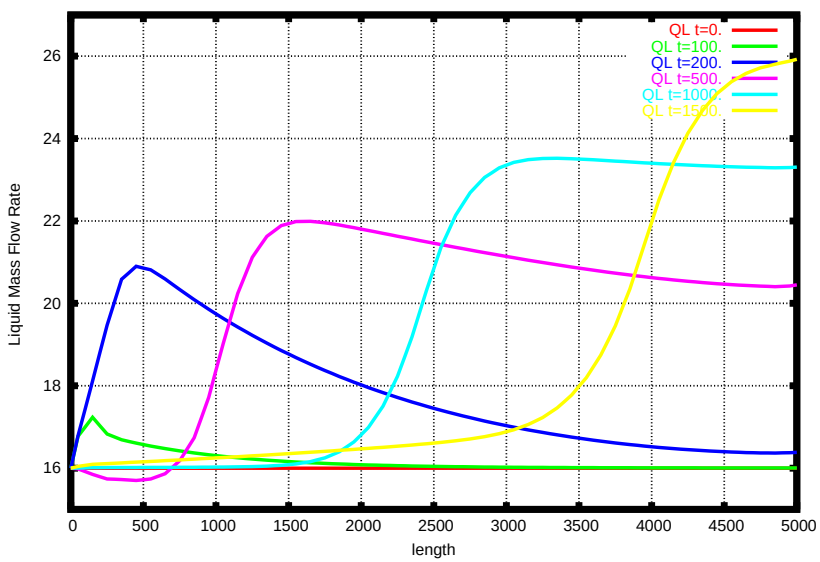
- Time discretization

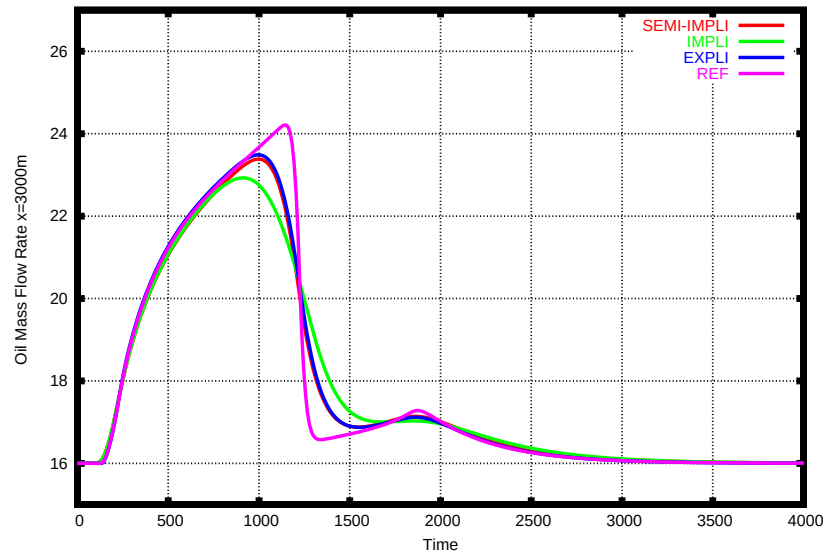
  - BC solved during the "explicit" step of the scheme

  - W_0,W_{I+1} kept constant during the semi-implicit step
  Loss of accuracy as H^{n+1} does not satisfy the BC
  Reasonnable results

