Physics and insects require compressible low Mach number flow

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Radial, laminar, plane, low velocity flow represents the simplest, non-linear fluid dynamics problem. Ostensibly this apparently trivial flow could be solved using the incompressible Navier-Stokes equations, universally believed to be adequate for such problems. Most researchers would, however, simply use the incompressible flow assumption to estimate the velocity as a function of radius and the incompressible Bernoulli equation to compute the pressure gradient.

Because this approach does not dynamically determine the velocity, it is fair to ask whether the solution exactly represents such flow or whether the actual laminar velocity will be greater or less than the incompressible velocity. To answer this, it is eminently appropriate to solve the more rigorous compressible problem. If the incompressible flow assumption is robust, it will show that in the limit of zero velocity any compressible contribution decreases more rapidly in importance than incompressible contribution, leading to the incompressible flow limit.

A Lagrangian compressible-flow analysis is used to obtain an analytical solution to the problem. The exact solution shows that for the most natural flow, adiabatic flow, and isothermal and other flows, the true velocity is different from the incompressible velocity, even at the limit of zero velocity. Thermodynamically speaking, only for the very special incompressible case is this untrue. However, it is not difficult to show that assuring incompressible flow would require extraordinary efforts to develop the techniques to precisely modify the flow by adding or extracting heat from the system. Free flow, of which adiabatic flow is the most representative, will not maintain constant density. In fact, in the radial flow problem it can be shown that for adiabatic flow between two parallel plates the pressure gradient is proportional to the divergence of the velocity and there cannot be a pressure gradient without it.

The analytical solution is based on applying Newton's second law of motion, not the Navier-Stokes equations. A robust Navier-Stokes approach must be able to duplicate this result given the same laminar flow assumptions. What it shows is that pressure and density remain linked at low velocity through the equation of state. There is no uncoupling between pressure and density at low velocity. The analytical solution does not contain an incompressible term that becomes large compared to the compressible term at the limit of zero velocity. Given the thermodynamic constraint (adiabatic, isothermal or other flows), or temperature, solving the density problem is equivalent to solving the pressure problem, and vice versa. However, in many instances, density, not pressure, is the independent variable.

Finding a single low-velocity flow phenomenon that cannot be derived from the incompressible paradigm will prove this is more than an academic argument. Invoking the low-velocity Mach number criterion, one frequently finds claims in the literature that the incompressible Navier-Stokes equations adequately represent insect flight. But the compressible model suggests that hovering insects use compression-decompression cycles to gain lift on the downstroke and an assist on the upstroke. This work examines insect flight and offers a glimpse of the future of fluid dynamics after incompressible theory assumes a subordinate role as a special case of compressible flow theory.