Two temperature Euler equations for a plasma with slowing down of Suprathermal Particles

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Abstract

In the framework of the modeling of the plasma by the compressible Euler equations, we consider the transport of suprathermal particles created by fusion, whose modeling is made by a kinetic equation. We focus on the coupling between the fluid model and the kinetic equation ; the momentum and energy deposit has to be consisered precisely and a coherent treatment of the electric plasma field has to be made. We emphasize some details of the numerical simulations.

INTRODUCTION

In hot plasmas such as stellar plasmas or plasmas produced by laser in Inertial Confinement Fusion, the fusion of Deterium and Tritium creates Helium ions whose initial velocity is very large compared to the thermal velocity of plasma ions (so they are called *suprathermal particles*). It is crucial to deal correctly with the slowing down of these particles due to the Colomb interaction with the plasma and to perform the coupling of these phenomena with the hydrodynamics of the plasma. Moreover, it is well known that for the plasma, one has to take into account a two-temperature model, with an electron temperature T_e different from the ion temperature T_0. For a relevant physical modeling, one has also to consider the electric plasma field $\mathbf{E}$ (see [1]).

The modeling of the transport of the suprathermal (ST) particles by a Vlasov-Fokker-Planck equation and the numerical simulation have been studied for a very long time by physicists see for example [7],[4] specially in a homogeneous plasma. But the momentum and energy deposit in the momentum and energy balance equations of the fluid model has to be consisered precisely and a coherent treatment of the electric plasma field has to be made (which was not the purpose of the mentioned litterature). So we focus here one these coupling aspects.

The outline of the paper is the following. Section 1 is devoted to the statement of the model, specially the coupling between the plasma and the ST particles. In section 2, we give some enlightments on the Monte-Carlo method for the ST particles. In the last section, some numerical results are given.

1 THE MODEL

For the shake of simplicity, the plasma is assumed to contain only one species of ions.

Notations : m_e, m_0, m_s are the mass of the electrons, the ions and the ST particles ; q_s the ST particle charge, Z the ionization level of the plasma ion ; $\mathbf{U}$ the plasma velocity ; ε_0 which is proportional to T_0 and $\varepsilon_e = \frac{3}{2}ZT_e$ are the internal energies ; P_0 and $P_e = ZNT_e$ the ion and electron pressures (the relation between P_0 and ε_0 is given by an equation of state); Ω the energy exchange term between ions and electrons which is proportional to $(T_e - T_0)$. Denote also by $f(t, x, \mathbf{v})$ the distribution function of the ST particles, where $\mathbf{v} \in \mathbf{R}^3$ and x belongs to a bounded domain in $\mathbf{R}^3$.

The Vlasov-Fokker-Planck equation. For the sake of simplicity, one assumes that the ST particles undergo Coulomb interactions on electron population only, so the evolution equation of f reads

$$\frac{\partial}{\partial t} f + \mathbf{v}.\nabla f = -\frac{q_s}{m_s}\mathbf{E}.\frac{\partial f}{\partial \mathbf{v}} + ZN\frac{\partial}{\partial \mathbf{v}}.((\mathbf{S}f)(\mathbf{v} - \mathbf{U})) \tag{1}$$

and the simplest form of the operator $\mathbf{S}$ reads as (by defining $\mathbf{w} = \mathbf{v} - \mathbf{U}$)

$$\mathbf{S}f(\mathbf{w}) = Y\mathbf{w}f + O_e(\mathbf{w}).\frac{\partial f}{\partial \mathbf{v}}, \qquad O_e(\mathbf{w}) \simeq 3Y\frac{T_e}{m_s}\left(1 - \frac{\mathbf{ww}}{|\mathbf{w}|^2}\right).$$

