Augmented projection methods for incompressible and dilatable flows

M. Jobelin [1], C. Lapuerta[1], Ph. Angot[2], J.-C. Latché[1], B. Piar[1]

[1] Institut de Radioprotection et de Sûreté Nucléaire (IRSN)
Direction de Prévention des Accidents Majeurs (DPAM)
BP 3 - 13115 Saint-Paul Lez Durance CEDEX

[2] Université de la Méditerranée
LATP - IMT - 38 rue F. Curie - F13451 Marseille CEDEX 20

We present in this paper a new numerical scheme for the solution of the mass and momentum balance equations governing dilatable flows. We proceed in two steps.

First, we build a Galerkin incremental projection scheme for the problem at hand, developped within the framework set up in [3]. It involves three independent variables, namely an intermediate velocity, the pressure and the end-of-step mass flow-rate: the intermediate velocity is searched in a discrete space V_h included in $(H^1(\Omega))^d$ and the pressure in $M_h \subset L^2(\Omega)$, whereas the mass flow-rate is searched in $X_h = V_h + \nabla M_h$, which *in fine* allows to recover the usual Poisson problem for the computation of the pressure.

In a second step, we derive an augmented projection scheme.

The augmented projection method is a novel scheme belonging to the class of the projection methods introduced by Témam and Chorin [7, 2], which has been recently proposed for the solution of incompressible Navier-Stokes equation. Since the publication of this work is only underway, we review here its main results. The basic idea originates, to our knowledge, from a paper of Caltagirone & Breil [1] and consists in adding in the first step of the usual incremental projection method (the momentum balance equation) the same penalisation term as in Augmented Lagrangian methods. Using for the projection step a more standard algorithm than in this original work, we obtain a method which can be seen as a generalization of the rotational pressure correction scheme [6, 5], thus enjoying the same properties (by comparison to the standard projection scheme, suppression of pressure boundary layers and much better behaviour in problems with Neumann boundary conditions [5, 4]) with an enhanced stability. Moreover, for large values of the penalisation parameter, the method is shown to behave as the usual coupled Backward Euler algorithm (scheme obtained when solving in a coupled way the momentum balance equations and the divergence constraint), which allows significant gains in accuracy. The price to pay is then to solve an ill-conditionned algebraic problem in the first step of the algorithm.

The error analysis of the augmented method is performed here on a model linear problem, namely the unsteady Stokes problem for a fluid with a time and space varying density. We restrict ourselves to density fields such that the corresponding steady Stokes problem is coercive. The conclusion is that, under this assumption, the scheme behaves as in the incompressible case.

Numerical tests are performed, first on problems admitting an explicit solution, and then on the second and third benchmark tests proposed in the conference (natural convection with large temperature difference and free convection). The space discretisation is performed using the Taylor-Hood finite element. A formally second order in time scheme is obtained by using the usual BDF2 method. The results confirm the theoretical error analysis.

References

[1] Jean-Paul Caltagirone and Jérôme Breil. Sur une méthode de projection vectorielle pour la résolution des équations de Navier-Stokes. *Comptes Rendus de l'Académie des Sciences – Série II b*, 327:1179–1184, 1999.

[2] Alexandre Joel Chorin. Numerical solution of the Navier-Stokes equations. *Mathematics of Computation*, 22:745–762, 1968.

[3] J.-L. Guermond. Some implementations of projection methods for Navier-Stokes equations. *Mathematical Modelling and Numerical Analysis*, 30(5):637–667, 1996.

[4] J.L. Guermond, P. Minev, and Jie Shen. Error analysis of pressure-correction schemes for the navier-stokes equations with open boundary conditions. TICAM Report 03-08, Texas Institute for Computational Engineering and Sciences, The University of Texas, 24th Street, ACE 6.412, Austin, Texas 78712., mar 2003. to appear in SIAM J. Numer. Anal.

[5] J.L. Guermond and Jie Shen. On the error estimates for the rotational pressure-correction projection methods. TICAM Report 02-09, Texas Institute for Computational Engineering and Sciences, The University of Texas, 24th Street, ACE 6.412, Austin, Texas 78712., apr 2002. to appear in Math. Comp.

[6] Steven A. Orszag, Moshe Israeli, and Michel O. Deville. Boundary conditions for incompressible flows. *Journal of Scientific Computing*, 1(1):75–111, 1986.

[7] R. Témam. Sur l'approximation de la solution des Équations de Navier-Stokes par la méthode des pas fractionnaires (II). *Archives of Rational Mechanics Analysis*, 33:377–385, 1969.