Application : Inlet Gas Flowrate Increase

Horizontal pipeline of $5000m$ length, immiscible two-phase flow





Oil Mass Flowrate at different times

Oil Flowrate /time at x = 3000m

Explicit, Implicit and Semi-implicit

Nb Step Expli = 50 * nbStep Semi-Impli

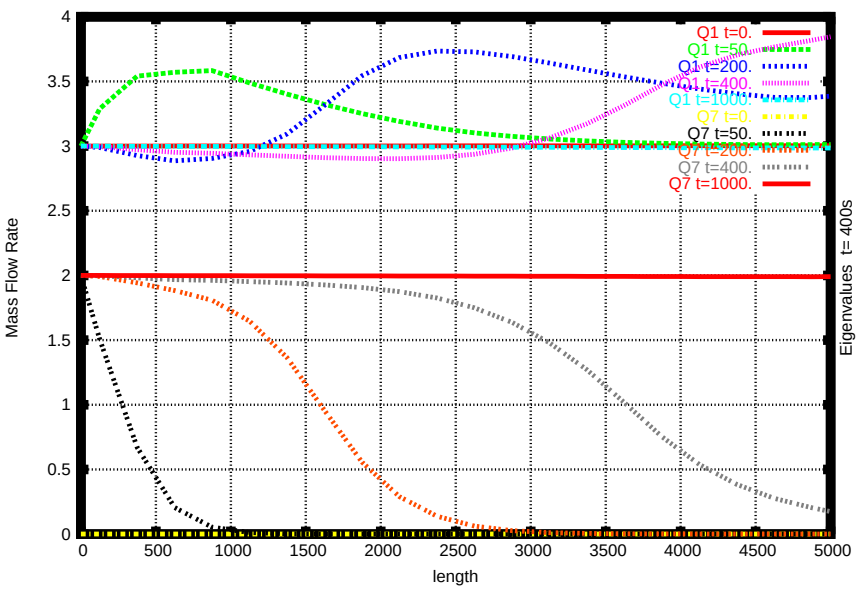
CPU Expli = 45 * CPU Semi-Impli

Application : 7 components

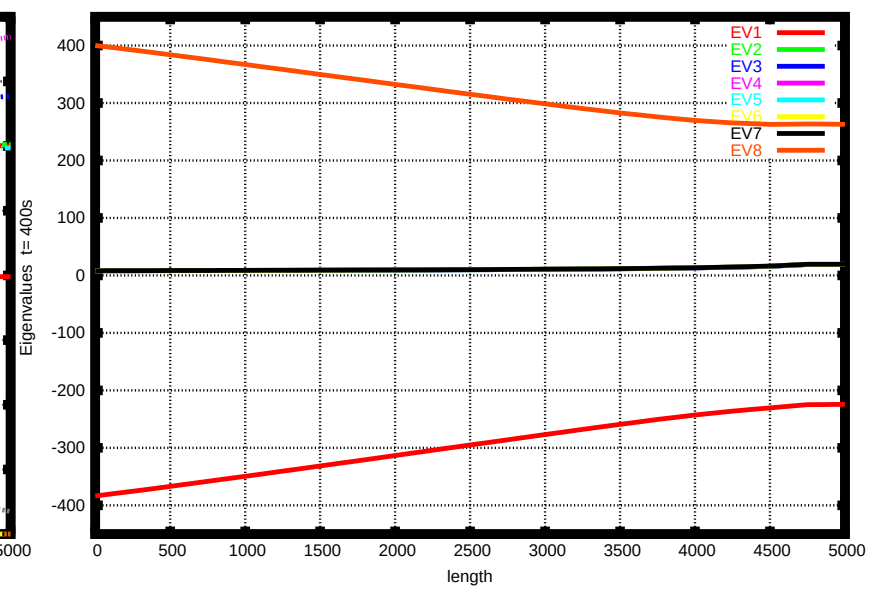
Ascending pipeline of $5000m$ length and $0.146m$ diameter.

