A numerical accuracy study for level set formulations

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The motion of interfaces is an important aspect of low Mach flow investigations. The accuracy of the numerical representation of this motion can play a central role in the global accuracy. Now that many schemes allow second or higher-order accuracy for low Mach flows without interfaces, it is necessary to investigate higher order numerical accuracy for interface motion.
Many recent progress has been realized by capturing discontinuous interfaces. In that case, the global accuracy is limited by a necessary compromise between the discontinuity diffusion and tha ability of the numerical scheme to move accurately a quasi-discontinuity.

At the contrary, smooth data are easier handled by numerical schemes. Lagrangian methods do not capture discontinuities, but data smoothness can be lost in many occasion and in particular when changes in interface topology happen.
The Eulerian Level set approach ([1,2]) also proposes a smooth formulation, since the dependant variable representing the interface is a smooth field that can be advected with higher-order accuracy. The accuracy of interface motion itself can be analysed in terms of characteristic function convergence ([3]). It appears that both level set function smoothness and level set function gradient norm determine the error magnitude.

The ideal level set method should then satisfy the following criteria:
- second-order accuracy in space and time,
- scheme should be monotone, or at least safe of oscillations,
- mass of each phase should be conserved,
- gradient of level set function should be maintained close to unity.

The purpose of the proposed communication is the investigation of several novel level set schemes applying to unstructured tetrahedrizations and their evaluation with respect to the previous criteria.
Comparison will be performed on a set of free interface flws with sloshing and falling liquid masses.

References.

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