A Preconditioning Technique for Biphasic Flows with Interfaces

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The present paper describes a methodology for constructing and numerically resolving a reduced model for isothermal biphasic incompressible flows. The incompressibility constraint is handled through a two-phase preconditioning technique that requires consistent modifications of the standard one-phase preconditioning techniques.

The main flow features considered are: incompressible flow in the dense phase (liquid), low Mach number or incompressible flow in the light phase (gas) and an interface confined in thin regions where the dense and light fluids are mixed.

Several challenges have to be handled. The first challenge is connected to the numerical scheme which should prevent the interface smearing during its evolution characterized by gross topological changes. The second challenge looks for the appropriate modeling of the physics associated to the interface. The third the challenge is related to the difficulty of the numerical handling of the incompressibility constraint and the computation of the pressure field in density based algorithms, in presence of very large density variations over a moving interface.

The Eulerian capturing techniques, such as Level Set (LVS) or Volume of Fluid (VOF) methods offer the framework for handling successfully these challenges. Our experience with LVS and VOF methods combined with a preconditioning technique showed that new insights in the pressure wave propagation over the interface, called here acoustic phenomena, should be gained.

Indeed, because the preconditioning technique recovers the pressure field by employing an advection equation for pressure the effect on the speed of pressure waves, induced by the homogeneous mixture contained in the interface, has to be quantified. This quantification, in the form of the speed λ of the pressure waves over the moving interface, is retrieved in the formulation of the pseudo-sound speed β_m, the key parameter of any preconditioning technique.

In order to devise our two-phase preconditioning technique the following 3-steps methodology was employed:

- work out an expression for the *speed of the pressure waves* propagating through a stationary interface also known as *mixture sound speed* a_m;
- work out an expression for the *speed of the pressure waves* λ propagating through a moving interface.
- construct an advection equation for pressure and precondition it in order to have the same order of magnitude for the sound speeds and the convective velocities.

The first two steps are handled with a homogenization technique also known as multiple scale asymptotic technique (Kevorkian and Bosley (1998), Lurie (1997)) applied to a 1D layered periodic structure, modeling the interface, in the space domain x and space-time domain (x,t) respectively. The outcomes of these steps are closed formulas for a_m:

$$\frac{1}{\rho a_m^2} = \frac{C}{\rho_1 a_1^2} + \frac{1-C}{\rho_2 a_2^2}$$

and λ

$$\lambda = \frac{|\vec{V}|\left[\hat{a}^2 - \hat{k}\left(\hat{1}/\rho\right)\right] - a_1 a_2\sqrt{\left(\hat{\rho}V^2 - \hat{k}\right)\left[\left(\hat{1}/k\right)V^2 - \left(\hat{1}/\rho\right)\right]}}{V^2 - \hat{k}\left(\hat{1}/\rho\right)}$$

where C is the VOF fraction of the dense phase, $k = \rho a^2$ the bulk modulus and $\wedge$ denotes the following average for a variable ω:

$$\hat{\omega} = (1-C)\,\omega_1 + C\omega_2$$

The preconditioned form of the governing system of our model is:

$$\frac{1}{\rho\beta_m^2}\frac{\partial p}{\partial\tau}+\vec{\nabla}\cdot\vec{V}=0 \quad (1,a)$$

$$\rho\frac{\partial\vec{V}}{\partial t}+\rho\nabla\left(\vec{V}\otimes\vec{V}\right)+\vec{\nabla}p=\rho\vec{f}_e \quad (1,b)$$

$$\frac{\partial\left(\rho C\right)}{\partial t}+\vec{\nabla}\cdot\left(\rho C\vec{V}\right)=0 \quad (1,c)$$

The pseudo-sound speed of the mixture β_m is defined by a Turkel-like formula:

$$\beta_m^2=min\left[max\left(u^2,\alpha^2u_{ref}^2\right),a_m^2\right]\frac{\lambda^2}{a_m^2}$$

The αu_{ref} is a flow dependent cutoff value and the min function allows using the formula also for "supersonic" regimes in the interface region. The last ratio measures the kinematical effects induced by the interface movement. It is found that this ratio is critical for the robustness of two-phase preconditioning technique.