The coefficient Y is roughly speaking proportional to $T_e^{-3/2}$, see [4]. For any function ϕ defined on the whole space $\mathbf{R}^3$ one sets $\langle\phi\rangle = \int \phi(\mathbf{v})d\mathbf{v}$. The balance relations related to (1) are

$$m_s(\frac{\partial}{\partial t}\langle\mathbf{v}f\rangle + \nabla\langle\mathbf{vv}f\rangle) = q_s\mathbf{E}\langle f\rangle - m_sZN\langle\mathbf{S}f\rangle,$$

$$\frac{m_s}{2}\frac{\partial}{\partial t}\langle|\mathbf{v}|^2 f\rangle + \frac{m_s}{2}\nabla.\langle\mathbf{v}|\mathbf{v}|^2 f\rangle = q_s\langle\mathbf{v}f\rangle.\mathbf{E} - m_sZN\langle\mathbf{v}.\mathbf{S}f(\cdot - \mathbf{U})\rangle.$$

The Euler system. Consider now the plasma model. The continuity equation is not changed by the coupling with the ST particles

$$\frac{\partial}{\partial t}N + \nabla(N\mathbf{U}) = 0. \tag{2}$$

In the momentum equation, one has to add by a natural way the counterpart of the momentum change of the ST particles ($m_s ZN \langle \mathbf{S} f \rangle$) added to ($q_s \mathbf{E} \langle f \rangle$), that is to say, with $P_{tot} = P_e + P_i$

$$m_0 \left(\frac{\partial}{\partial t} + \nabla(\mathbf{U}\bullet) \right) (\mathbf{U}) + \nabla P_{tot} = m_s ZN \langle \mathbf{S} f \rangle - q_s \mathbf{E} \langle f \rangle . \tag{3}$$

Define $\tilde{f}(\mathbf{w}) = f(\mathbf{w} + \mathbf{U})$. Since $\langle \mathbf{v}.\mathbf{S} f(\cdot - \mathbf{U}) \rangle = \left\langle (\mathbf{w} + \mathbf{U}).\mathbf{S}\tilde{f} \right\rangle$, if the plasma were characterized by only one internal energy $\varepsilon_{tot} = \varepsilon_e + \varepsilon_0$, the plasma energy equation would read as

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \nabla(\mathbf{U}\bullet) \right) \left(N\varepsilon_{tot} + \frac{m_0}{2} N|\mathbf{U}|^2 \right) + \nabla.(P_{tot}\mathbf{U}) + \nabla.\mathbf{q}_{ther} = \\ m_s ZN(\left\langle \mathbf{w}.\mathbf{S}\tilde{f} \right\rangle + \mathbf{U}. \left\langle \mathbf{S}\tilde{f} \right\rangle) - q_s \mathbf{E}. \langle \mathbf{v} f \rangle . \end{aligned}$$

where $\mathbf{q}_{ther}$ is the Spitzer thermal flux proportional to the gradient of T_e, see [6]. So for the internal energy equation, the source term would reduce to $m_s ZN \left\langle \mathbf{w}.\mathbf{S}\tilde{f} \right\rangle + Q$, where :

$$Q = -q_s \mathbf{E}. \left\langle \mathbf{w}\tilde{f} \right\rangle .$$

As a matter of fact, two energy evolution equations are to be considered for a classical modeling of the plasma. Let us recall that without any coupling with the ST particles, they read as (see [5]).

$$\left(\frac{\partial}{\partial t} + \nabla(\mathbf{U}\bullet) \right) (N\varepsilon_i) + P_i \nabla.\mathbf{U} - \Omega = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \nabla(\mathbf{U}\bullet) \right) (N\varepsilon_e) + P_e \nabla.\mathbf{U} + \nabla.\mathbf{q}_{ther} + \Omega = 0 \tag{5}$$

