Equilibrium and stability calculations for flux-core-spheromak configurations

Numerical Flow Models for Controlled Fusion
16-20 april 2007, Porquerolles

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Motivations: low aspect ratio, high beta

What is "aspect ratio"?

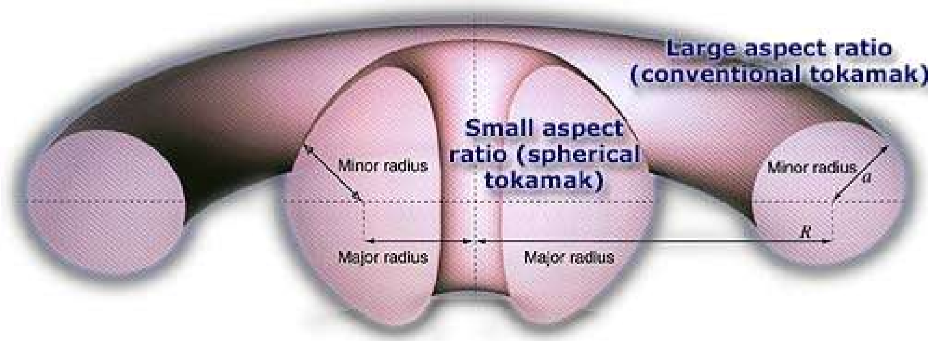
Aspect ratio is a measure of how "tight" the torus ring-shape is. It is defined as the ratio of the major radius to the minor radius.

$$A = R / a$$

So a small aspect ratio means a tighter toroidal shape.

Typical values for a conventional tokamak are 3 or more (JET is A = 3.1), for a spherical tokamak 1.5 or less (MAST is A = 1.4).

Close



What is beta (ß)?

Plasma pressure is given by the *plasma density* multiplied by its *temperature*. To confine the plasma, this pressure must be balanced by that produced by magnetic fields generated by electric coils called the *toroidal field coils*.

The ratio of plasma pressure to this magnetic field pressure is called beta (designated by the Greek character **ß**).

A high value of ß indicates good performance. START has achieved ß = 40%, three times the previous world record.

Close

- The toroidal magnetic field (supplied by the current flowing in the central column) needed to keep the plasma stable can be a factor of 10 less in a spherical tokamak than that of a conventional tokamak carrying the same plasma current.
- Efficiency is measured in values of the parameter ß .

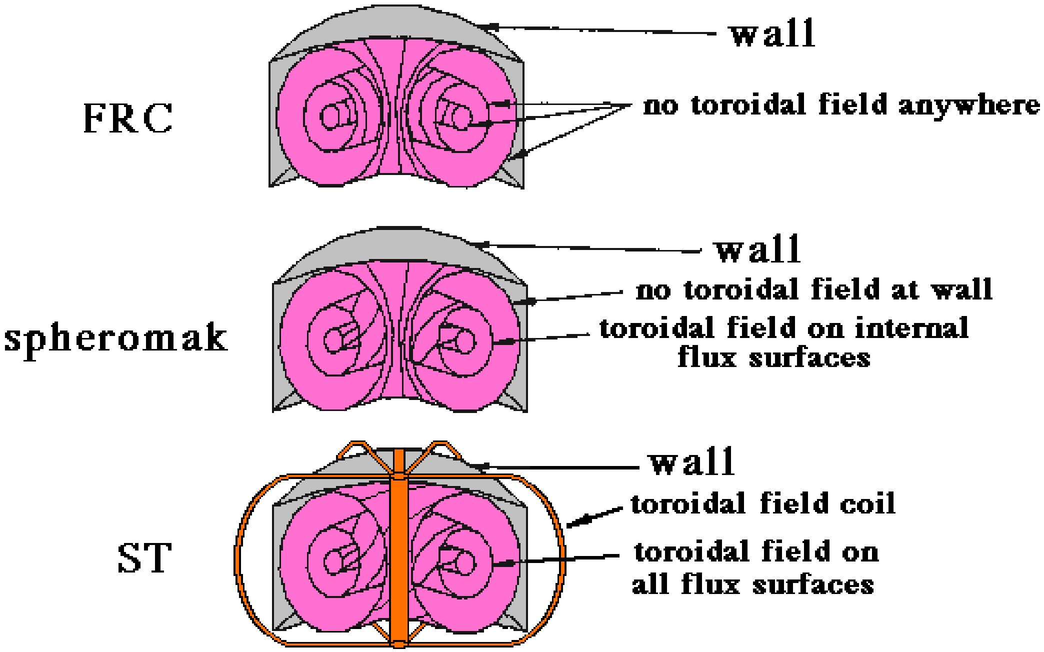
Ref : homepage of Start project

Spherical Tori, Spheromaks and FRCs

- Spheromaks are low β toroidal confinement configurations where currents flowing in the plasma produce the magnetic field almost entirely; they have a finite internal toroidal magnetic field, which vanishes at the plasma surface; hence no external field coils link the plasma.
- Field Reverse Configuration's are high β toroidal confinement configurations with zero toroidal magnetic field everywhere and so, like spheromaks, do not have coils linking the plasma.

Spheromaks manage to have a toroidal field without having toroidal field coils;
FRCs do not have toroidal field coils, but also do not have a toroidal field

- ST is a modification of the conventional tokamak and differs by having a much smaller aspect ratio

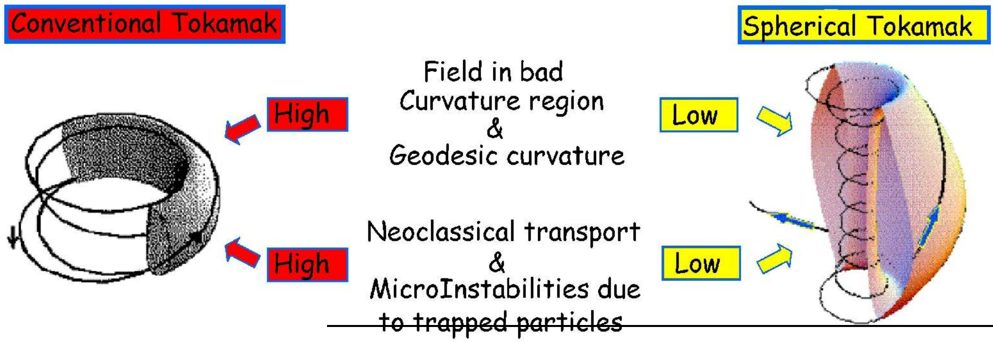


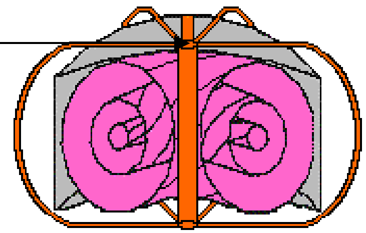
Compact tori in the world :
NSTX (US), START(UK), Globus (Rus.), TS 3 (Japan),..

PROTO-SPHERA

The Spherical Tokamak (ST) is a magnetic confinement configuration that provides plasma β much higher than conventional tokamaks

The advantage stems from the nearly-geodesic lines of force in ST



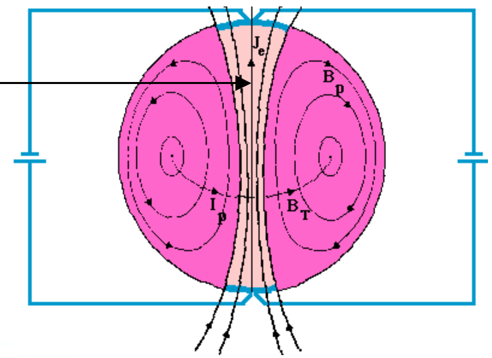


Severe space constraints for material center column:

- No space for central solenoid
- No neutrons shield (no superconductor/high dissipation)

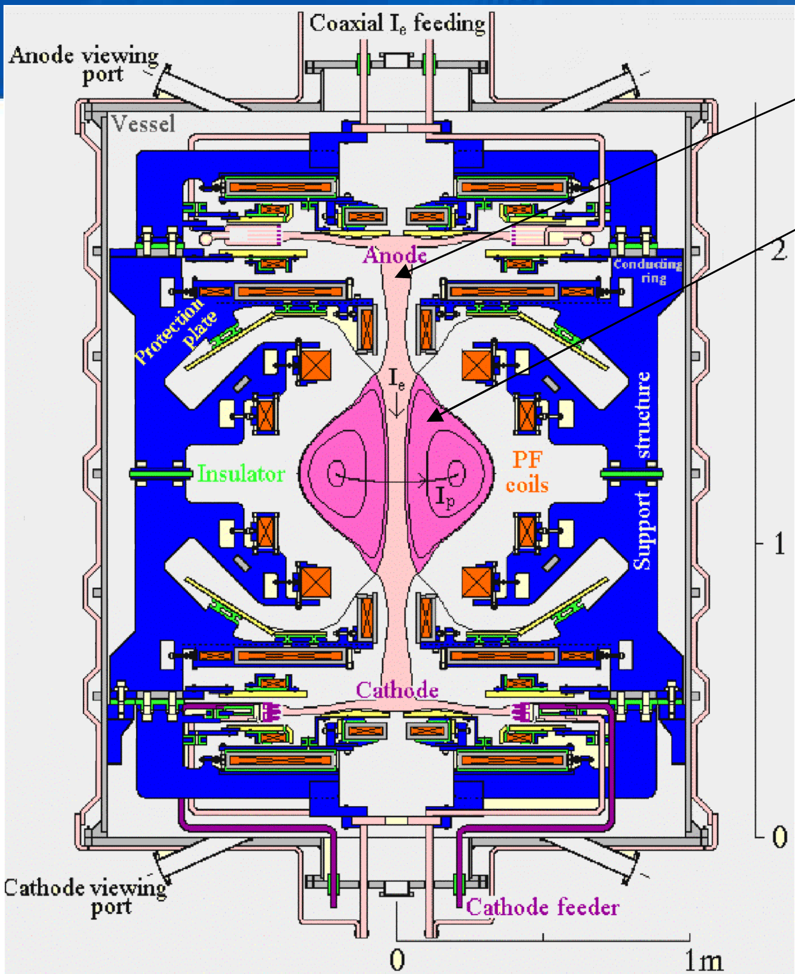
PROTO-SPHERA[1]: a **plasma central column** (**ST**-**PCC**)[2]

Requires electrodes and plasma self-organization!



[1] F. Alladio, et al., **Nucl.Fus. 46** S613 (2006)

[2] S.C. Hsu & X.Z. Tang, **J.Fus.En.** (2006) **Complementary proposal-LANL**



PROTO-SPHERA

PCC - ScrewPinchPlasma (SP), electrode-driven

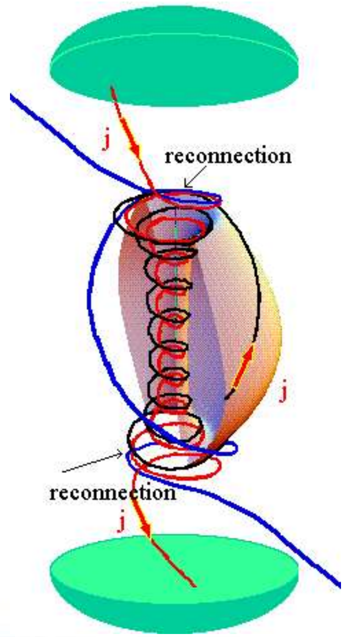
Elongated ST ($k \geq 2.3$), aspect ratio A~1.2

to get an MHD safety factor q_0~ 1, q_{edge}~ 3

PCC electrode current:	I_e=60 kA
ST toroidal current (limited by ideal MHD):	I_{ST}=240 kA
ST diameter	R_{sph}=0.7 m

Driven relaxation of Plasma Central Column forms & sustains the ST

"Smoke-ring like self-organization": sustainement by Helicity Injection



- electrode plasma - open field lines - has **j** || **B**
- the open field lines also wind toroidally →
- magnetic reconnections convert open **j**,**B** lines into closed **j**,**B** lines winding on a spherical torus

Proto-Sphera experiment: aim

•The PROTO-SPHERA experiment aims at sustaining a flux-core-spheromak,while exploring the configuration space that connects spherical tori and spheromaks.
•The compression of the central pinch, while decreasing the total longitudinal pinch current, would lead, if successful, to the formation of a field reversed configuration.
• PROTO-SPHERA could also explore a new technique for setting up an FRC.
• PROTO-SPHERA could also aim at exploring the novel Chandrasekhar-Kendall-Furth configuration consisting of a spherical torus enclosed within a spheromak.

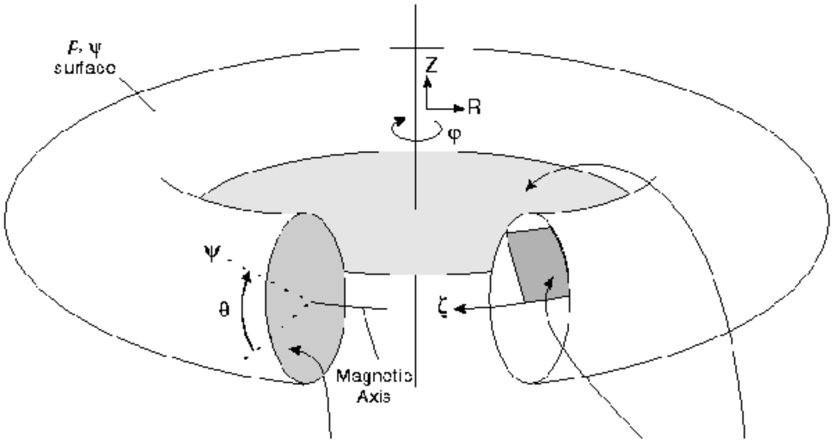
Objectives of the numerical modeling

• validate different scenarii of plasma formation
• help to conceive realistic CKF configuration
• study the ideal MHD stability of configurations

Outline

- MHD equilibrium equations
- Boozer coordinates
- Ideal MHD stability: numerical modelling
- CKF configurations
- Proto-SPHERA
- Future

