An adaptive finite element solver for compressible flows: Application to heat-driven cavity benchmarks in 2D and 3D

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We present a finite element discretization which is stable for compressible flows for a wide range of Mach number: from the incompressible limit up to subsonic Mach numbers ($Ma < 1$). The finite element space consists of equal-order quadratic ansatz functions. The stability is achieved by local projections of small-scale fluctuations of pressure and velocities [2]. For reducing the necessary amount of unknowns, local mesh refinement controlled by a posteriori error estimation is applied [5, 4]. The discrete equations are solved coupled by Newton iteration with multigrid for the linear systems [1]. This leads to a robust and very efficient solver.

In order to validate accuracy and performance, we present results for the heat-driven cavity benchmark of Le Quere and Paillere [7] for low Mach number flows where especially Nusselt numbers are of interest [3]. Due to adaptive refinement we are able to compute these quantities up 5 digits accuracy on a PC.

Since numerical methods for a wide range of Mach number are still of scientific interest [6, 8], we illustrate the performance of the approach by considering a configuration with higher Mach number. Furthermore, we formulate an extension to the 3D case, which remains to be a challenge. However, we show that local mesh refinement is a powerful tool to solve 3D problems on PC's in a couple of minutes.

References

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