Linear Stability of Bursting Solutions in non-Boussinesq natural convection

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This study focuses on natural convection flows of air in a two-dimensional differentially heated cavity under large temperature gradients with the fluid viscosity a nonlinear function of temperature. The transition to time-dependence appears to be subcritical and numerical simulations in the vicinity of transition exhibit intriguing time-dependent solutions. These solutions are intermittent with periodic bursts separating quasi-steady states. To understand these observations we perform a linear stability analysis of steady and time-dependent base solutions.

The model equations are the low Mach approximation equations obtained by Paolucci [1] allowing for filtering of sound waves. We integrate these using a finite volume method with fractional time stepping derived from the projection method used to compute incompressible flows. To understand the intermittent bursting dynamics, we have developed a pseudo-linearization method which allows us to compute linearized solutions using only the original nonlinear time stepping code. The pseudo-linearization is combined with Arnoldi method to allow us to compute efficiently leading eigenmodes of both steady and unsteady states.

The computations are performed using the configuration previously described [2], corresponding to a relative horizontal temperature difference (with respect to the mean temperature) of 120%, in a cavity of vertical aspect ratio 4. The Prandtl number is constant equal to 0.71, the Rayleigh number is $Ra = 2.3 \times 10^5$, and the viscosity coefficient is given by Sutherland's law. Horizontal boundaries are adiabatic. The computational mesh is 128×256.

Figure 1 shows a time series with two bursts of the intermittent dynamics found near onset. Figure 2 shows a snapshot of the fluctuation of temperature with respect to the mean temperature field within a burst. Our linear computations of flows at points P1 - P5 along the quiescent part of the solution reveal that the real part of the leading eigenvalue varies from negative (P1 and P2) to positive (P3, P4, and P5). The eigenvalues are complex with imaginary part corresponding closely with the burst frequency. Figure 3 shows the temperature component (real and imaginary parts) of the leading eigenmode computed at time labeled P3. This locally unstable eigenmode of the slow flow has the form of the fast bursting phase of the dynamics. This can be understood in terms of a slow passage through a Hopf bifurcation [3].

In addition, our method is used to determine leading eigenvalues of steady flows prior to onset and from these the critical Ra for transition $Ra = 2.43 \times 10^5$. This verifies that the transition is subcritical.

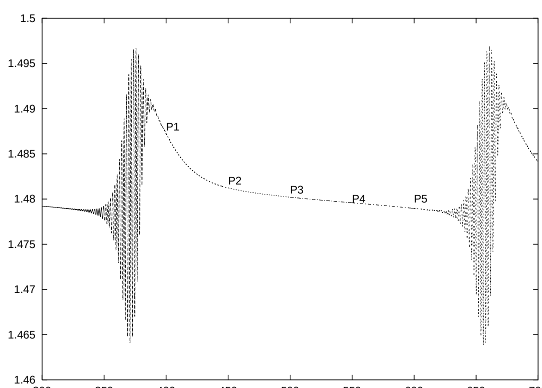

Figure 1

Figure 2

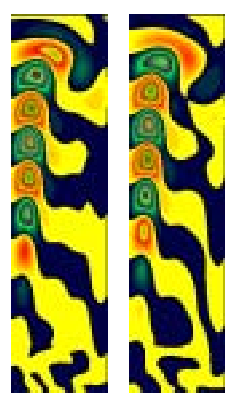

Figure 3

References

[1] Paolucci,*Sandia National lab. Report SAND 82-8257*, 1982 (non published)
[2] C. Weisman, L. Calsyn, C. Dubois, P. Le Quéré, *C.R.Acad.Sci.*, 329, série IIb, 343-350, 2001.
[3] C. Weisman, D. Barkley, P. Le Quéré, *2002 APS DFD 55th annual meeting, Nov., 2002.*