Derivation of Grad-Shafranov equilibrium MHD equations



Magnetic field obey to equilibrium MHD equations:

$$\nabla . \mathbf{B} = 0,$$
$$\nabla \wedge \mathbf{B} = \mu_0 \mathbf{J},$$
$$\nabla p = \mathbf{J} \wedge \mathbf{B},$$

in axisymmetric and using the potential vector :

$$\mathbf{B} = \nabla \Psi \wedge \mathbf{e}_\phi + B_\phi \mathbf{e}_\phi$$

Derivation of Grad-Shafranov equations

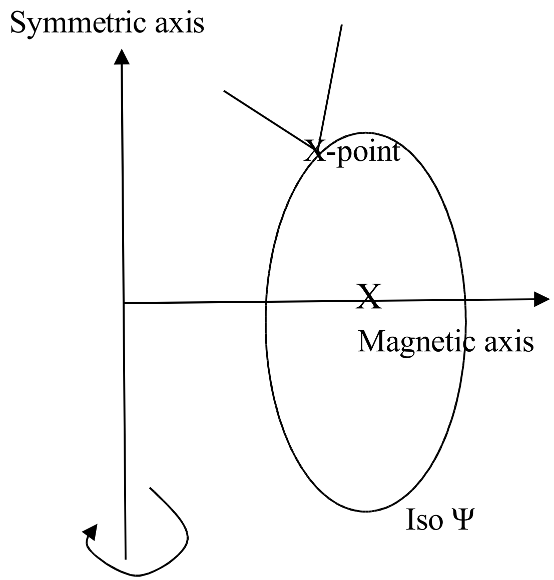
iso- pressure are orthogonal to field lines, then :

$$\nabla(rB_\phi).\mathbf{B} = -\mu_0 \nabla p.\mathbf{B} = 0$$

$$P = P(\Psi), rB_\phi = (rB_\phi)(\Psi)$$

Ψ is the poloidal magnetic flux

using pressure equilibrium relation :



$$\nabla p = \frac{j_\phi}{r}\nabla\Psi - \frac{B_\phi}{r\mu_0}\nabla rB_\phi$$

$$j_\phi = rP'(\Psi) + \frac{1}{r\mu_0} rB_\phi (RB_\phi)'(\Psi)$$

rearranging terms we arrive at the **Grad-Shafranov** equation:

$$-\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\Psi}{\partial r} - \frac{\partial}{\partial z}\frac{1}{r}\frac{\partial\Psi}{\partial z} = \mu_0 P'(\Psi) + \frac{1}{\mu_0 r} B_\phi (rB_\phi)'(\Psi)$$

1. $P(\Psi), rB_\varphi(\Psi)$ are given functions of Ψ.
2. β poloidal and total toroidal current are given
3. Plasma boundary (ψ=cste) pass trough X-point (B poloidal=0)

Nonlinear free-boundary problem

Numerical methods for solving GS equation

Some families of numerical methods:

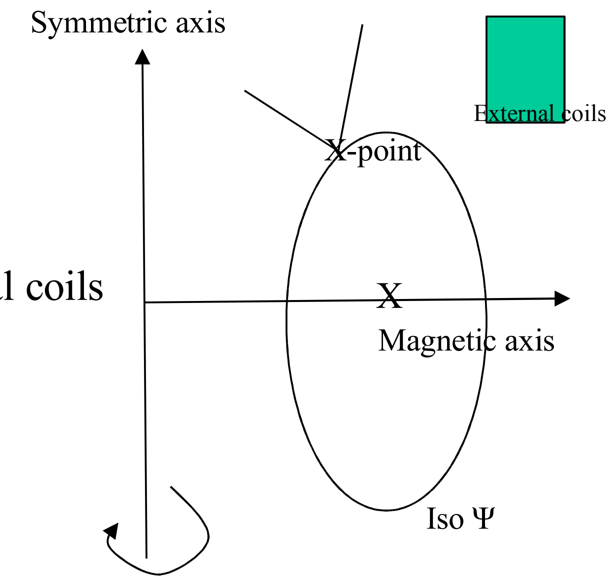
2. Iterative reconstruction of metric defined by magnetic surfaces:
   - require a topology of plasma described by magnetic surfaces
3. FEM or FD methods need to solve a linear system into a nonlinear loop (J. Blum)
   - unstructured meshes, boundary conditions
4. Semi-analytical expansion : spherical or toroidal coordinates
   - efficient for quasi-spherical (toroidal) configurations (Alladio, Chrisanti)
5. *Green functions (Kerner)*
   - *accurate, require CPU and large memory, need optimization*

see Takeda's review (JCP 84)

Green's function
Int. J. of Comp. Eng. Sci., (2004)

Formulation of GS by integral representation :

$$\boxed{\psi = G * j_t(\psi) + \psi_S}$$



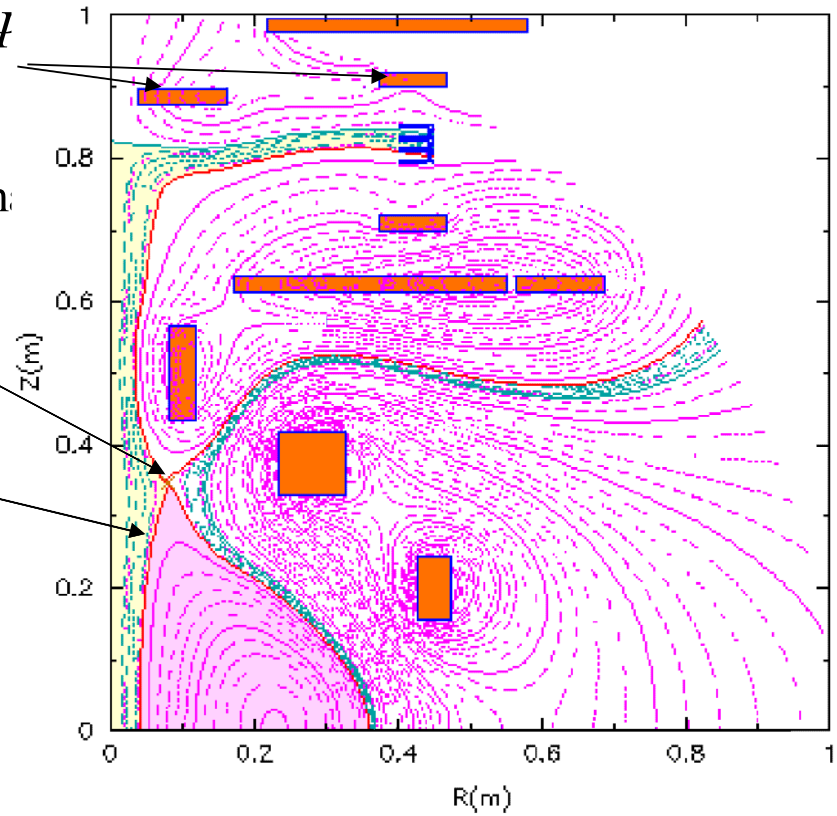
Ψ_S is the poloidal magnetic flux created by magnetic external coils
G is the Green function of GS operator

Ingredients :

- Galerkin method with constant or Q1 elements on $N=Nr_x Nz$ cells recovering a rectangular estimation of the plasma: $$\boxed{\psi_h = M^{-1} K j_t(\psi_h) + \psi_{S,h}}$$
- Over relaxation scheme : $$\boxed{\psi^{n+1}{}_h = \varpi \psi^n{}_h + (1-\varpi) M^{-1} K j_t(\psi^n{}_h) + \psi_{S,h}}$$
- Expansive computational cost of *K(jt(Ψ)),* FFT accelerate the computation: $\boxed{2\log(N) N^{3/2}}$

Green's function : implementation

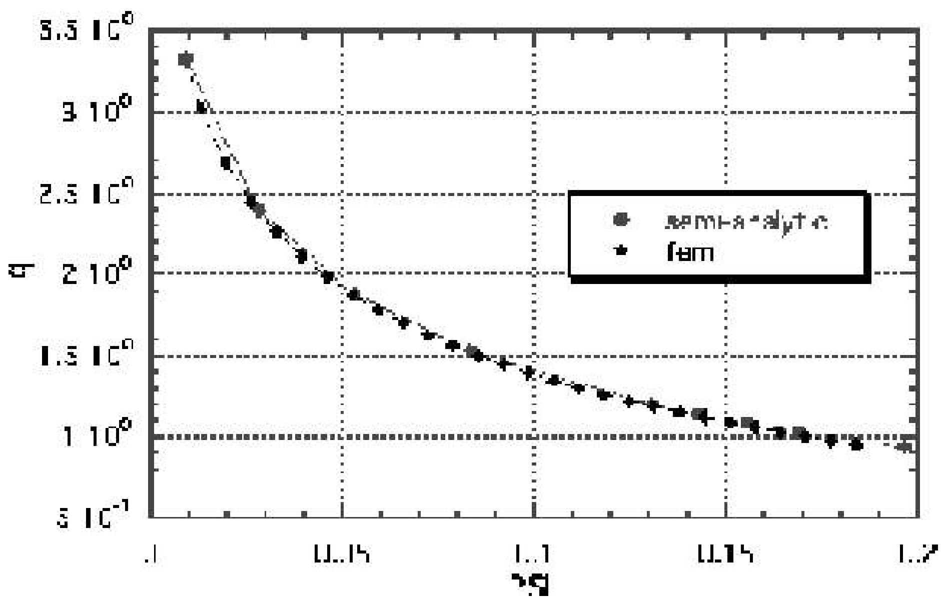
1. Initialization with calculation of matrix K and Ψ
2. From Ψ_n compute poloidal field $\mathbf{B}_n$
3. Search the point of the mesh (in a given zone) th realizes the minimum of the modulus of $\mathbf{B}_n$
4. Compute the plasma shape
5. Compute the toroidal current density and normalize to respect constraints.
6. Apply FFT matrix product K
7. Apply over-relaxation algorithm
8. Compute residual and stop the iterations if necessary



Green's function : numerical results , conventional tokamak

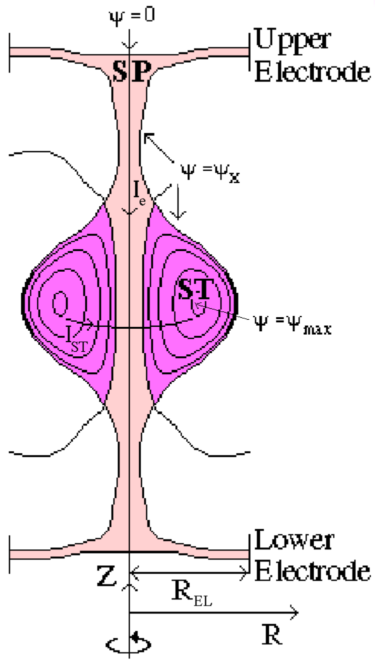
Comparisons with semi-analytical methods:
Tokamak with large aspect-ratio (~6),
results deal for a shot characterized by the following plasma's parameters :Ip=250kA , β=1.5
Q: safety factor : number of turns in the toroidal/poloidal

	GF	Semi-analytical
Triangularity	0.69	0.68
Aspect ratio	6.3	6.28
Volume	0.9	0.93
R axis	0.6	0.6
X-point	0.304-0.93	0.303-0.93



safety factor

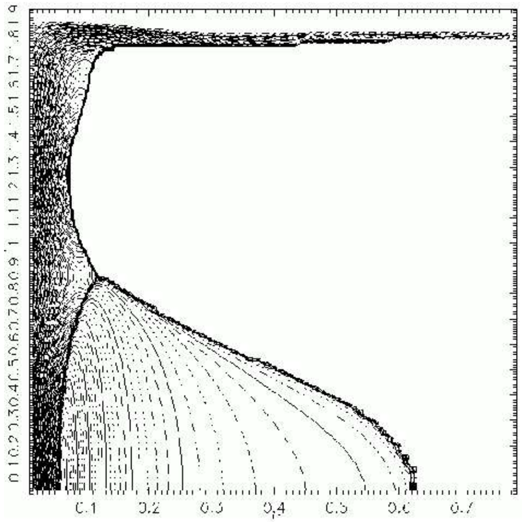
Specific conditions for Proto- Sphera experiment



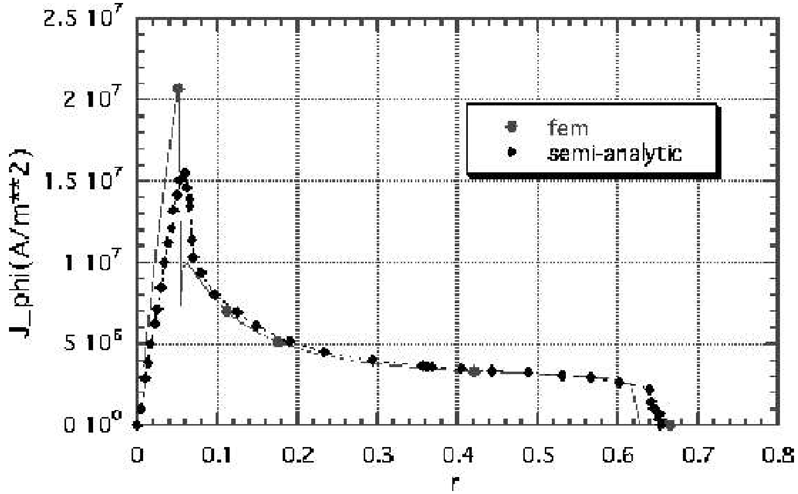
1. The SP is a homogeneous force-free plasma : p=constant
2. The ST-SP interface is defined by the separatrix passing through X-point
3. Inside ST: $P(\Psi) = \alpha + \beta(\Psi - \Psi_X)^{\chi}$, $rB_{\phi}(\Psi) = \alpha' + \beta'(\Psi - \Psi_X)^{\chi'}$
4. Total toroidal current flowing inside the ST and beta poloidal are inputs

Green's function : numerical results , Sphera + Pinch

Predictive calculations for Sphera: Ip=2.7MA,β=0.07,Ie=0.38MA, Nr*Nz=100*128, CPU time=2-3mn



toroidal current



Triangularity	Aspect ratio	Volume	R-axis	X-point
0.73	1.20	1.08	0.396	0.118-0.868

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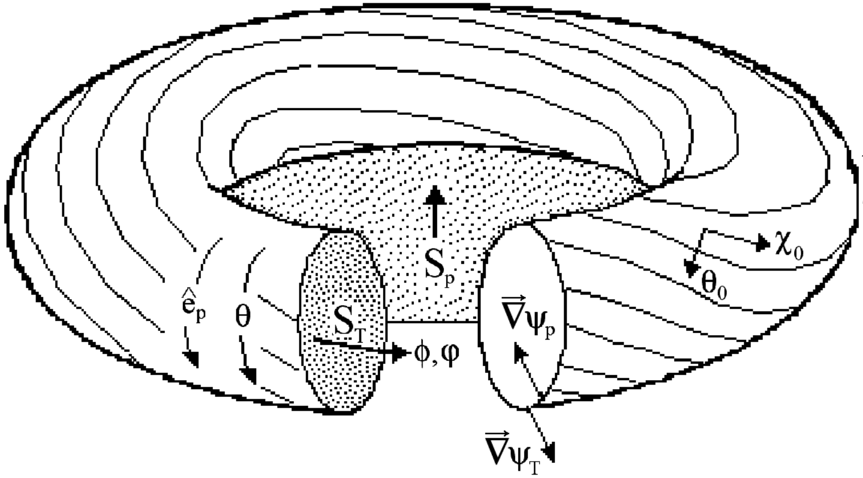
Boozer coordinates