Boundary conditions : $P_{outlet}(t) = 10\,bar$

At the inlet: Q1..Q5 constant, Q6,Q7 increased from 0 to 2 in 50 s

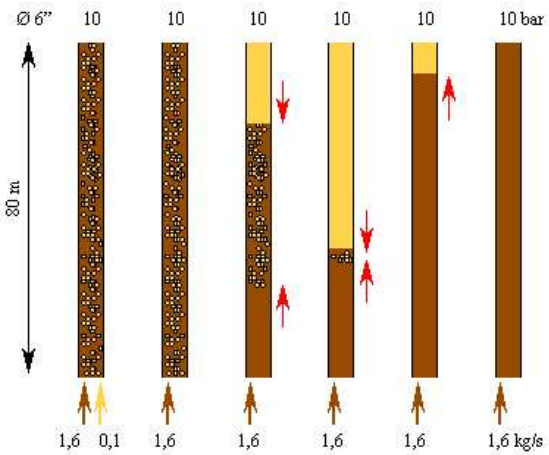


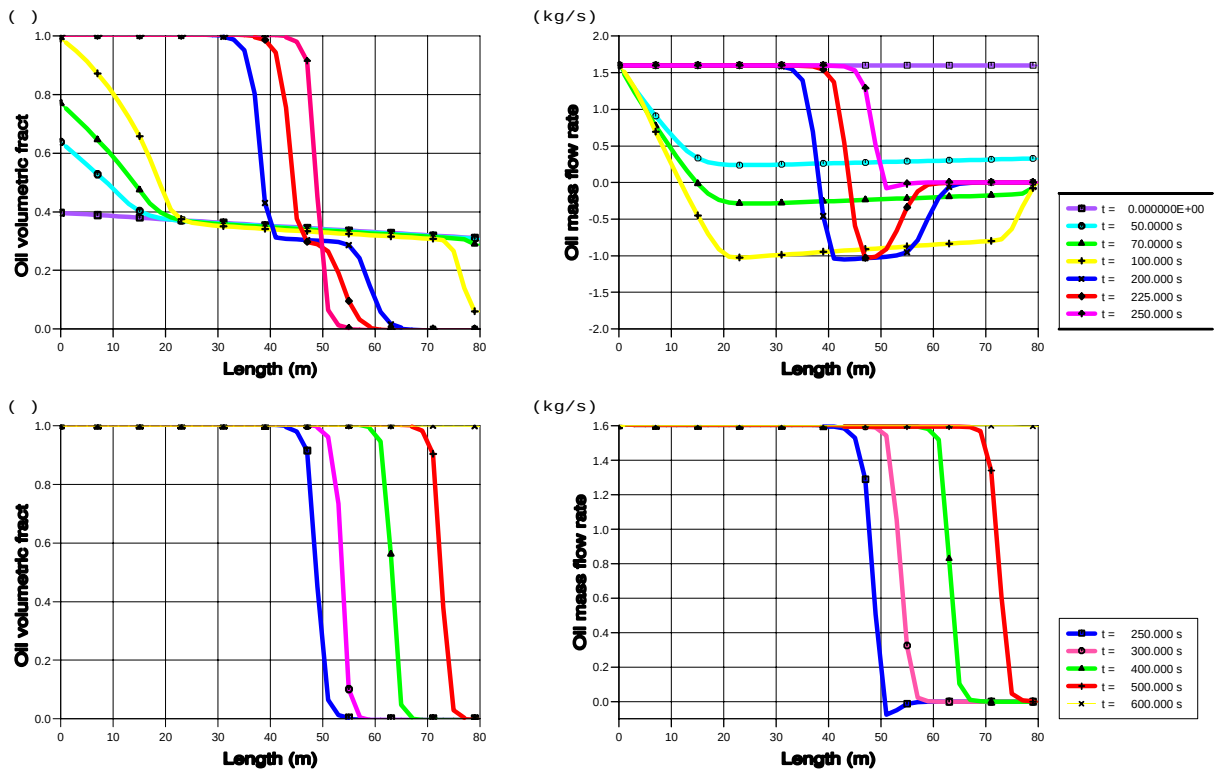
Q_1 and Q_7 at different times



Eigenvalues at time t = 400s

Vertical pipe : liquid fraction and flowrate





"No Pressure Wave" Model

Two phase immiscible flow

- Modify the DFM model to account for the low Mach Number flow
- Analytical study for a simplified slip law (H. Viviand)

