A Mach-uniform pressure correction algorithm

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We present a collocated finite volume pressure correction method with excellent efficiency over the whole Mach number range, including the low Mach number regime. The spatial discretization is AUSM+ with the interface mass flux preconditioned for good low Mach number accuracy and with added pressure-velocity coupling for low Mach number stability.

The low Mach number asymptotics of the Navier-Stokes equations reveal that pressure is composed of three components: a zeroth-order thermodynamic pressure p_0, a first-order acoustic pressure p_1, and a second-order pressure p_2 which is related to velocity through the momentum equations. We consider here solutions for larger time steps, so that the acoustic pressure p_1 is filtered out. The energy equation forms a constraint on the divergence of the velocity, where the latter is affected by the heat conduction and heat release and the time change of the zeroth-order pressure. It is important to note that the energy equation forces the velocity divergence, and not the continuity equation, as is often believed. In our pressure correction method we do not use the low Mach number equations, but they form the basis of the construction of the Mach-uniform algorithm.

First we consider the Euler equations, thus neglecting heat transfer and friction. The low Mach number analysis shows that the divergence of the momentum equation, together with the constraint forced by the energy equation, leads to a Poisson equation for the second-order pressure p_2, which we use as a pressure correction equation. When p_2 is determined in this way, acoustic waves are removed as it happens in a pressure correction method for the incompressible flow equations. An acoustic CFL-limit on the time step is therefore avoided, leading to a Mach-uniform convergence rate.

For non-adiabatic flow, conduction and heat input terms have to be added to the energy equation. Consequently, temperature appears into the energy equation, though so far we considered the latter equation as pressure determining. A simple method consists in putting the heat transfer terms into the right hand side of the energy equation as source terms, thus treating them explicitly. This is consistent with the low Mach number analysis where the heat terms affect the divergence constraint. The energy equation is then still used to determine pressure, without any acoustic time step limit.

However, due to the explicit treatment of the temperature diffusion terms, the maximum allowable time step is determined by the von Neumann number. For low speed flow, where conduction is more important than convection, this diffusive time step restriction can lead to poor convergence rates. In such a case, the temperature terms in the energy equation have to be treated implicitly. The energy equation then becomes an equation for both pressure and temperature. Therefore, corrections for both variables are introduced. A second equation containing temperature and pressure corrections is derived from the continuity equation. Both the energy and continuity equations are then solved in a coupled manner. The momentum equation is still treated in a segregated way. Doing so, we are able to remove the diffusive time step limit and again reach Mach uniform efficiency.

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