1. Axisymmetric MHD equilibrium calculations are usually carried out in terms of the poloidal flux or toroidal flux enclosed within each magnetic surface:

[illegible] [illegible]

[illegible]



the normalized toroidal current: [illegible]

The normalized poloidal currrent [illegible]

The magnetic field is expressed in terms of the contravariant basis vectors as

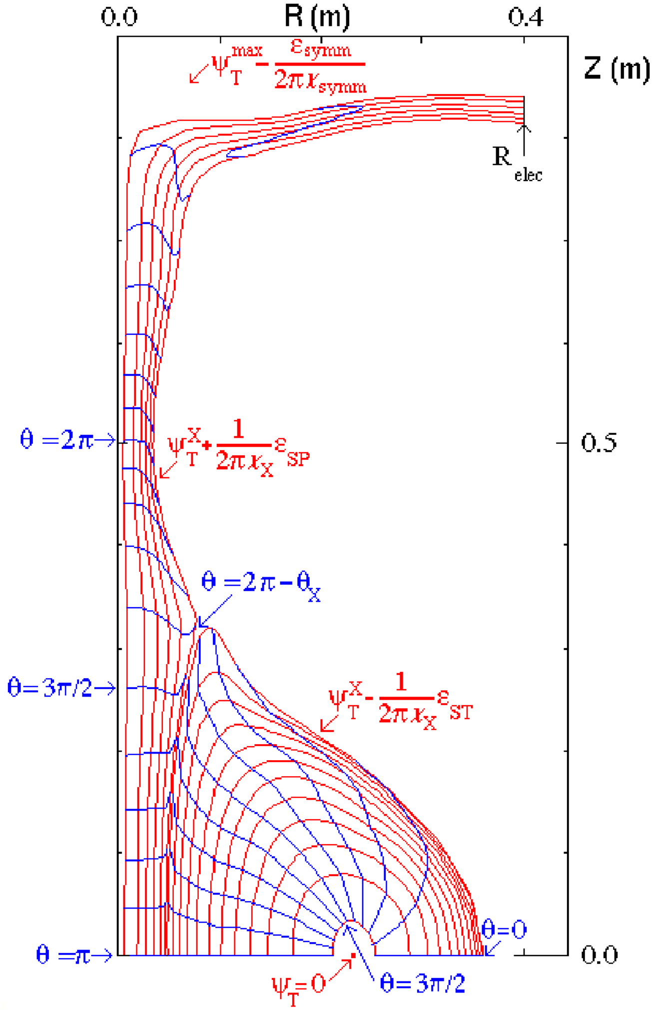
[illegible]

Or of covariant basis vectors or in terms as :

[illegible]

In the plane θ,φ field lines are straight

Boozer coordinates : extension to proto-sphera configurations
(Phys.Plasmas,2005)



Open field lines in the screw pinch: $\psi_X < \psi < \psi_{max}$

Closed field lines in the spherical torus: $0 < \psi < \psi_X$

Boozer coordinates are extended to the Pinch in the following way :

3. Continuity at the separatrix of the metric
4. Continuity of toroidal cuurent

It can be shown the following expression of θ,φ

Divergences of the metric at the symmetry axis, at the magnetic axis **and** at the separatrix have to be carefully analyzed.

Linear MHD stability equations : derivation

Linearization of MHD equations around the equilibrium state $P_0, \mathbf{J}_0, \mathbf{B}_0$

$$\rho_0 \frac{\partial \delta \mathbf{v}}{\partial t} + \nabla(\delta p) = (\nabla \wedge \delta \mathbf{B}) \wedge \delta \mathbf{B} + \mathbf{J}_0 \wedge \delta \mathbf{B}$$

$$\frac{\partial \nabla \delta p}{\partial t} + \mathbf{v}\nabla(p_0) + \gamma p_0 \nabla.\mathbf{v} = 0$$

$$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \wedge (\mathbf{v} \wedge \delta \mathbf{B})$$

Introducing $\mathbf{v} = \frac{\partial \xi}{\partial t}$, we obtain $\rho_0 \frac{\partial^2 \xi}{\partial t^2} = R(\xi), \xi = \xi \exp(i\varpi t)$

The variational principle (Lundquist 51, Bernstein 58) states that ξ makes stationary the Rayleigh quotient :

$$-\varpi^2 = \frac{\delta W_p}{\delta W_k}, \delta W_p = \frac{1}{2}\int \frac{|\mathbf{C}|^2}{\mu_0} + \Gamma_p (\nabla.\xi)^2 - D(\xi.\nabla\psi)^2$$

$$\delta W_k = \frac{1}{2}\int \rho_0 |\xi|^2$$

$$C = \nabla \wedge (\xi \wedge \mathbf{B}_0)$$

$$D = 2\frac{\mathbf{J}_0 \wedge \nabla\psi}{|\nabla\psi|^2} B_0.\nabla \frac{\nabla\psi}{|\nabla\psi|^2}$$

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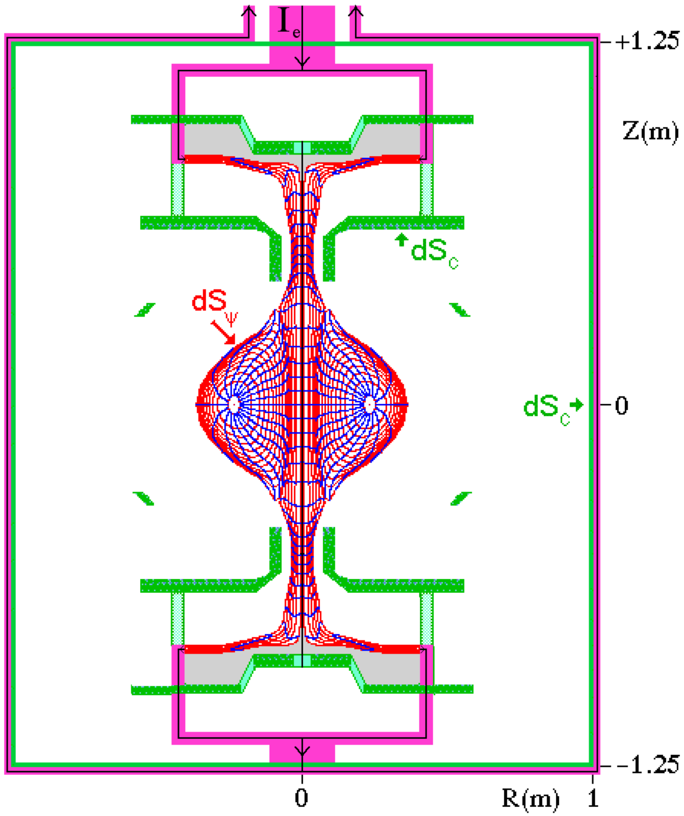
Ideal MHD stability: perturbed vaccum energy

The normal-mode equation is modified by the free-boundary vacuum magnetic energy through an additional term present only upon the last point :

$$\delta W_p\left(\vec{\xi}^*,\vec{\xi}\right)+\delta W_v\left(\xi^{\psi*}\left(\psi_T^{N_\psi}\right)\xi^{\psi}\left(\psi_T^{N_\psi}\right)\right)=\omega^2\delta W_k\left(\vec{\xi}^*,\vec{\xi}\right)$$



$$\delta W_v=\frac{\mu_0}{2}\oint\!\!\oint_{S_\psi}\Phi\vec{\nabla}\Phi\cdot d\vec{\mathbf{S}}_\psi$$

$$\Delta\Phi_n=0$$

$$\frac{\partial\Phi_n}{\partial n}=0, S_{vessel}$$

$$\frac{\partial\Phi_n}{\partial n}=\mathbf{B}.n, S_{plasma}$$

Continuity of the normal component of **B** :

$$\left.\frac{\partial\Phi}{\partial n}\right|_{S_\psi}=\frac{1}{\mu_0\sqrt{g}\left|\vec{\nabla}\psi_T\right|}\left[\iota\frac{\partial\xi^\psi}{\partial\theta}+\frac{\partial\xi^\psi}{\partial\phi}\right]_{\psi_T^{edge}}$$

Linear MHD Stability analysis

The ideal MHD stability of plasma equilibria is usually undertaken expanding perturbed plasma displacement in magnetic coordinates and then solving the normal mode equation.

The actual vector components of the displacement are most easily expressed t hrough the three (normal, binormal and parallel) vectors:

$$\vec{\xi} = \left[\frac{\vec{\xi}\cdot\nabla\psi}{|\nabla\psi|^2}\right]\nabla\psi + \left[\frac{\vec{\xi}\cdot(\vec{B}\times\nabla\psi)}{|\nabla\psi\times\vec{B}|^2}\right](\vec{B}\times\nabla\psi) + \left[\frac{\vec{\xi}\cdot\vec{B}}{B^2}\right]\vec{B}$$

the key argument for removing the ambiguity is that in the ideal MHD stability problem the normal plasma displacement $\vec{\xi}\cdot\nabla\psi$ being the displacement vector away from the equilibrium, must be continuous at the ST-SP interface

Stability analysis

Phys.Plasmas(2006)

The perturbed displacement variables away from the equilibrium are expanded in a trigonometric Fourier series of modes; each mode is labeled by an index l, which corresponds to a poloidal number m and a (fixed as separable) toroidal number n.

Up down symmetry is supposed

Ideal MHD stability code requires to treat configuration with:

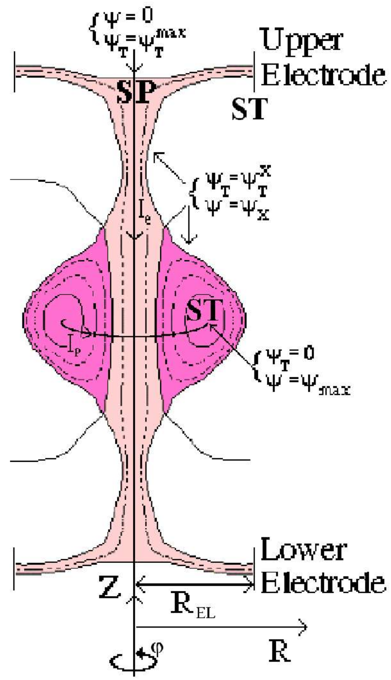
2. open and close field lines

3. separatrices between ST and SP plasma.

4. Vaccum between plasma and camera vessel

Constraints :

- the metric coefficients near the separatrix exhibits singular behavior.
- Symmetric axis is contained in the plasma => singularity of the metric when R->0



Ideal MHD stability: solver

A remarkable property of this representation is that neither the parallel component appear in the derivatives terms.

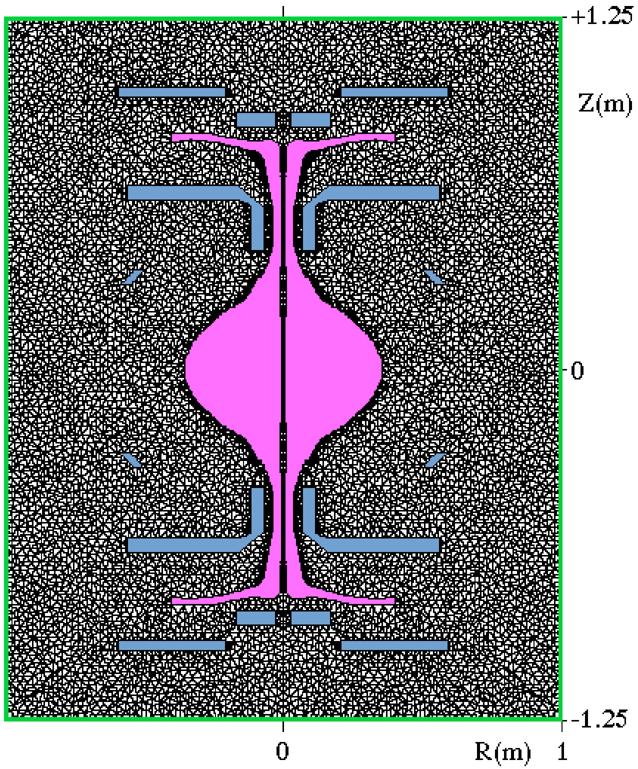
$$
\frac{\partial}{\partial \psi_T}\left(\frac{2}{4\pi^2}\delta W_p^i\right)\Bigg|_{\psi_T^e} = \frac{1}{2\pi\mu_0}\sum_{\ell,k}\delta_{n_\ell n_k}
$$

$$
\left\{
\begin{aligned}
&\left(\xi_\ell\left(\psi_T^e\right)\right)\left[\left(\iota m_\ell - n_\ell\right)\left(\iota m_k - n_k\right)\int_0^{2\pi} d\theta' \frac{g_{\psi\psi}}{\sqrt{g}}\, e^{i\left(m_k - m_\ell\right)\theta'}\right]\left(\xi_k\left(\psi_T^e\right)\right) \\
&-\left(\frac{\partial \xi_\ell}{\partial \psi_T}\right)\Bigg|_{\psi_T^e}\left[\left(\iota m_k - n_k\right)\int_0^{2\pi} d\theta' i\beta_* e^{i\left(m_k - m_\ell\right)\theta'}\right]\left(\xi_k\left(\psi_T^e\right)\right) \\
&+\left(\xi_\ell\left(\psi_T^e\right)\right)\left[\left(\iota m_\ell - n_\ell\right)\int_0^{2\pi} d\theta' i\beta_* e^{i\left(m_k - m_\ell\right)\theta'}\right]\left(\frac{\partial \xi_k}{\partial \psi_T}\right)\Bigg|_{\psi_T^e} \\
&+\left(\xi_\ell\left(\psi_T^e\right)\right)\left[\left(\iota m_\ell - n_\ell\right)\left(m_k\right)\int_0^{2\pi} d\theta' i\beta_* e^{i\left(m_k - m_\ell\right)\theta'} - \left(\iota m_\ell - n_\ell\right)\left(\iota m_k - n_k\right)\int_0^{2\pi} d\theta' i \frac{g_{\psi\theta}}{\sqrt{g}}\, e^{i\left(m_k - m_\ell\right)\theta'}\right]\left(\eta_k\left(\psi_T^e\right)\right) \\
&-\left(\eta_\ell\left(\psi_T^e\right)\right)\left[\left(\iota m_k - n_k\right)\left(m_\ell\right)\int_0^{2\pi} d\theta' i\beta_* e^{i\left(m_k - m_\ell\right)\theta'} - \left(\iota m_k - n_k\right)\left(\iota m_\ell - n_\ell\right)\int_0^{2\pi} d\theta' i \frac{g_{\psi\theta}}{\sqrt{g}}\, e^{i\left(m_k - m_\ell\right)\theta'}\right]\left(\xi_k\left(\psi_T^e\right)\right) \\
&-\left(\xi_\ell\left(\psi_T^e\right)\right)\left[\frac{d\iota}{d\psi_T}\left\{\left(\iota m_k - n_k\right) - \left(\iota m_\ell - n_\ell\right)\right\}\int_0^{2\pi} d\theta' i \frac{g_{\psi\theta}}{\sqrt{g}}\, e^{i\left(m_k - m_\ell\right)\theta'}\right]\left(\xi_k\left(\psi_T^e\right)\right)
\end{aligned}
\right\}
$$

The radial behavior of the displacement variables is approximated by an hybrid Finite Element Method (Rappaz Gruber (85)).

