LOW MACH NUMBER LIMIT OF THE NON-ISENTROPIC NAVIER–STOKES EQUATIONS

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This work is devoted to the study of the low Mach number limit for classical solutions of the compressible Navier-Stokes equations for polytropic fluids in the whole space with general initial data, allowing for large density and temperature variations. In particular, we take into account the combined effects of large temperature variations and thermal conduction. This study rigourously justify well-known formal computations (see [2, 5]) and answer a question addressed in [3]. In addition we can include chemical reactions so as to rigourously justify the equations of low Mach number combustion (see [1] and the lectures notes of R. Klein).

More precisely, the system we consider is the following:

$$\left\{\begin{aligned} &\frac{\partial \varrho}{\partial t} + u\cdot\nabla\varrho + \varrho \operatorname{div} u = 0,\\ &\varrho\Big(\frac{\partial u}{\partial t} + u\cdot\nabla u\Big) + \nabla\left(\frac{1}{\gamma \mathrm{Ma}^2}\varrho\theta\right) = \frac{1}{\mathrm{Re}}\Big(\operatorname{div}(2\mu Du) + \nabla(\lambda \operatorname{div} u)\Big),\\ &\varrho\Big(\frac{\partial \theta}{\partial t} + u\cdot\nabla\theta\Big) + (\gamma-1)\varrho\theta\operatorname{div} u = \frac{\gamma}{\mathrm{PrRe}}\operatorname{div}(k\nabla\theta).\end{aligned}\right. \tag{1}$$

where $0 < \mathrm{Ma} \leqslant 1$ and $1 \leqslant \mathrm{Pr}, \mathrm{Re} < \infty$. As concern the initial data, we just assume that they satisfy

$$(\log\varrho_0, v_0, \log T_0) \in H^s(\mathbf{R}^d)^{d+2} \quad \text{and} \quad p_0 := \varrho_0 T_0 = \text{constant} + \mathcal{O}(\mathrm{Ma}). \tag{2}$$

where s is an integer large enough and $d \geq 1$.

We prove that the solutions of (1)-(2) exist and are uniformly bounded for a time interval which is independent of Ma, Re and Pr. Once this is granted, we can prove that, as the Mach number Ma goes to 0, the solutions converge to the solutions of the corresponding zero Mach number equations by using a Theorem of G. Métivier and S. Schochet [4].

References

[1] P. Embid – "Well-posedness of the nonlinear equations for zero Mach number combustion", *Comm. Partial Differential Equations* **12** (1987), no. 11, p. 1227–1283.

[2] P.-L. Lions – *Mathematical topics in fluid mechanics. Vol. 1*, Oxford Lecture Series in Mathematics and its Applications, vol. 3, The Clarendon Press Oxford University Press, New York, 1996, Incompressible models, Oxford Science Publications.

[3] A. Majda – *Compressible fluid flow and systems of conservation laws in several space variables*, Applied Mathematical Sciences, vol. 53, Springer-Verlag, New York, 1984.

[4] G. Métivier & S. Schochet – "The incompressible limit of the non-isentropic Euler equations", *Arch. Ration. Mech. Anal.* **158** (2001), no. 1, p. 61–90.

[5] C.-D. Munz – "Computational fluid dynamics and aeroacoustics for low Mach number flow", in *Hyperbolic partial differential equations (Hamburg, 2001)*, Vieweg, Braunschweig, 2002, p. 269–320.

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