  - $\epsilon = \frac{U}{a_G}$, $\frac{U}{a_L} = \frac{\epsilon}{K}$

  a_α sound velocity in phase α, U characteristic velocity

  - asymptotic expansion of the solution with respect to ϵ
- **Simplified momentum conservation without momentum time derivative and flux terms**

NPW

- Mass and simplified momentum conservation

$$\frac{\partial}{\partial t}(\rho_\alpha R_\alpha) + \frac{\partial}{\partial x}(\rho_\alpha R_\alpha V_\alpha) = 0 \quad \alpha = G, L$$

$$\frac{\partial}{\partial x}(P) = T_w - \rho g sin\theta$$

  - thermo, slip laws

- Properties

$$B\frac{\partial}{\partial t}W + A\frac{\partial}{\partial x}W = Q$$

B is singular. Find (a, b) s.t. $det(aB - bA) = 0$:

  - $\lambda = \frac{a}{b} \simeq u$: one "fluid wave" velocity
  - $b = 0$: one "double" infinite wave velocity

Sonic waves approximated by infinite velocity waves

Mixed parabolic/hyperbolic system of PDE

Compositional NPW Model

- **Mass conservation of each component i**

$$\frac{\partial}{\partial t}\Big(\sum_{\alpha=G,L} C_i^\alpha \rho_\alpha R_\alpha\Big) + \frac{\partial}{\partial x}\Big(\sum_{\alpha=G,L} C_i^\alpha \rho_\alpha R_\alpha V_\alpha\Big) = 0 \;\; i = 1 \ldots N$$

- Thermo :

  Phase properties ρ_α, R_α, C_α^i as a function of (P, T, C_i)

- **Simplified momentum conservation**

$$\frac{\partial}{\partial x}(P) = T_w - \rho g sin\theta$$

- Algebraic slip equation : $dV = V_G - V_L$

$$\Phi(V_M, x_G, \Gamma(P), dV, x) = 0$$

- Same initial and Boundary conditions as DFM

Numerical scheme

- VF scheme on staggered mesh

  - implicit centered scheme for the parabolic part (sonic waves)

  - explicit upwind scheme for the hyperbolic part (slow waves)

  - CFL based on phase velocities (0.4)

- Mass conservation + thermo : cell $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$

$$\frac{\Delta x_i}{\Delta t}\left((C_k^\alpha \rho_\alpha R_\alpha)_i^{n+1} - (C_k^\alpha \rho_\alpha R_\alpha)_i^n\right) + H_{\alpha\, i+\frac{1}{2}} - H_{\alpha\, i-\frac{1}{2}} = 0$$

$$H_{\alpha\, i+\frac{1}{2}} = \begin{cases} (C_k^\alpha \rho_\alpha R_\alpha)_i^n V_{\alpha\, i+\frac{1}{2}}^{n+1} & \text{if} \quad (V_\alpha)_{i+\frac{1}{2}} > 0 \\ (C_k^\alpha \rho_\alpha R_\alpha)_{i+1}^n V_{\alpha\, i+\frac{1}{2}} & \text{otherwise} \end{cases}$$

- Momentum conservation + slip law : cell $[x_i, x_{i+1}]$

$$P_{i+1}^{n+1} - P_i^{n+1} = \Delta x_i Q_{i+\frac{1}{2}}^{n+1} \qquad \text{i=1..I-1}$$

Boundary conditions

Two-phase immiscible flow

- Inlet Boundary : given mass flowrates

$$H_{\alpha\,\frac{1}{2}} = Q_\alpha$$

- Outlet Boundary

  - Mass Fluxes :