For ξ the hat functions are used. For η and μ the piecewise constant functions are used.

Ideal MHD stability: perturbed vaccum energy



Continuity of the normal component of the perturbated magnetic field **B** :

$$\left.\frac{\partial \Phi}{\partial n}\right|_{S_\psi} = \frac{1}{\mu_0 \sqrt{g}\,\left|\vec{\nabla}\psi_T\right|}\left[\iota \frac{\partial \xi^\psi}{\partial \theta} + \frac{\partial \xi^\psi}{\partial \phi}\right]_{\psi_T^{edge}}$$

For each toroidal mode

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Phi_n}{\partial R}\right) + \frac{\partial^2 \Phi_n}{\partial Z^2} - \frac{n^2}{R^2}\Phi_n = 0$$

$$\frac{\partial \Phi_n}{\partial n} = 0, S_{vessel}$$

$$\frac{\partial \Phi_n}{\partial n} = \mathbf{B}.n, S_{plasma}$$

A 2D finite element method is used that can fit shape of plasma and of the surrounding conductors.

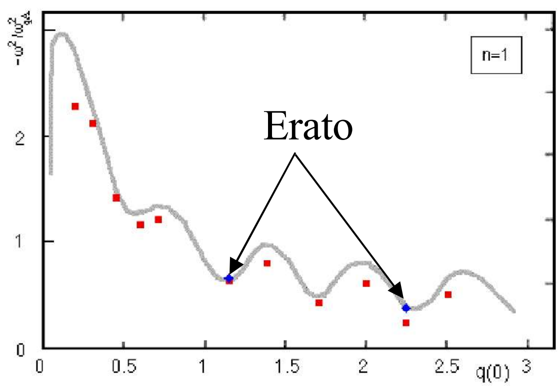
Synthesis

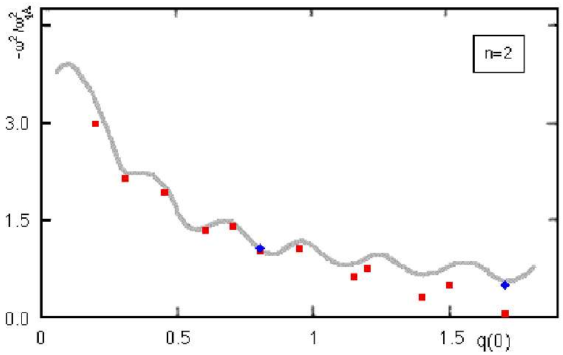
1. Solve the equilibrium MHD equations
2. Calculate the Boozer coordinates
3. Expand in trigonometric series the displacement
4. Discretize the radial variable by FEM
5. Compute the Vaccum contribution to potential energy by a 2D FEM
6. Compute the eigenvalues (LAPACK)

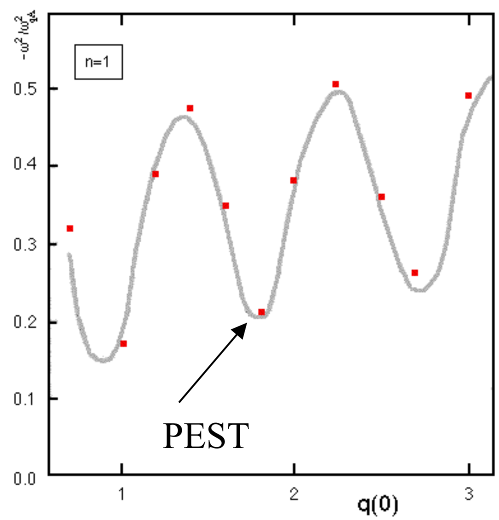
Ideal MHD stability: comparisons with Solovev's testcase

The axisymmetric analytical Solovev MHD equilibrium solutions
have since long time been chosen as a benchmark
for all ideal MHD stability codes. The reason is that they permit a configurational variety of "approximate inverse
aspect ratio" , cross-section elongation and safety factor
at the magnetic axis.

$$\psi = \frac{\pi B_0}{q_0}\left\{\varepsilon^2 \kappa R_0^2 - \frac{1}{kR_0^2}\left[R^2 Z^2 + \frac{\kappa^2}{4}\left(R^2 - R_0^2\right)^2\right]\right\} \qquad \frac{a}{R} = \frac{2}{\left(\sqrt{1+2\varepsilon} + \sqrt{1-2\varepsilon}\right)^2}$$





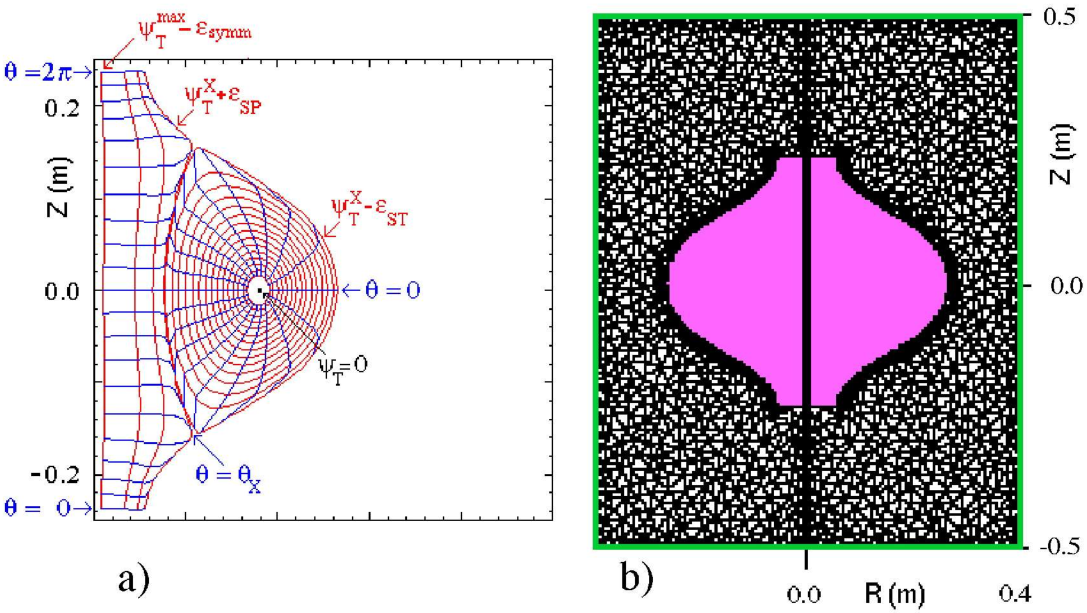


Normalization with the poloidal alfven rate $\omega_{qA}^2 = B_0^2 / \mu_0 \rho_0 q_{edge}^2 R_0^2$

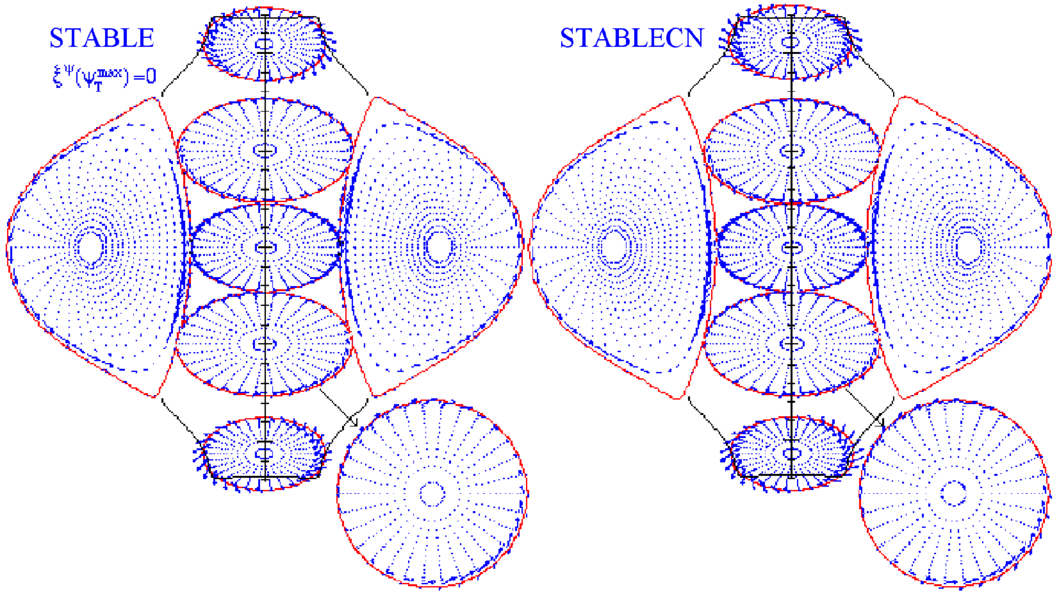
Ideal MHD stability: flux –core tokamak

Tokyo University Spherical Torus No. 3 (TS-3) flux-core-spheromak experiment, No-singular X-point,). The first equilibrium of TS-3 considered for the ideal MHD stability calculation has a longitudinal pinch current Ie=40 kA, a toroidal ST current I_{ST}=50 kA and a volume averaged βinside the ST 12.0% .

The positive eigenvalue ω_2 provided by the solver is $\omega 2/\ \omega 2_{Alfven}$ =+1.33•10-7



Ideal MHD stability: flux –core tokamak

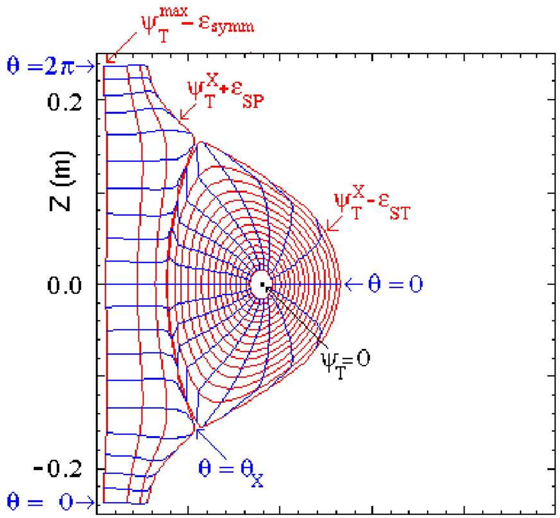


Comparisons with different formulations of ξ on the symmetric axis.
The solver calculate an oscillating motion around the q=1 resonance, which is present inside the ST as well as inside the SP.
Two eigenfunctions are similar when the displacement plots are shown upon the poloidal cross-section, inside both the ST and the SP.

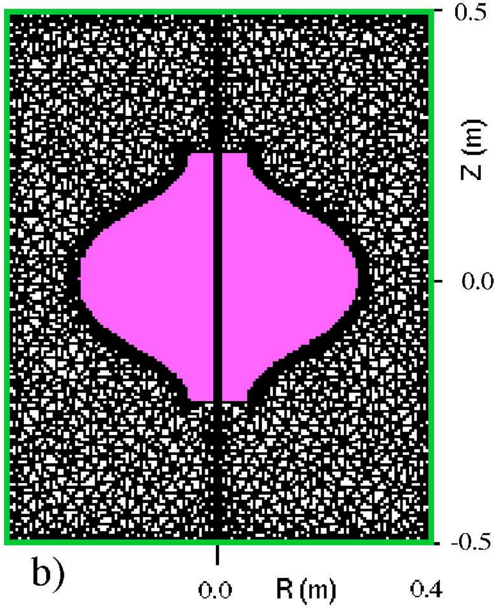
Ideal MHD stability: flux –core tokamak

Tokyo University Spherical Torus No. 3 (TS-3) flux-core-spheromak experiment, No-singular X-point,). The second equilibrium of TS-3 considered for the ideal MHD stability calculation has a longitudinal pinch current Ie=40 kA, a toroidal ST current I_{ST}=100 kA and a volume averaged βinside the ST 14.0% .

The negative eigenvalue ω_2 provided by the solver is $\omega 2/\ \omega 2_{Alfven}$ =-3.92

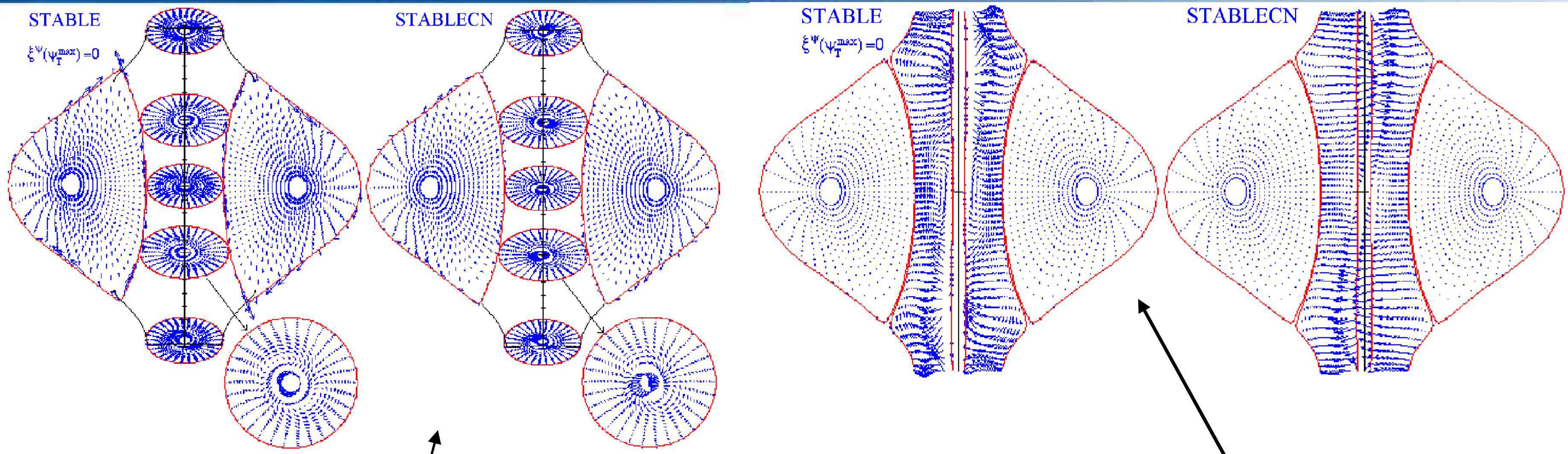


a)



b)

Ideal MHD stability: flux –core tokamak



upon a poloidal cross-section inside both the ST and the SP.

upon some horizontal plane cross-sections inside the screw pinch.