The simplest defintion for $\mathbf{E}$ is $\nabla P_e + ZNq_e\mathbf{E} = 0$, see [1]. Consider now the coupling with the ST particles : we must add at the right hand side of (5) all the terms coming from this coupling

$$\left(\frac{\partial}{\partial t} + \nabla(\mathbf{U}\bullet) \right) (N\varepsilon_e) + P_e \nabla.\mathbf{U} + \nabla.\mathbf{q}_{ther} + \Omega = m_s ZN \left\langle \mathbf{w}.\mathbf{S}\tilde{f} \right\rangle + Q \tag{6}$$

Up to our knowledge, system (2)(3)(4)(6) has not been considered up to now. Let us notice that in [2] such a system is written in 1D without the term Q. This term is the counterpart of the work of the electric field. It is crucial (and not intuitive) to notice that this work has to be evaluated in the matter reference frame.

Of course one get a global momentum balance relation and a global energy balance relation.

The coupling terms. The momentum deposition term within the matter reference frame reads as

$$\left\langle \mathbf{S}\tilde{f} \right\rangle = Y \left(\left\langle \mathbf{w}\tilde{f} \right\rangle + 3\frac{T_e}{m_s} \left\langle \frac{2\mathbf{w}}{|\mathbf{w}|^2}\tilde{f} \right\rangle \right),$$

The ST particle energy $m_s|\mathbf{w}|^2$ is generally large compared to the electron temperature, so the second term is negligeable compared to the first one and $\left\langle \mathbf{S}\tilde{f} \right\rangle \simeq Y \left\langle \mathbf{w}\tilde{f} \right\rangle$. Since $\mathbf{w}.O_e(\mathbf{w}) = 0$, the energy deposit term within the matter reference frame reads as $\left\langle \mathbf{w}.\mathbf{S}\tilde{f} \right\rangle = Y \left\langle |\mathbf{w}|^2\tilde{f} \right\rangle$

2 NUMERICAL METHOD.

The Euler system. One uses a Lagrange type code based on the classical Wilkins method.

At each time step, this method consists of two stages : firstly move each node according to the force due the pressure gradient, secondly solve the internal energy equations, see [5]. These stages are followed by a mesh regularization.

To perform the coupling between the two models, one must evaluate in each cell the electric field $\mathbf{E}$ on one hand and the momentum and energy deposit by the ST particles on the other hand.

The transport equation. A Monte-Carlo method is used. The method is based on the approximation of the solution $f(t, x, \mathbf{v})$ by a sum of Dirac measures, see [3]. For instance at the begining of the simulation, one sets

$$f(0, x, \mathbf{v})dxd\mathbf{v} \simeq \sum_{p=1}^{N_{part}} \omega_p \delta_{\mathbf{v}_p}(d\mathbf{v})\delta_{x_p}(dx).$$

where the weights ω_p of the particles are such that in each cell M, one has

$$\sum_{p,\ \text{s.t.}\ x_p \in M} \omega_p = \int_M \int f(0, x, \mathbf{v})d\mathbf{v}dx, \qquad \sum_{p,\ \text{s.t.}\ x_p \in M} \omega_p \mathbf{v}_{p,i} = \int_M \int f(0, x, \mathbf{v})\mathbf{v}_i d\mathbf{v}dx$$

It is usefull for a good implementation to have a probabilist interpretation of the dual operator of $f \hookrightarrow -\frac{q_s}{m_s}\mathbf{E}.\frac{\partial f}{\partial \mathbf{v}} + ZN\frac{\partial}{\partial \mathbf{v}}.((\mathbf{S}f)(\mathbf{v} - \mathbf{U}))$ that is to say (with $Y_D = 3ZNY\frac{T_e}{m_s}$)

$$\varphi \hookrightarrow \frac{q_s}{m_s}\mathbf{E}.\frac{\partial \varphi}{\partial \mathbf{v}} - ZNY(\mathbf{v} - \mathbf{U})\frac{\partial \varphi}{\partial \mathbf{v}} + Y_D\frac{\partial}{\partial \mathbf{v}}\left((1 - \frac{\mathbf{w}\mathbf{w}}{|\mathbf{w}|^2})\frac{\partial \varphi}{\partial \mathbf{v}} \right)$$

