Direct resolution of the Vlasov equation on a moving grid

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Outline

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Non relativistic Vlasov-Maxwell equations

Distribution function $f(\mathbf{x}, \mathbf{v}, t)$ solution of the **Vlasov** equation :

$$\frac{\partial f}{\partial t} + \mathbf{v}.\nabla_x f + F(\mathbf{x}, \mathbf{v}, t).\nabla_v f = 0$$

where $F(\mathbf{x}, \mathbf{v}, t) = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ is the Lorentz force, $E(t, \mathbf{x})$ and $B(t, \mathbf{x})$ electric and magnetic fields solutions of the Maxwell's equations.

$$\left\{ \begin{array}{rcll} \frac{\partial E}{\partial t} - c^2 \text{curl } B & = & -j(x, t) & = q \int_{\mathbb{R}} f(\mathbf{x}, \mathbf{v}, t) \mathbf{v} \, d\mathbf{v}. \\ \text{div} E & = & \rho(t, \mathbf{x}) & = q \int_{\mathbb{R}} f(\mathbf{x}, \mathbf{v}, t) \, d\mathbf{v}. \\ \frac{\partial B}{\partial t} - \text{curl } E & = & 0 & \\ \text{div} B & = & 0 & \end{array} \right.$$

Vlasov-Poisson equations

Simplified model : magnetic field B is assumed known and external $\Longrightarrow$ Vlasov-Poisson model.

$$\begin{cases} \nabla . E &= \rho \\ \nabla \times E &= 0 \end{cases}$$

$$\begin{cases} E &= -\nabla \phi \\ -\Delta \phi &= \rho \end{cases}$$

State of the art

Two main methods :

- Particle-in-Cell (PIC) codes (the most popular)
  - Noisy method.
  - The macro-particles concentrate in regions where the probability of finding particles is important and not where density of particles is low but where important physical phenomena arise.
    e.g : tail of the distribution , halo, ...
- Direct simulation on a phase space grid
  - Noiseless.
  - Same resolution in the whole domain.
  - Numerical cost : N^d points for d dimensions (for Vlasov equation $d \leq 6$!).
  - Moreover, most of the points are useless as the distribution function is zero on many points.

Idea(s)

- Adaptive mesh - decrease the computational cost by refining some grid regions where thin structures are expected to develop and derefining zones where the distribution function is smooth :
  $\longrightarrow$ hierarchical meshes (Michel Mehrenberger, Martin Campos-Pinto, Eric Violard, Olivier Hoenen)
  $\longrightarrow$ wavelets (Michaël Gutnic, Matthieu Haefele, Guillaume Latu)
  Drawbacks : complex data structure difficult to handle with parallelisation.
- Moving grid : try to use the minimal uniform mesh containing the distribution function.

Outline

1. Vlasov equation
2. Semi-lagrangian method
3. Moving grid
4. Numerical results

Semi-lagrangian method : property

"The distribution function f is conserved along the characteristics."
For time s and t,

$$f(\mathbf{x}, \mathbf{v}, t) = f(\mathbf{X}(s; \mathbf{x}, \mathbf{v}, t), \mathbf{V}(s; \mathbf{x}, \mathbf{v}, t), s)$$

$(\mathbf{X}(s; \mathbf{x}, \mathbf{v}, t), \mathbf{V}(s; \mathbf{x}, \mathbf{v}, t))$ characteristics of Vlasov equation:

$$\begin{cases} \dfrac{d\mathbf{X}}{ds} & = & \mathbf{V}(s) \\ \dfrac{d\mathbf{V}}{ds} & = & \mathbf{E}(\mathbf{X}(s) + \mathbf{V}(s) \times \mathbf{B}(\mathbf{X}(s))) \\ \mathbf{X}(\mathbf{t}) & = & \mathbf{x} \\ \mathbf{V}(\mathbf{t}) & = & \mathbf{v} \end{cases}$$

Semi-lagrangian method : algorithm

Uniform mesh of the phase-space.
Given the distribution function $f_n = f(t_n)$ and field $E_n(B_n)$, compute f at time $t_{n+1} = f_{n+1}$.
Step 1 Advection: Solving backward the characteristics equations.

Step 2 Interpolation: Interpolate on the grid at time t_n the value of f.

N.B The grid can be different at each time step !
Only compute points where the distribution function is non zero: changing the mesh at each time step, following the evolution of f.