The ideal MHD stability calculations broadly agree **with the experimental results**, confirming the experimental observation that the TS-3 flux-core-spheromak did not achieve a toroidal ST current much higher than IST=50 kA and that its ST was limited to an aspect ratio not lower than A=1.6.

Ideal MHD Current Limits in Proto-Sphera

- Three cases of PROTO-SPHERA have been selected with growing ratio between the toroidal ST and the longitudinal SP current, respectively Ip/Ie = 2, 3 and 7/2 and corresponding to some steps of the Protosphera scenario.
- All of these cases have a volume averaged beta of the spherical torus β $=2\mu_0 \langle p \rangle_{vol} / \langle B^2_{T0} \rangle_{vol}$ ~20%.

4. **T5 (with Ie=60 kA, Ip=120 kA and β =23.1%) ->** $\omega^2/\omega^2_{Alfven}$ **=+2.3110-6 : It is a stable oscillating motion around the q=1 and q=2 resonances inside the ST and around the q=3 resonance inside the SP.**
5. **T6 : (with Ie=60 kA, Ip=180 kA and β =22.4%), a globally unstable mode, which takes the character of a kink mode inside the SP and of a tilt mode inside the ST.** $\omega^2/\omega^2_{Alfven}$ **=-4.89**
6. **T7 (with Ie=60 kA, Ip=210 kA and β =20.6%),** $\omega^2/\omega^2_{Alfven}$ =**-3.2210-3 :two unstable eigenfunctions represents a globally unstable mode, which takes the character of a kink mode inside the SP and of a tilt mode inside the ST.**

Proto-Sphera : stability results

n=1

- **n=1,2,3 investigated for all equilibria**
- **m=[-5:10]**
- **Main constraint I_{ST}/I_e ,**

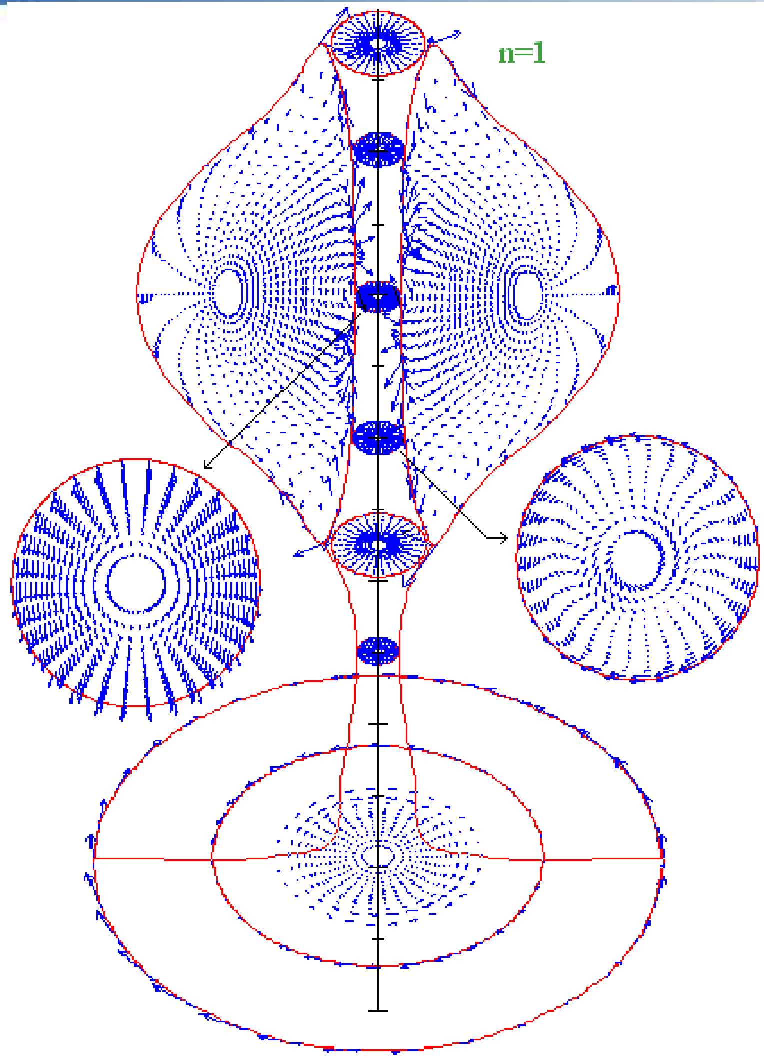
- **β = 21÷26% , I_{ST}/I_e=0.5 - 1**
- **β = 14÷15% , I_{ST}/I_e=4 B^2_{T0}**

With $\beta_{T0}=2\mu_0<p>_{vol}/< \quad >_{vol}$

- **β_{T0}=28÷29% , I_{ST}/I_e=0.5 - 1**
- **β_{T0}=72÷84% , I_{ST}/I_e= 4**

The dominant instabilities are :

- **up to $I_{ST}/I_e \approx 3$ The Spherical Torus instabilities**
- **I_{ST}/I_e> 3 the Screw Pinch kink instabilities**

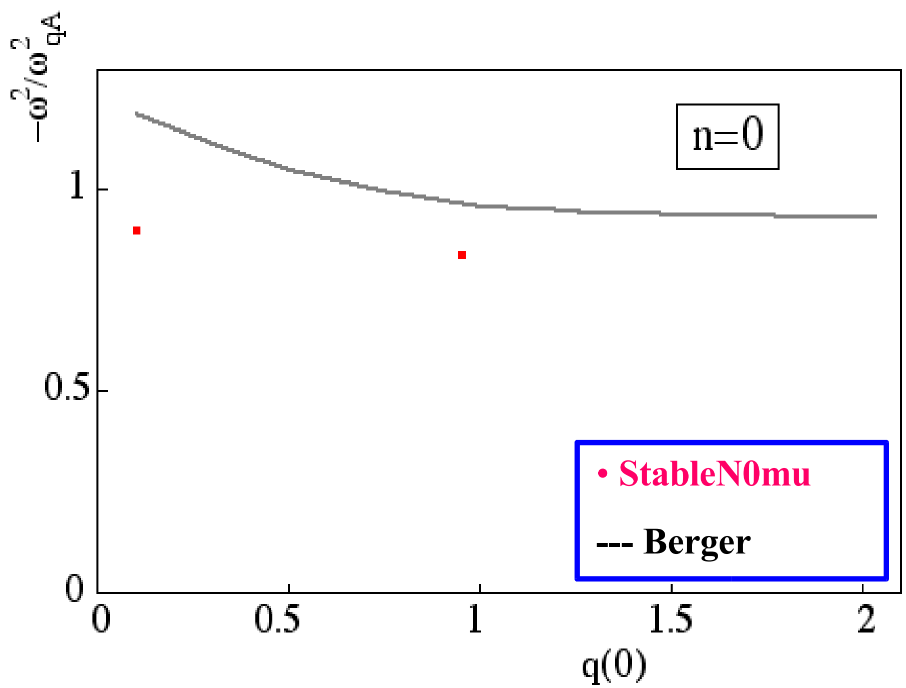


I_{ST}/I_e = 5 and $\beta \approx 15\%$

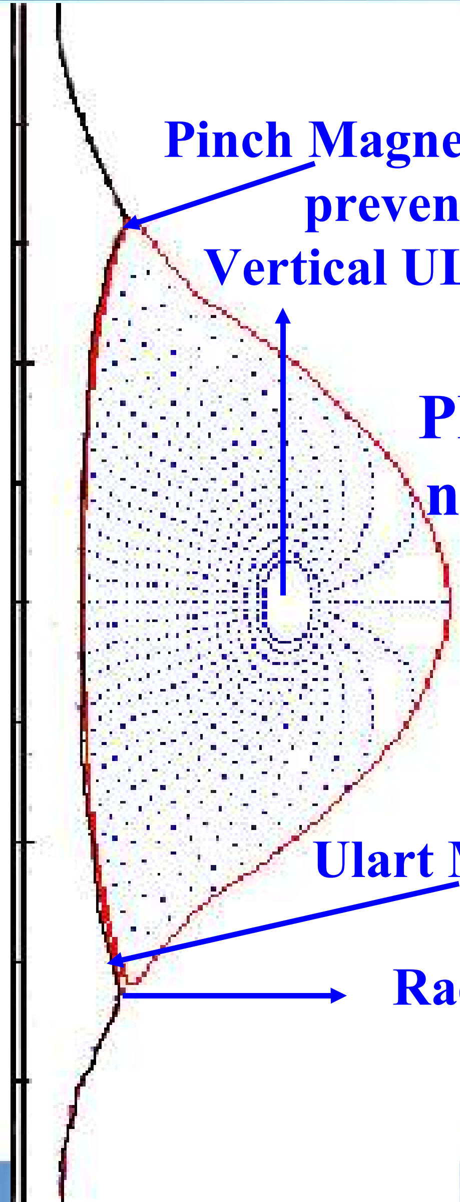
Unstable

ENEA
ONERA

Proto-Sphera stability results



Comparison of the instabilities growth rate for free boundary codes Mode numbers n=0, m=[-15,+15].



PROTO-SPHERA n=0 stable for any scenarios up to R/a = 1.2

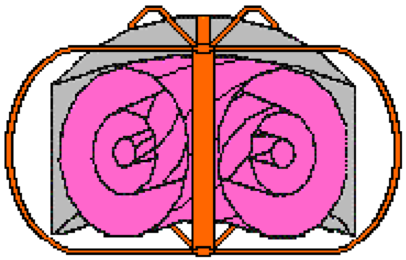
$\kappa < 4$

Linear MHD Stability analysis

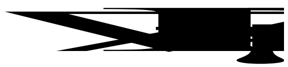
1. The use of magnetic coordinates, defines the simplest approach to the numerical study of the **ideal MHD stability of magnetoplasma equilibria**, but the magnetic coordinates become **singular** in presence of magnetic separatrices (particularly relevant in the stability analysis of a simply connected axisymmetric plasmas (Flux-Core-Spheromak or Chandrasekhar-Kendall-Furth configurations)).
2. The approach taken is that of maintaining the 1-dimensional radial Finite Elements, while using an asymptotic analysis of the perturbed plasma displacement near the separatrices.
3. The permissible asymptotic limits for the perturbed displacement are derived in Boozer magnetic coordinates.
4. Numerical results confirm that the formation sequence of PROTO-SPHERA is ideal MHD stable, provided that it occurs at $\beta<20\%$.

Chandrasekhar-Kendall-Furth force free fields

The problem of finding a stable force-free axisymmetric magnetic field, by minimizing the magnetic energy , has received as a tentative answer a singular domain: an extreme "apple", in which north and south pole are pressed and joined together (spheroidal surface enclosing a double connected volume). So the spheromak has the right topology, but needs to be embedded in a stabilizing plasma, which provides to the spheromak an inner flux-core.



A **simply** connected magnetic confinement scheme can be obtained superposing two axisymmetric homogeneous force-free fields, each with



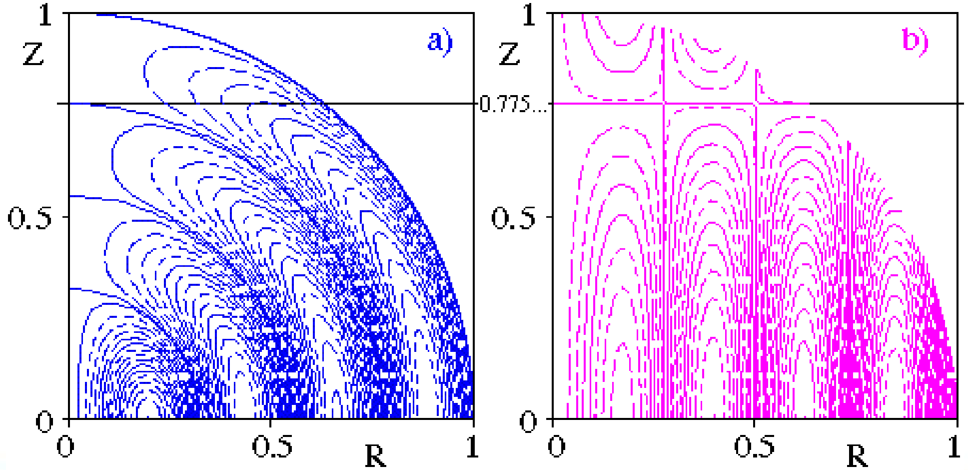
Chandrasekhar-Kendall and Furth force free fields

The first is the Chandrasekhar-Kendall force-free field of order-1 which in spherical coordinates admits the poloidal flux:

[redacted]

The second homogeneous force-free field is the Furth square-toroid s (embedding stabilizing plasma), whose poloidal flux can be written as:

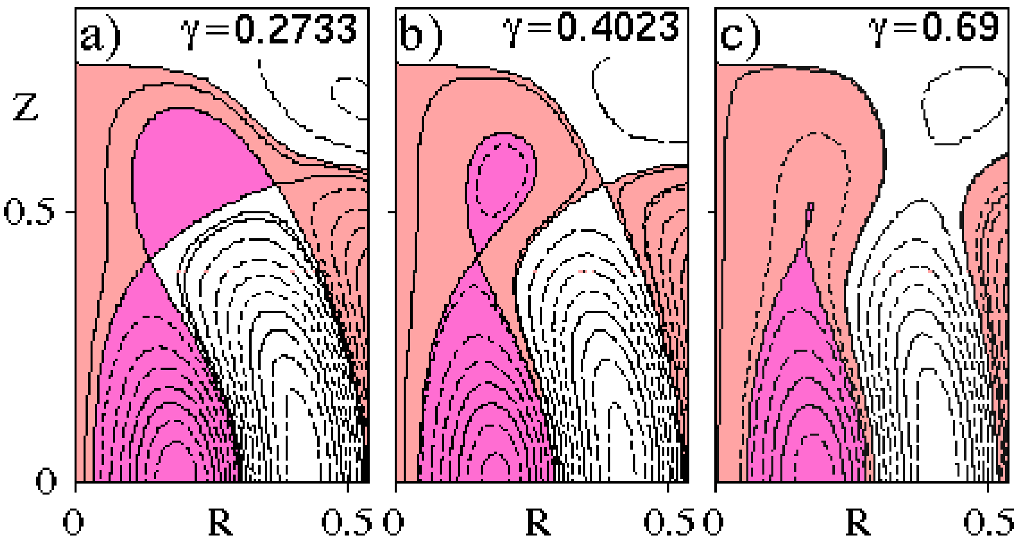
[redacted]



Chandrasekhar-Kendall-Furth force free fields

The superposition of the two force-free fields is written as:

[redacted]