Notice that

1. $\mathbf{E}\frac{\partial}{\partial \mathbf{v}}\varphi$ corresponds to an acceleration in the direction of $\mathbf{E}$

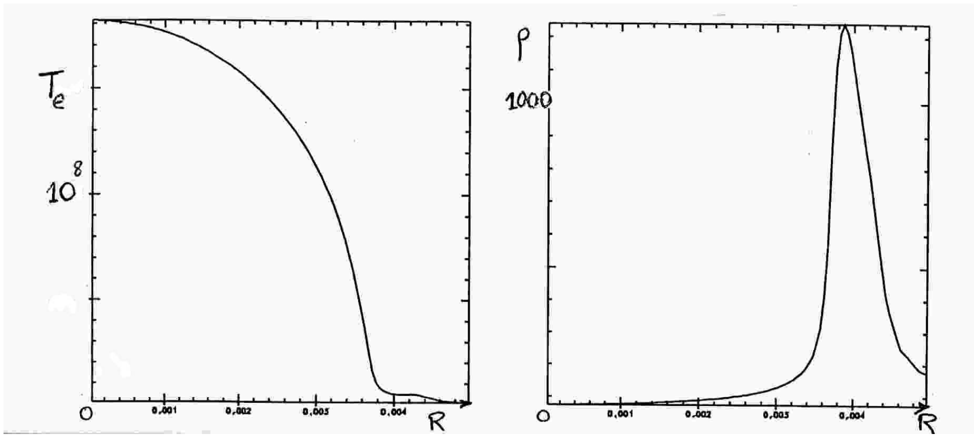

Figure 1: Temperature and density profiles versus radius

2. $-\mathbf{w}\frac{\partial}{\partial \mathbf{v}}\varphi$ corresponds to a straight line slowing down (in the matter reference frame)
3. the deflection operator $\frac{\partial}{\partial \mathbf{w}}(1-\frac{\mathbf{ww}}{|\mathbf{w}|^2})\frac{\partial \varphi}{\partial \mathbf{w}}$ corresponds to a diffusion on a sphere, indeed one can check that the solution of the elementary equation (of Laplace-Beltrami type)

$$\frac{\partial \varphi}{\partial t} - \frac{\partial}{\partial \mathbf{w}}(1-\frac{\mathbf{ww}}{|\mathbf{w}|^2})\frac{\partial \varphi}{\partial \mathbf{w}} = 0, \qquad \varphi(0,\mathbf{w}) = \delta_{\mathbf{w}_0}$$

satisfies $\int \varphi(t,\mathbf{w})|\mathbf{w}|^2 d\mathbf{w} = |\mathbf{w}_0|^2$ for any t. As a matter of fact, the solution of this equation is an analytic function depending only on the angular variable $\mathbf{w}.\mathbf{w}_0$ and its support is the sphere of radius $|\mathbf{w}_0|$. So the Monte-Carlo method consists in a tracking of the particles in the mesh used by the hydrodynamics of the plasma. In each cell M, where the mean velocity is $\mathbf{U}_M$, the particles move with their relative velocity $\mathbf{w}_p = \mathbf{v}_p - \mathbf{U}_M$, and their velocity are changed according to the three modifications listed above.

Moreover, when the particle p goes from cell M to cell M', its velocity $\mathbf{w}_p$ has to be corrected by the following way

$$\mathbf{w}'_p + \mathbf{U}_{M'} = \mathbf{w}_p + \mathbf{U}_M.$$

In each cell M, one has to estimate the quantities $\left\langle \tilde{f} \right\rangle|_M$, $\left\langle \mathbf{w}\tilde{f} \right\rangle|_M$, $\left\langle |\mathbf{w}|^2\tilde{f} \right\rangle|_M$, with in the matter reference frame. For instance, we get $\left\langle \mathbf{w}\tilde{f} \right\rangle|_M \simeq \sum_{p,\text{s.t.}x_p\in M} \mathbf{w}_p L_p^M \omega_p$, where L_p^M denotes the distance of particle p in the cell M.