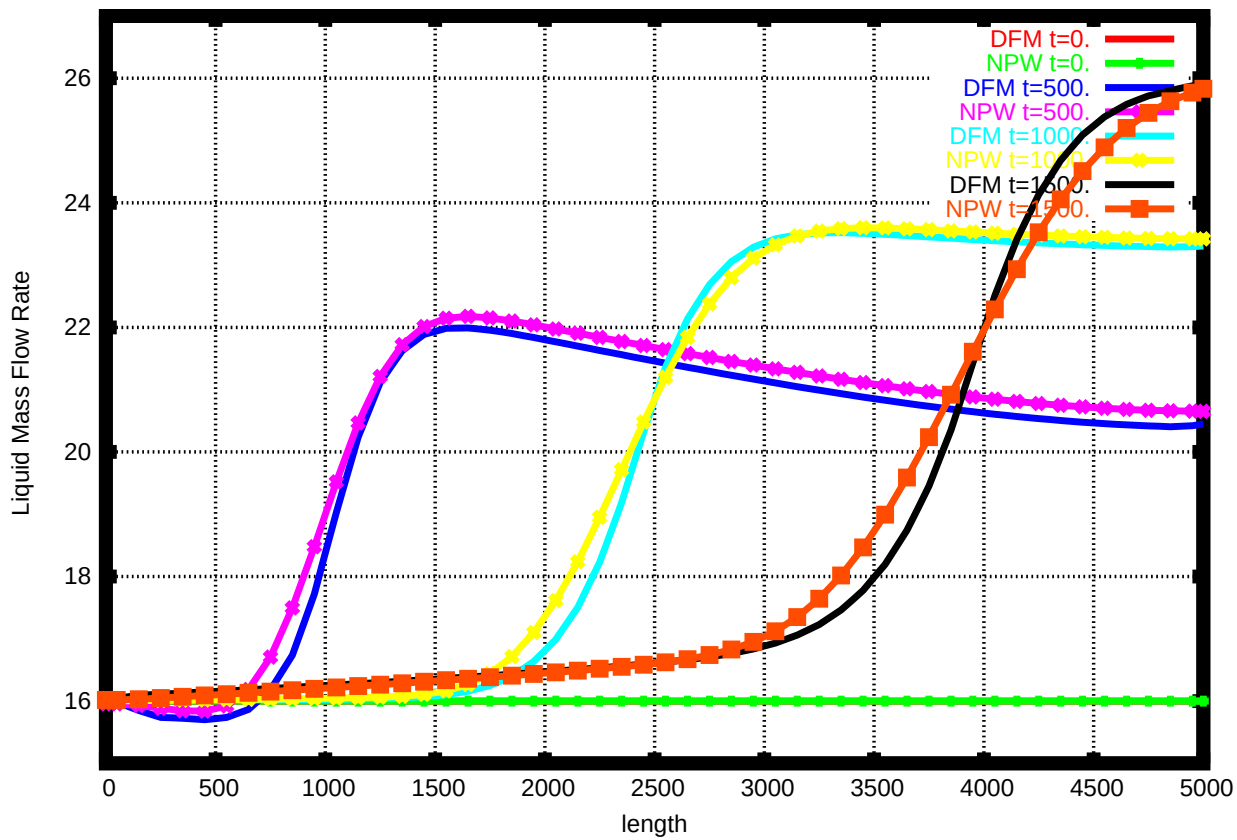
$$H_{L\,I+\frac{1}{2}} = \begin{cases} \rho_{L\,I}(R_L)_I V_{L\,I+\frac{1}{2}} & \text{if} \quad (V_L)_{I+\frac{1}{2}} > 0 \\ 0 & \text{otherwise} \end{cases}$$