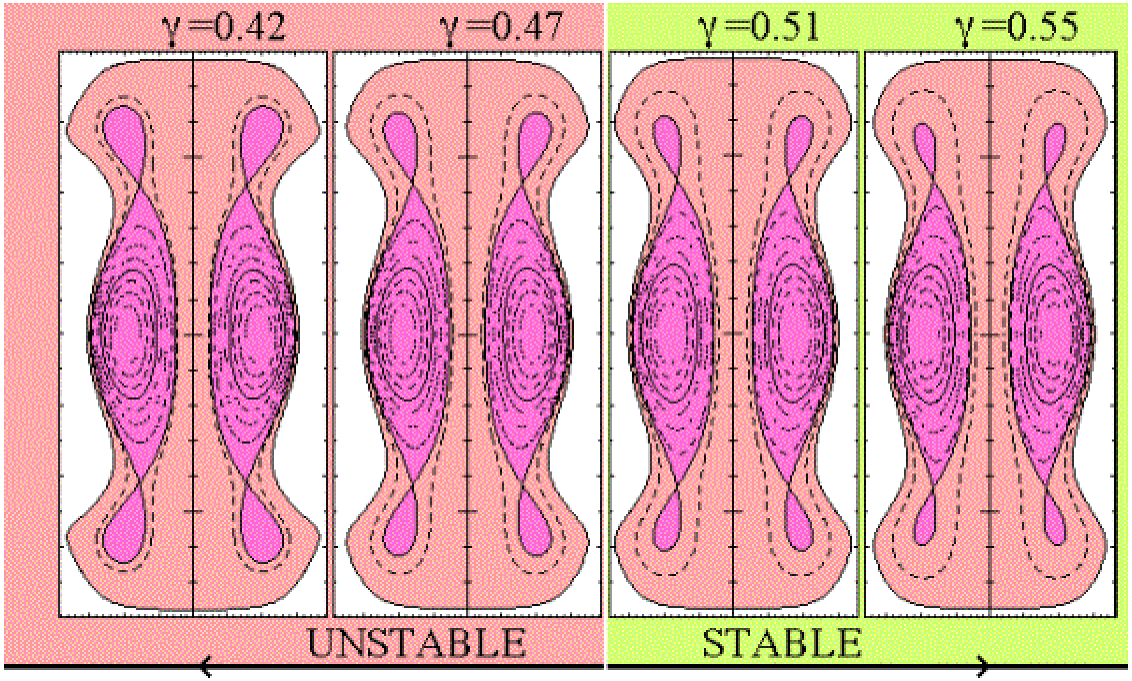


The main torus (ST) has a safety factor q with value ≈1.0 on the magnetic axis (y=ymax), and ≈1.5 at the "edge“

When the superposition constant exceeds γ=0.69… the secondary tori disappear .

The outermost toroidal shell surrounding the three tori and extending up to the symmetry axis is indicated by SP and has a larger safety factor (always the ratio between the toroidal and the poloidal turns of a field line respectively ≈1.5 on the symmetry axis and ≈3.7 at the "separatrix" (=0.95•yX).

Chandrasekhar-Kendall-Furth force free fields: Stability



The result of the ideal MHD stability calculations for low toroidal mode numbers (n=1,2,3), assuming fixed boundary conditions at the edge of the plasma: $\xi=0$, is that the Chandrasekhar-Kendall-Furth force-free fields are stable when the value of the superposition parameter is greater than $\gamma=0.5$.

The unrelaxed CKF equilibrium

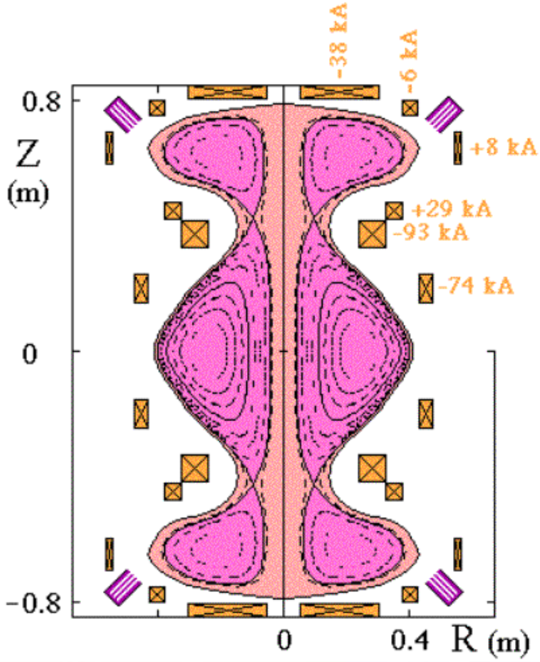
CKF homogeneous force-free fields have ∇p=0, and are so unable to confine plasmas of fusion interest. Unrelaxed equilibrium, similar to homogeneous CKF force-free fields, are calculated imposing that the relaxation parameter μ is constant only at the edge of the plasma.

Hypothesis

4. the pressure gradient vanishes at the edge of the plasma: [illegible]
5. the toroidal current density j vanishes at the edge of the plasma: [illegible]
6. the relaxation parameter is constant at the edge of the plasma, where both the field as well as the current density is purely poloidal: [illegible]
7. The parameter [illegible] [illegible] represents the drop of the relaxation parameter and the pressure from the edge of the plasma to the axis of the ST

The unrelaxed CKF equilibrium

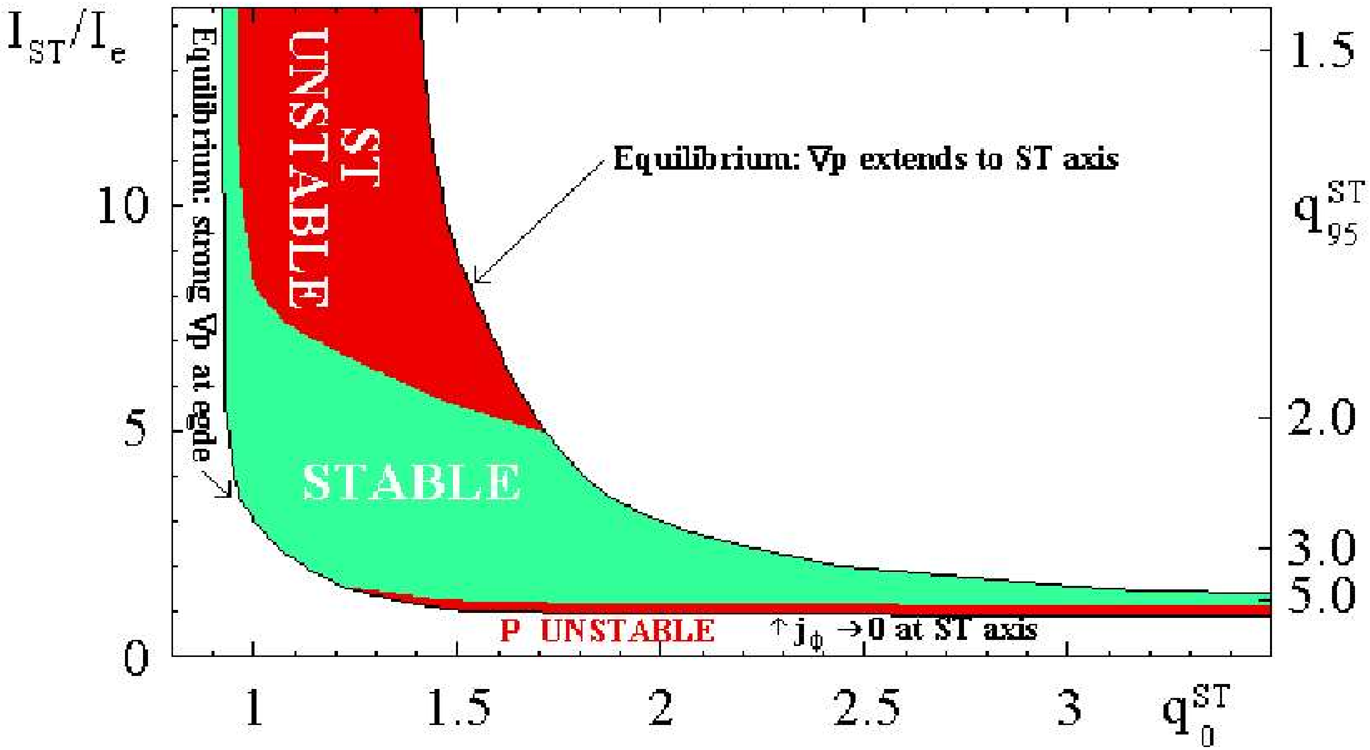
The case shown here corresponds to setting the geometrical choices to γ=0.55 and μ =0.35, the pressure peaking parameter to R/A=2, and Ist=836 kA.
It has a very high total plasma beta in the spherical torus: 1.02 .
It refers to an unrelaxed CKF equilibrium which has roughly the same geometrical dimension as PROTO-SPHERA. The total poloidal spheromak current is Ie=60 kA
(the same as the longitudinal pinch current of PROTO-SPHERA),
whereas the total toroidal current in the main spherical torus is Ip=451 kA
(much larger than in the case of PROTO-SPHERA, which has Ip=120/240 kA).
The unrelaxed CKF configurations can be obtained by simple external poloidal coils:



The unrelaxed CKF equilibrium: stability results

CKF CONFIGURATIONS

CKF, with this kind of <μ> and p profiles, are stable <u>in free boundary</u> to ideal MHD perturbations with low toroidal mode numbers (n=1, 2, 3), at $\beta_{ST}=2\mu_0\langle p\rangle_{ST}/\langle B^2\rangle_{ST}\approx 1/3$

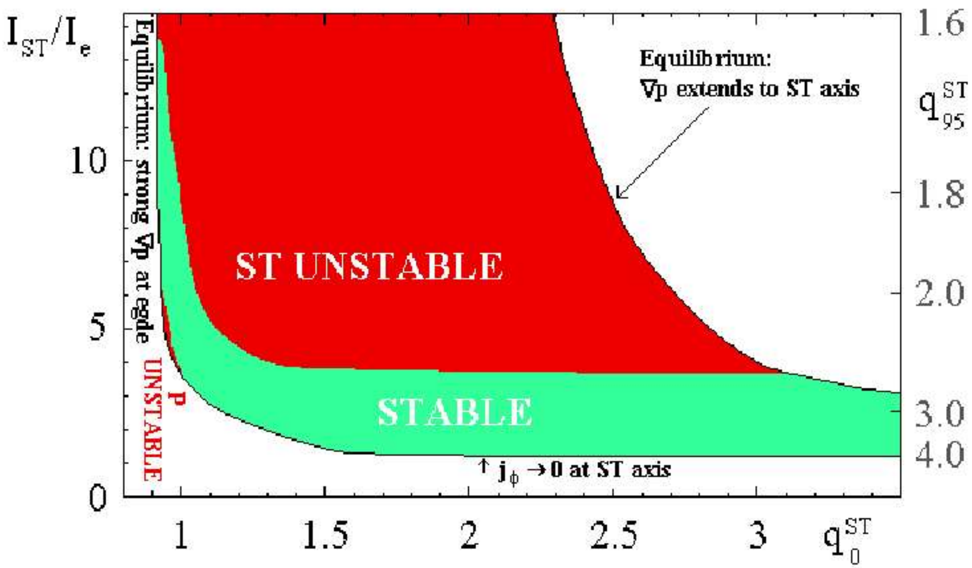


Trend of MHD stability with I_{ST}/I_e: same as in PROTO-SPHERA

The unrelaxed CKF equilibrium: stability results

CKF CONFIGURATIONS

and even <u>in free boundary</u> up to $\beta_{ST}=2\mu_0<p>_{ST}/<B^2>_{ST}\approx 1$



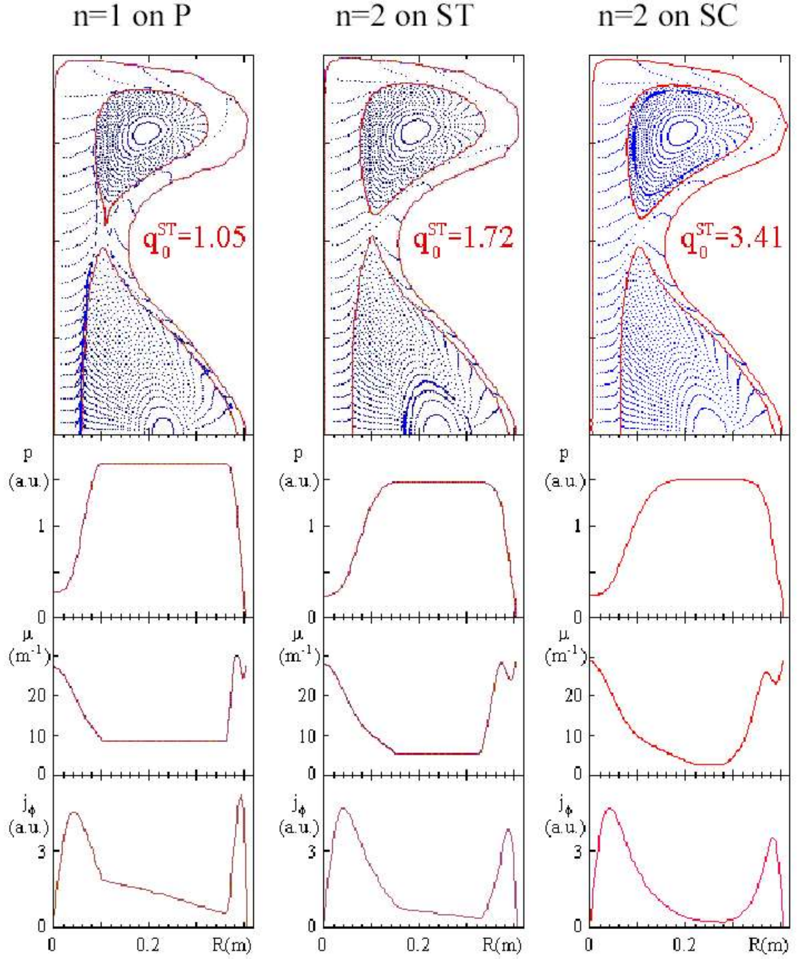
Trend of MHD stability with β: same as in PROTO-SPHERA

IMPORTANCE of high β for a reactor: reduces cost and size
P_{fusion}~$\beta^2 B^4$ therefore higher $\beta \Rightarrow$ lower B
$nT\tau_E$~β/χ $\{a^2B^2\}$ therefore higher $\beta \Rightarrow$ lower a at same χ

The unrelaxed CKF equilibrium: stability results

CKF CONFIGURATIONS

$\beta_{ST}=1$, ratio $I_{ST}/I_e=3$, stable oscillations:



Equatorial profiles: p(R), μ(R) & j_ϕ(R) at $\beta_{ST}=1$, $I_{ST}/I_e=3$

The unrelaxed CKF equilibrium: conclusions

1. Chandrasekhar-Kendall-Furth (CKF) equilibrium are innovative simply connected magnetic confinement configurations.

2. They embed a magnetic separatrix with regular X-points, which divides a main spherical torus ,two secondary tori on top and bottom of the main torus and an outermost toroidal shell surrounding plasma (characterized by its total poloidal current Ie and with $q \approx 3$ on the symmetry axis and $q \approx 5$ at the "separatrix").