3 NUMERICAL RESULTS

Numerical example 1.

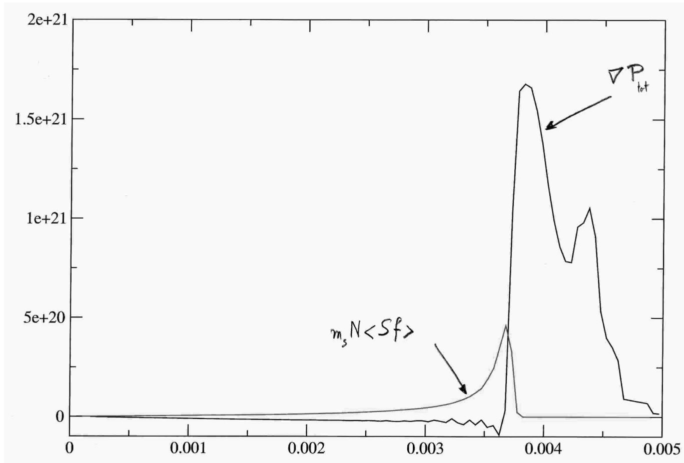

Figure 2: Profiles of ∇P_{tot} and of $m_s N \left\langle \mathbf{S}\tilde{f} \right\rangle$ versus radius

One addresses a dense and hot spherical plasma with a source of ST particles in the center of the sphere. The initial density and temperature profiles are very stiff (conditions of Inertial Confinement Fusion plasma), see Fig. 1. At a given time, one compares the profile of the momentum deposit with the profile of ∇P_{tot}, (Fig. 2). One notices that the momentum deposit $m_s N \left\langle \mathbf{S}\tilde{f} \right\rangle$ is important in the zone where the pressure gradient is large, which is the crucial zone for energy deposit.

Numerical example 2.

The same spherical plasma is considered with the same T_e profile, but with ρ much lower. The comparison of Q with $\mathcal{I} = m_s N \left\langle \mathbf{w}.\mathbf{S}\tilde{f} \right\rangle$ at a given time step is plotted on Fig. 3. One sees that in that case the influence of the term Q may be not negligeable.

CONCLUSION.

In the two-temperature Euler equations modeling a hot plasma, we have performed the coupling with a simplified transport equation which is relevant for the slowing down of the ST particles. This coupling has be made by a consistant manner in such a way that there is good momentum and energy balance. It is implemented in a plasma code and some preliminary numerical results are given.

References

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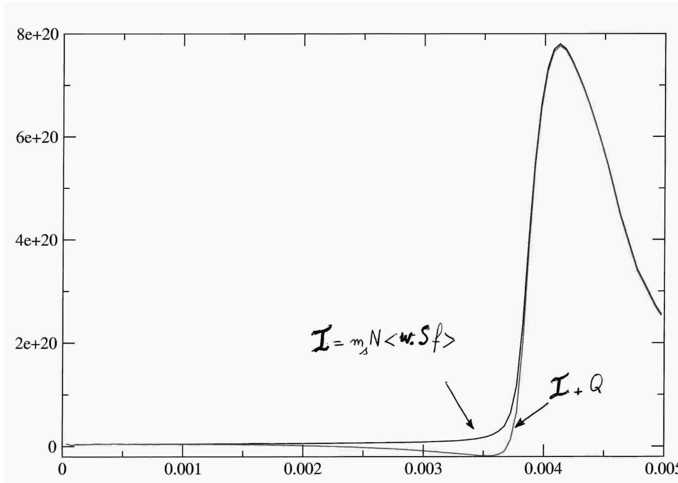

Figure 3: Profiles of $\mathcal{I}$ and $\mathcal{I} + Q$ versus radius

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