Separate temporal and spatial discretizations are used for the governing system of the reduced model. A dual-time stepping technique is employed to advance the solution in the physical time. At each physical time step a steady state problem is solved in the pseudo-time and acceleration techniques can be applied to speed up the convergence.

The discretization process is as follows:

- the spatial discretization uses the central scheme stabilized with an artificial dissipation of Jameson type, the VOF-type advection equation uses a first order upwind scheme
- the pseudo-time smoother is represented by a 4^{th} order Runge-Kutta scheme with optimized coefficients
- the dual-time stepping technique employs first or second-order accurate backward differencing in the physical time to advance the solution.

Validation is made against the broken dam problem. For this test case we added a level set advection equation in order to control the numerical diffusion of the interface only on its edge between the dense phase and the mixture. Figures 1 and 2 show the evolution of the free surface (marked by the zero level set) and the comparison with the experimental data provided by Martin and Moyce (1952).

Figures 3 and 4 emphasize the solution computed for an industrial application namely the hydroplaning of the 195/65R15 rigid slick tire (see Grogger (1996)). The application is challenging because one has violent impacts between the free surface and the tire with formation of sprays in its frontal part. In figure 3 one can find the density distribution in the computational domain and in figure 4 the frontal pressure distribution on the ground, on the symmetry axis, is given. The difference between the computations and experiment is due to the fact that the tire's deformability is not considered. Ongoing work is focused on coupling the flow solver with a finite element software which evaluates the deformation based on the computed pressure distribution acting on the tire's skin.

References

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Figures

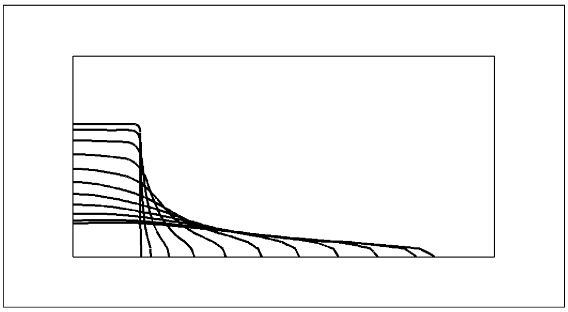

Fig. 1 Snapshots with the free surface position in the broken dam problem

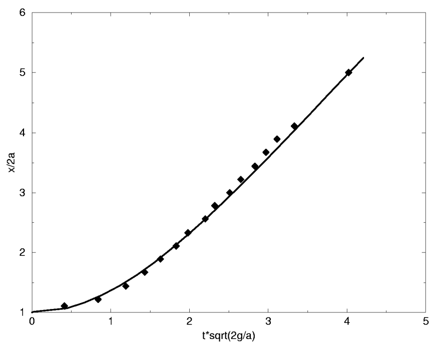

Fig.2 Broken dam problem: comparison between computation and experiment for the leading edge position

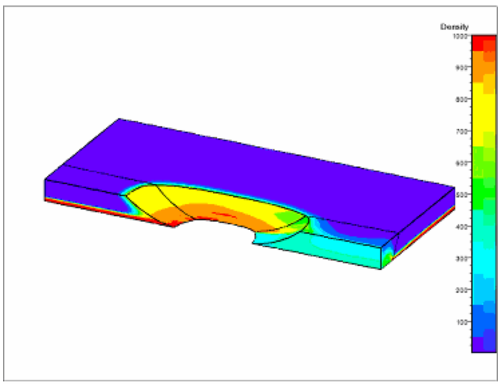

Fig. 3 Solution for slick tire: density field, regime velocity 60km/h

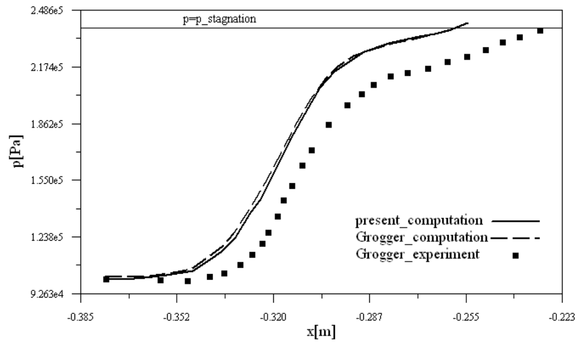

Fig. 4 Solution for slick tire: pressure field on the ground, regime velocity 60km/h