3. CKF equilibrium can be calculated with the boundary condition that the relaxation parameter is constant only at the edge of the plasma

4. If the surrounding plasma can be sustained by driving a poloidal current Ie on its flux surfaces, magnetic helicity, flowing down the gradient of , will be injected into the main spherical torus, through magnetic reconnections at the regular X-points. If the helicity injection is efficient enough it will sustain the toroidal current IST, while converting part of the magnetic energy into kinetic plasma energy.

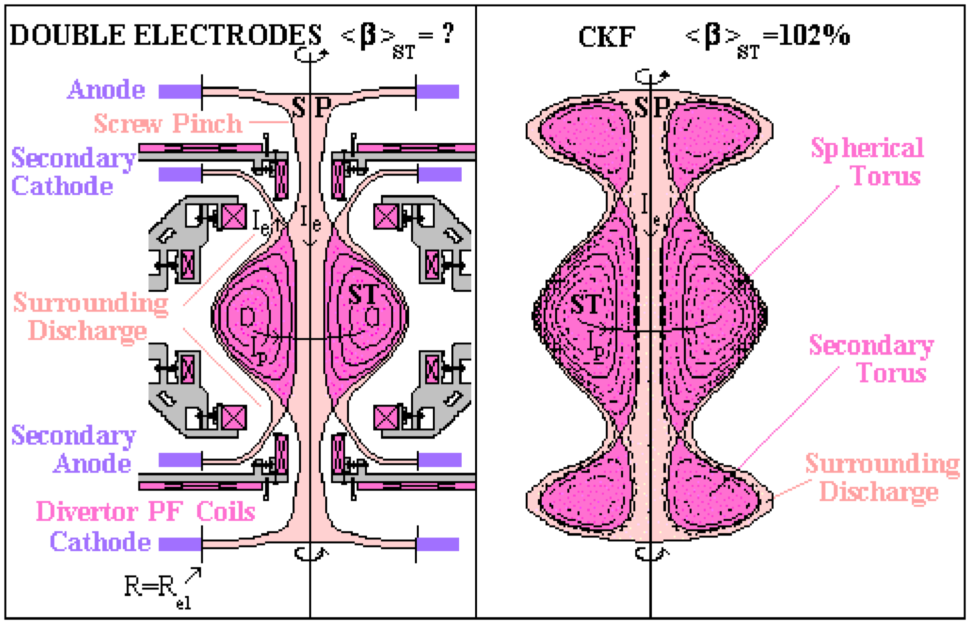
5. A parametric scan of unrelaxed CKF equilibria, at fixed shape of the plasma edge, has been calculated in terms of the current ratio Ist/Ie and of the safety factor at the ST magnetic axis . The vanishing of the toroidal current density jt on the ST axis, a too large at the plasma edge and ≠0 extending up to the ST axis respectively set the low Ist/Ie, the low and the high equilibrium limits. The transformation of each regular X-point into a pair of connected Y-points along the magnetic separatrix sets the equilibrium limit for the plasma beta in the spherical torus at a value of about one,

Towards a high power thruster ?

Similar to LANL proposal

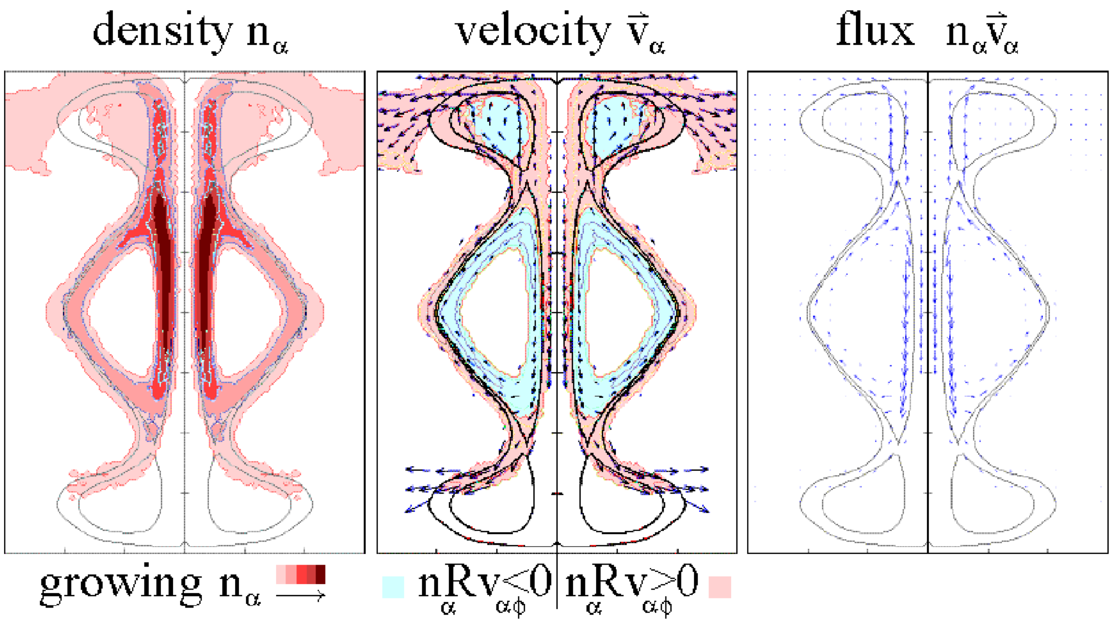
Reactor extrapolation: even if a PCC could be maintained at a few 100s eV, PCC power dissipation could be too large: a configuration initiated by electrodes but then sustained in absence of them?

CKF configurations[4]: variant with closed flux surface - ideal MHD stable at β=1! But how to form and to sustain them?



"Very imaginative" development: β=1 mandatory in D-^{3}He space thusters[5]: (weight forbids large B)

CKF channel promptly lost (~15%) charged fusion products (α): from ordered ($n_\alpha v_\alpha$) + magnetic nozzle ⇒ direct energy conversion & thrust



[4]F. Alladio, et al., **ICPP02**, Sydney (2002)

[5] NASA/TM-2005-213559 (2005)

Conclusions and future

1. Numerical tools for calculate equilibrium and stability of complex axisymmetric plasma configurations have been developed and validated.

2. Equilibrium and stability calculations have been proved that Proto-Sphera is ideally stable up to β =20%

3. This high β value of CKF configurations opens the possibility that plasma motions can sustain the magnetic field. PROTO-SPHERA experiment could host an unrelaxed Chandrasekhar-Kendall-Furth (CKF) configuration.

4. The PROTO-SPHERA project is in the framework of the research on Compact Tori (ST, spheromaks, FRC) and has the capability of exploring the connections between the three concepts. In particular its goal is to form and to sustain a flux-core- spheromak with a new technique.

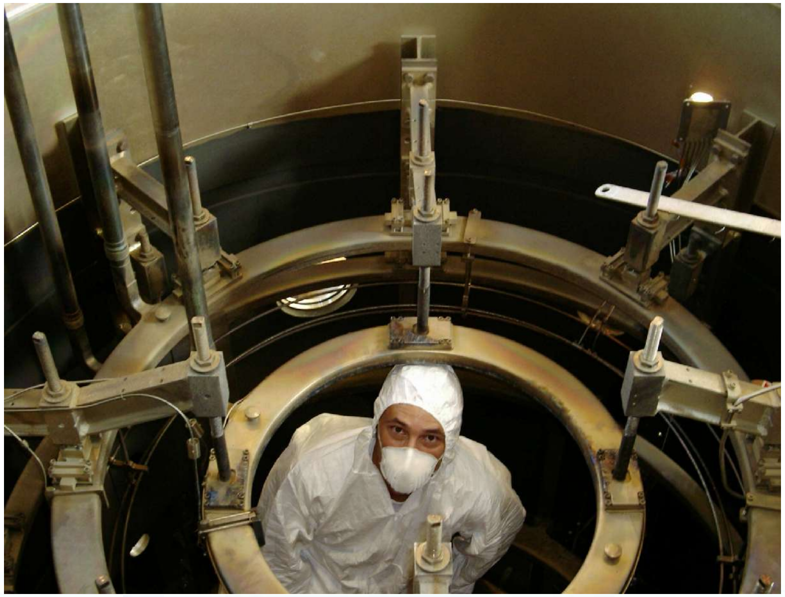
5. If all the major points of PROTO-SPHERA are successfully met, and if, in the meantime, a method for injecting current (or torque) into a CKF configuration is developed, the road toward small, compact, low field and simple maintenance fusion reactor (particularly suitable to direct energy conversion and to the use as space thrusters) could be possible.

6. Others simulation have to be performed to quantify the principle of helicity injection

7. Equilibrium must be reconstructed from measurements of magnetic field

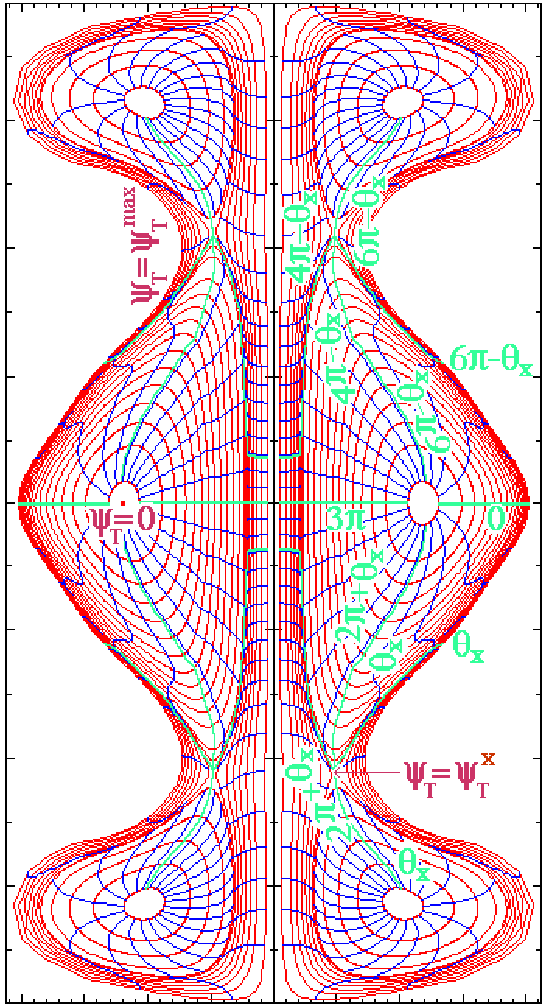
QUESTIONS ?

Progress of the Multi-Pinch construction



Correspondence of the poloidal between the tori and the surrounding plasma

1. the poloidal angle θ makes an excursion $[0,\theta_x]$ upon the lower outboard of the main ST
2. θ runs in the range $[\theta_x, 2\pi+\theta_x]$ upon the lower SC
3. θ continues through the range $[2\pi + \theta_x, 4\pi - \theta_x]$ upon the inboard of the main ST
4. θ runs in the range $[4\pi - \theta_x, 6\pi - \theta_x]$ upon the upper SC.
5. θ closes its run in the range $[6\pi - \theta_x, 6\pi]$ upon the upper outboard of the main ST.

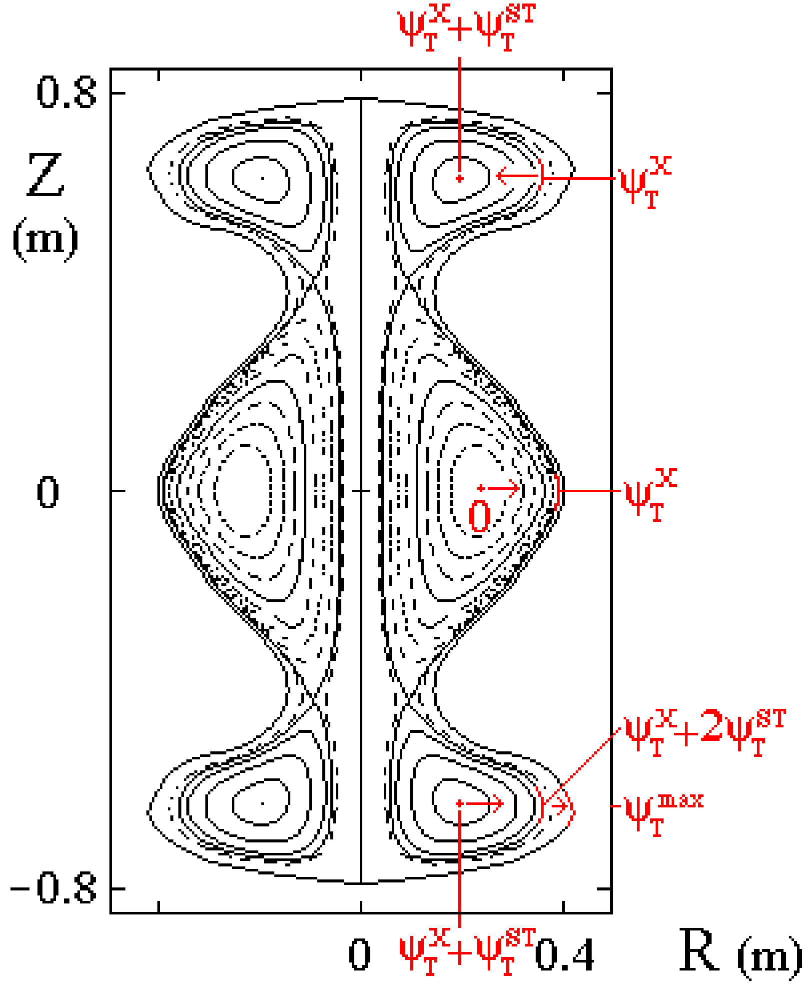


Toroidal flux ψ_T = constant

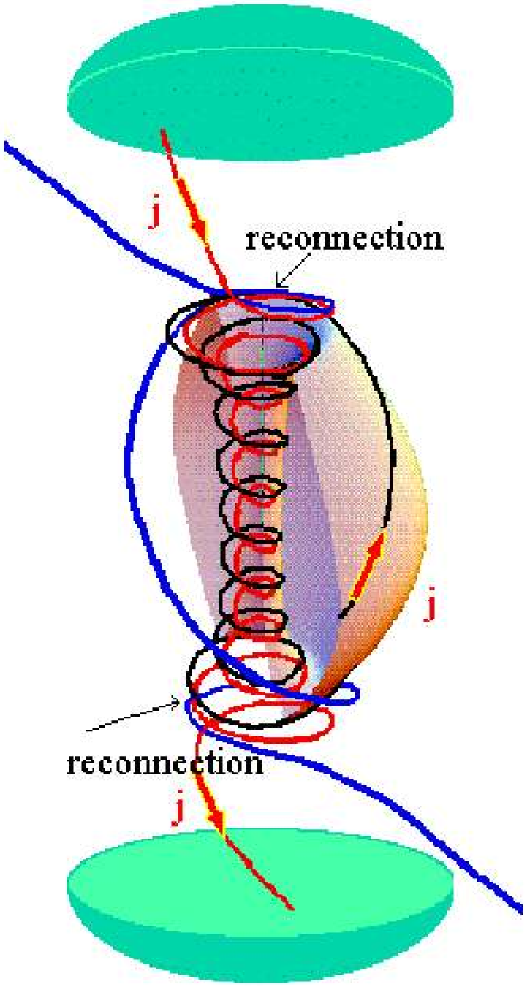
Poloidal angle θ = constant

Ordering of the radial variable: CKF configuration

1. Starts at the magnetic axis of the main spherical torus : = 0
2. reaches the value on the separatrix.
3. reenters the upper secondary torus from the edge ,
4. jumps to the magnetic axis of the lower secondary torus ,
5. and enters the surrounding plasma.