  - Pressure

$$P_{outlet} - P_I = \frac{\Delta x_I}{2} Q_{I+\frac{1}{2}}$$

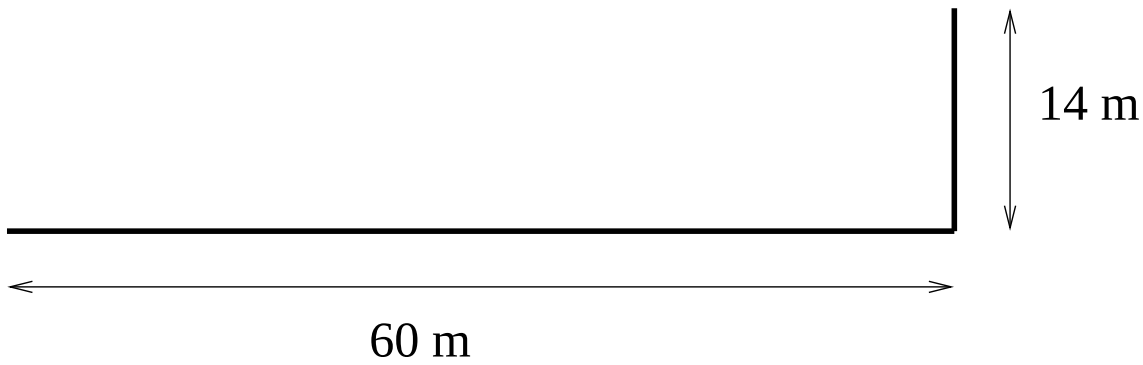
Application : Inlet Gas Flowrate Increase

Horizontal pipeline of $5000m$ length and $0.146m$ diameter.



Oil Mass Flowrate in the pipe : NPW/DFM comparison

Severe Slugging



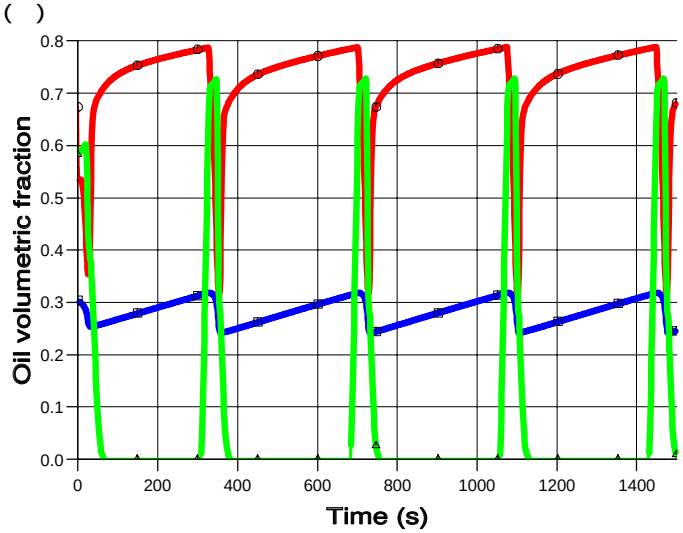
- 60m long horizontal pipe, 14m long riser
- D=5cm
- Constant inlet mass flowrates and outlet pressure
  - $Q_G = 1,9610^{-4} kg/s$, $Q_L = 2,8510^{-4} kg/s$
  - $P_{outlet} = 1bar$