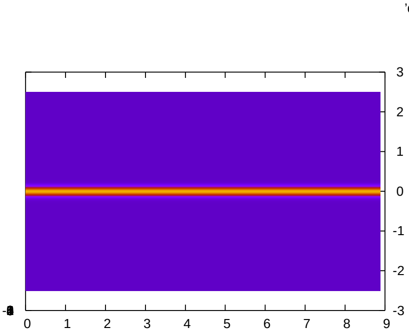
Outline

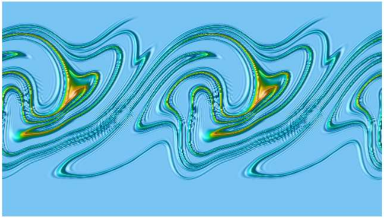
Relativistic Vlasov-Maxwell equations (1D)

$$\begin{cases} f(x, p_x, t), \quad (x, p_x) \in [0, L_x] \times [p_{min}(t), p_{max}(t)], \ (t \geq 0) \\ \dfrac{\partial f}{\partial t} + \dfrac{p_x}{m\gamma}\dfrac{\partial f}{\partial x} + e\left(E_x + \dfrac{P_y B_z - P_z B_y}{m\gamma}\right)\dfrac{\partial f}{\partial p_x} \qquad = \quad 0 \\ \gamma^2 = 1 + \dfrac{p_x^2}{m^2 c^2} + \dfrac{P_\perp^2}{m^2 c^2} \\ + \text{ Maxwell's eq.} \end{cases}$$

Relativistic Vlasov-Maxwell equations (1D)

Code developed by Alain Ghizzo.





Laser plasma interaction: method

Defining the p-projection of the distribution function:

$$F(t,p) = \int_0^{L_x} f(t,x,p)\,\mathrm{d}x, \quad p \in [p_{min}(t), p_{max}(t)].$$

Initialization: given a large impulsion domain

While $F(t=0, p_{max}(t=0)) < \varepsilon$,

$$p_{max}(t=0) = p_{max}(t=0) - 2\Delta p = \text{ impulsion domain decreasing.}$$

Time t_n:

If $F(t=t^n, p_{max}(t=t^n)) > \varepsilon$ then

$$p_{max}(t=t^{n+1}) = p_{max}(t=t^n) + 2\Delta p = \text{ impulsion domain increasing,}$$

else

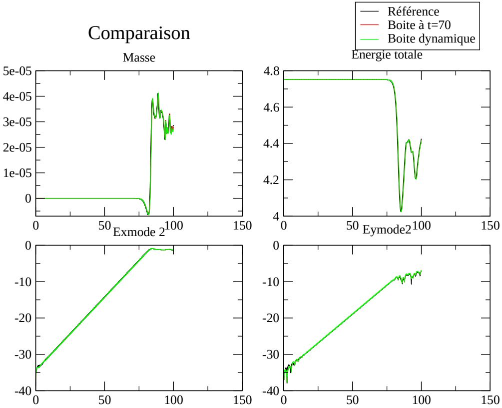
$$p_{max}(t=t^{n+1}) = p_{max}(t=t^n) = \text{ impulsion domain unchanged.}$$

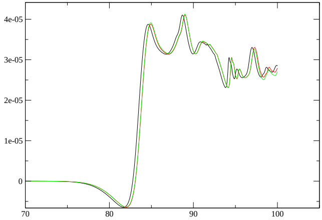
Outline

Laser plasma interaction: results

laser plasma interaction

Laser plasma interaction: results





Improving the code LOSS

LOcal Spline Simulator :
4D Code solving Vlasov-Poisson (Nicolas Crouseilles, Guillaume Latu, Eric Sonnendrücker).

- Semi-Lagrangian method,
- Local cubic splines for 2d advection.
- Parallel in velocity (domain decomposition, MPI).

First step : replace parallellism in velocity by a moving grid in velocity. (as in the Vlasov-Maxwell code)
Second step : cutting diagonally the moving grid to save points.

4D beam: results for LOSS

LOSS4D

4D beam: results for LOSS

Third step : Parallel in space (domain decomposition, MPI) AND moving grid in velocity.

- Very good results
- large gain in time because of the time step (less constrained).

Conclusion

- Easy to implement in all semi-lagrangian algorithm.
- Use more complex 1D results (with Edouard Oudet) to handle more complex beam movements predictible by the envelop equation

2D beam: results

Semi-gaussian beam, periodic field