Computation of Low Mach Number Flows with a Generalized Gibbs Relation

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Classic studies in fluid mechanics are generally divided into two categories, incompressible fluids and perfect gases. Although these idealizations are useful and have important practical applications, limiting our analysis to these two fluids loses much of the richness of the complete subject and bypasses some fascinating fluid mechanics problems. A pertinent aspect of many non-perfect gas flows is that the Mach numbers are small, so that the mathematical and numerical issues are dominant concerns in their solution. The thermodynamics of a complete set of fluids, however, can shed much light on the issues involved in the low Mach number perfect gas limit, while also providing an avenue for moving to multi-phase fluids.

In the present paper we present an approach that stresses issues involved when the thermodynamic behavior is such that the Maxwell relations of the fluid approach limiting values. Although we use a 'time-marching' solution method, we separate the formulation of the equations from the iterative method. In particular we formulate the equations as the space-time divergence of a tensor, $\nabla \cdot F = 0$, that includes both spatial and temporal derivatives. This formulation includes both compressible and incompressible fluids without distinction. As an explicit step in formulating the equations, we separate the partial differential equations that describe the conservation laws from the algebraic equations that describe the fluid behavior. This gives a perspective that is often lost when the two systems are combined.

Numerical solutions are obtained by imbedding the space-time equation in a pseudo-time term that is formulated in terms of primitive variables. The algebraic equations are then coupled with the solution procedure using artificial fluid properties for the pseudo time term and physical fluid properties for the space-time terms. Artificial fluid properties are defined in such a manner as to maintain efficiency during the iterative process for either steady or transient solutions. Representative results are shown for incompressible flows, perfect gases, supercritical fluids and multi-phase flows.