Severe Slugging : R_L

NPW

$R_L(t)$

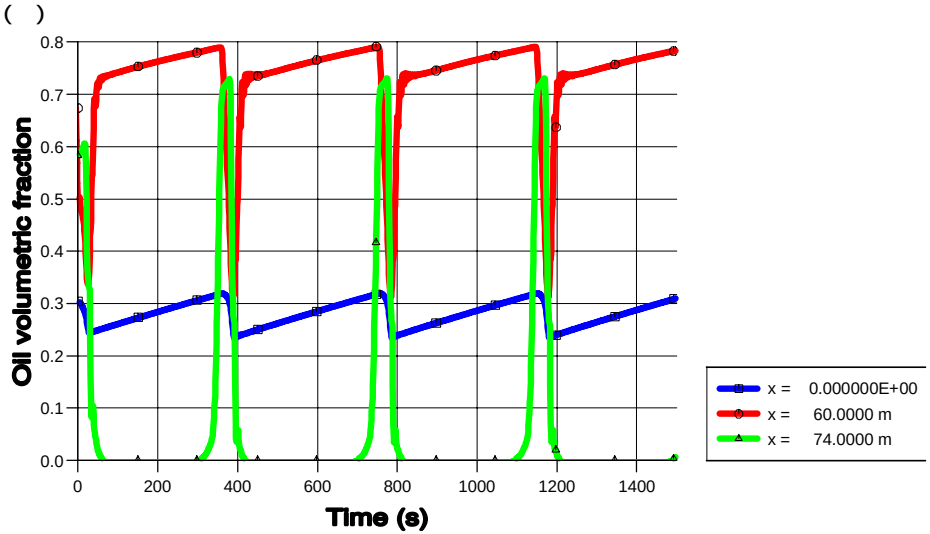
$x = 0, 60, 74m$



DFM

$R_L(t)$ pour

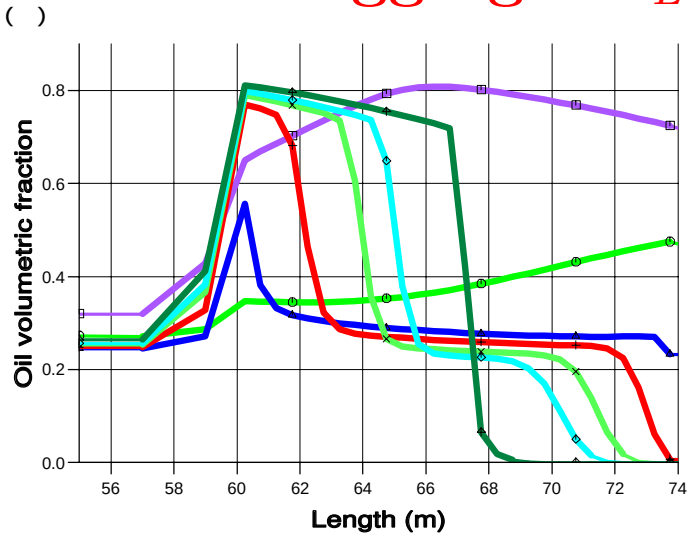
$x = 0, 60, 74m$



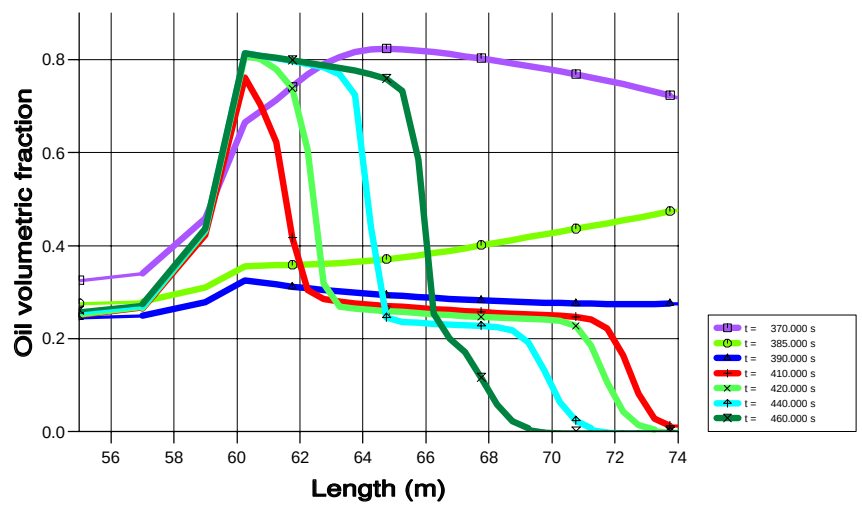
DFM CPU $\simeq$ 9 NPW CPU (single-phase flow)

Severe Slugging : R_L in the riser

NPW



DFM



Conclusion

- Two models and schemes adapted to low Mach Number compositional two-phase flow

  - DFM : scheme explicit for the "slow" wave and "implicit" for the fast waves

  - NPW : accoustic waves approximated by infinite valocity waves, semi-implicit scheme
  Larger time steps for NPW, less CPU time

- Extension to

  - more complex fluid flow :

    · Three-phase flow : water phase

    · Four-phase flow : "solid" phase (hydrate, wax) for cold deep sea production

  - Complex networks

The Girassol (West Africa) production network