Multigrid for Low Mach Number Flows Including Acoustic Modes

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Abstract:
Considering low Mach Number flows in acoustically non-compact computational domains, one cannot neglect acoustic modes travelling with the speed of sound.
The compressible Navier-Stokes equations naturally model these waves, but the equations become stiff for decreasing Mach Number M. In addition, one receives a multiscale coupled problem depending on the Mach Number.
The impedance condition

$$p_{acou} = Z v_{acou} \simeq M \; v_{acou}$$

provides insight in the relation between the acoustic pressure p_{acou} and the acoustic velocity disturbance v_{acou}. Hence, small errors in the pressure field, i.e. introduced by initial conditions, will be transformed into higher order velocity errors. In this sense also round-off errors become important and act as a natural limit for the decreasing Mach Number.

The presented Finite-Volume discretisation scheme is an extension to the compressible regime of an incompressible discretisation on vertex centered grids using collocated variables. Inaccurate initial conditions result into disturbing waves, for which the amplitude depends on the quality of the initial conditions and the Mach Number. For low Mach Numbers they might become dominant. Common Dirichlet or Neumann boundary conditions tend to reflect those travelling disturbing waves, rather than to let them flow out of the computational domain. Thus, boundary conditions have been implemented that assure a defined time averaged value at the boundary and simultaniously reduce the reflection at the boundary to a minimum.

Time discretisation is performed using an implicit multistep procedure in order to circumvent the restricting CFL number condition. Analytical studys for the linear case exhibit, that the second order Fractional Step method has even better properties with respect to numerical damping than the classic fourth order explicit Runge-Kutta method.

The stiffness of the system of equations for low Mach Numbers is a challenging task for the underlying iterative solver. GMRES shows good convergence properties for low Mach Numbers. However, numerical experiments revealed, that also ILU offers Mach Number independent convergence rates and can be used as a smoother in a multigrid procedure. Due to the multiscale character of the problem for low Mach Numbers one has to take care, that the interpolation property $|I - pr|$ is bounded in order to obtain a Mach Number independent approximation property for the multigrid process.