HELICITY INJECTION



- The plasma with open field lines (intersecting electrodes) has β~0, therefore $\vec{\mathbf{j}} \parallel \vec{\mathbf{B}}$

- Because of the twist of the open field lines, the current between the electrodes also winds in the toroidal direction near the closed magnetic flux surfaces

- Resistive MHD instabilities convert, through magnetic reconnections, open current/field lines into closed current/field lines, winding on the closed magnetic flux surfaces

- Magnetic reconnections necessarily break, through helical perturbations, the axial symmetry, as per Cowling's anti-dynamo theorem

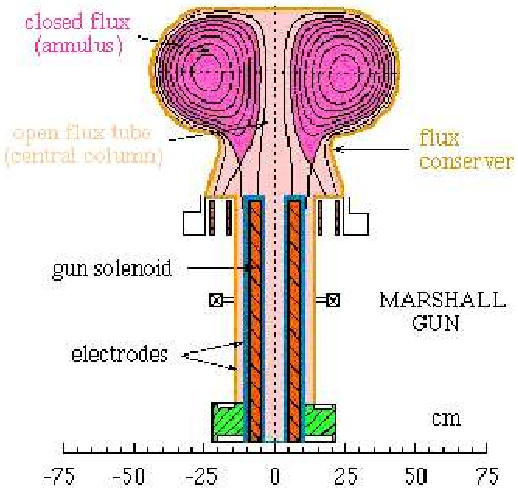
HELICITY INJECTION

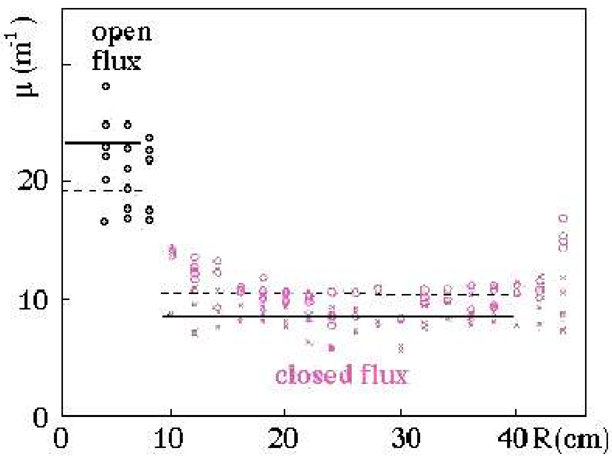
If helicity source (SP of PROTO-SPHERA) is physically separated from helicity sink (ST of PROTO-SPHERA) a gradient in the relaxation parameter $\mu=\mu_0 \vec{\mathbf{j}} \bullet \vec{\mathbf{B}}/B^2$ appears: resistive MHD instabilities produce helicity flow from larger μ to smaller μ regions

$$d(\Delta \int \vec{\mathbf{A}} \bullet \vec{\mathbf{B}}\, dV)/dt = - \int -\lambda \vec{\nabla}\mu \bullet \hat{\mathbf{n}}\, dS - 2\int \Phi_E \vec{\mathbf{B}} \bullet \hat{\mathbf{n}}\, dS - 2\int \vec{\mathbf{A}} \wedge \partial \vec{\mathbf{A}}/\partial t \bullet \hat{\mathbf{n}}\, dS - 2\int (\vec{\mathbf{E}} \bullet \vec{\mathbf{B}})\, dV$$

- $-\lambda \vec{\nabla}(\vec{\mathbf{j}} \bullet \vec{\mathbf{B}}/B^2)) = -\lambda \vec{\nabla}\mu$ helicity flux (Boozer fluctuation-averaged Ohm's law)

SPHEX at UMIST (Manchester) has explored $\vec{\nabla}\mu$ in a Flux-Core-Spheromak



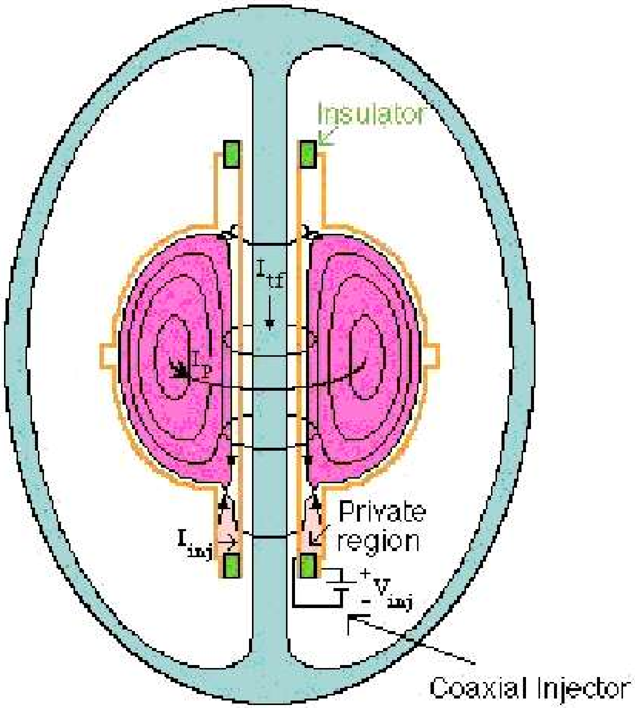


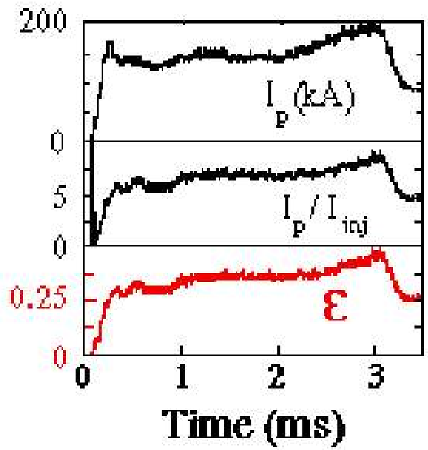
In agreement with the results of SPHEX, the PROTO-SPHERA equilibria have been calculated in order to get: $2.4 \leq \mu_{Pinch}/\langle\mu_{ST}\rangle(\psi_{axis}) \leq 3.3$

HELICITY INJECTION

Reconnection processes convert part of the magnetic energy into kinetic energy of the magnetized plasma, HIT Washington University (Seattle)

Energy efficiency (from HIT results) $\varepsilon = I_p V_{loop} / I_{inj} V_{inj} = 0.25$





Towards a high power thruster ?

In a CKF fusion space thruster, the large Larmor radii of the charged fusion products induce remarkable prompt losses (~15%), enhanced by the large variation in the magnetic field strength (and possibly by the helicity-injecting nonaxisymmetric magnetic reconnections). Different orbits of co/counter-circulating fusion products on the inboard/outboard of the surrounding discharge (fig. 1.49) drive, through the generalised Ohm law, a net current density. Order of magnitude evaluations indicate that such a current could be sufficient to sustain the plasma of the surrounding discharge, but detailed calculations are still in progress. Therefore, the prompt losses of charged fusion products from a CKF fusion space thruster could play the double role of transforming the fusion power into propulsive power and of self-sustaining the magnetic configuration of the thruster.

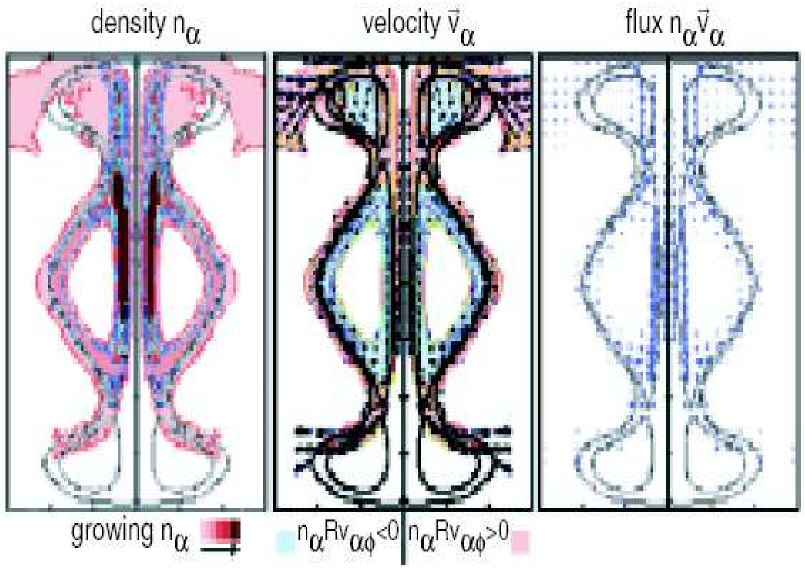


Fig. 1.49 - Promptly lost α-particles: contour plot of their density n_α; arrow plot of their poloidal velocity v_α and poloidal flow $n_\alpha v_\alpha$

Status & Perspectives of the PROTO-SPHERA Project

F. Alladio, A. Mancuso, P. Micozzi, S. Papastergiou

Stable SP I_e= 8.5 KA destabilized

by I_e-> 60 KA in 0.5 ms, forms ST

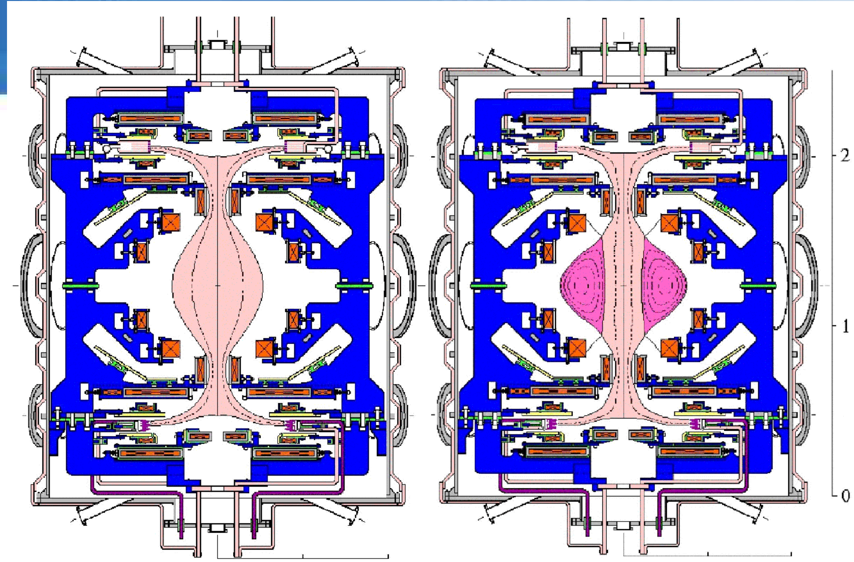
as in **TS-3** Tokyo University experiment[3]

Alfvén MHD growth time: τ_A~ 0.5 µs

Resistive diffusion time: τ_R~ 70 ms

$(\tau_R\tau_A)^{1/2}$~ 1 ms for I_{ST}=120 KA ($2xI_e$)

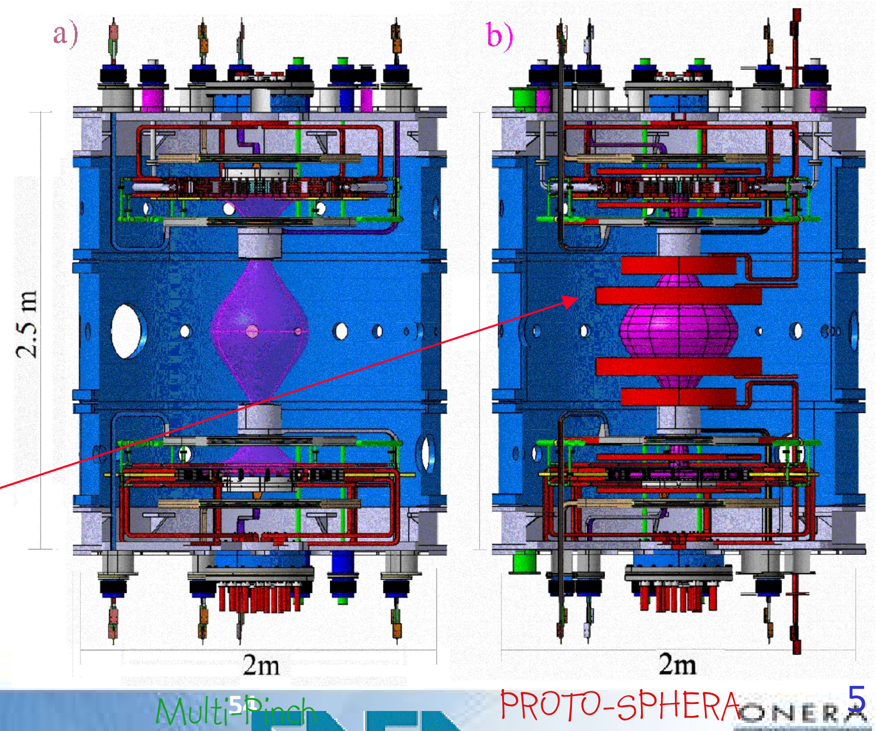
- I_{ST}= 240 KA ($4xI_e$) really achievable?
- Can it be sustained for τ_R~ 70 ms?
- Resistive MHD stability & confinement?



- A **reduced setup** (a) of **PROTO-SPHERA** (b) **is being built inside START vacuum vessel** (gifted by Culham UKAEA Science Centre) **to clarify PCC breakdown & stability with annular electrodes** (beware anode anchoring!)

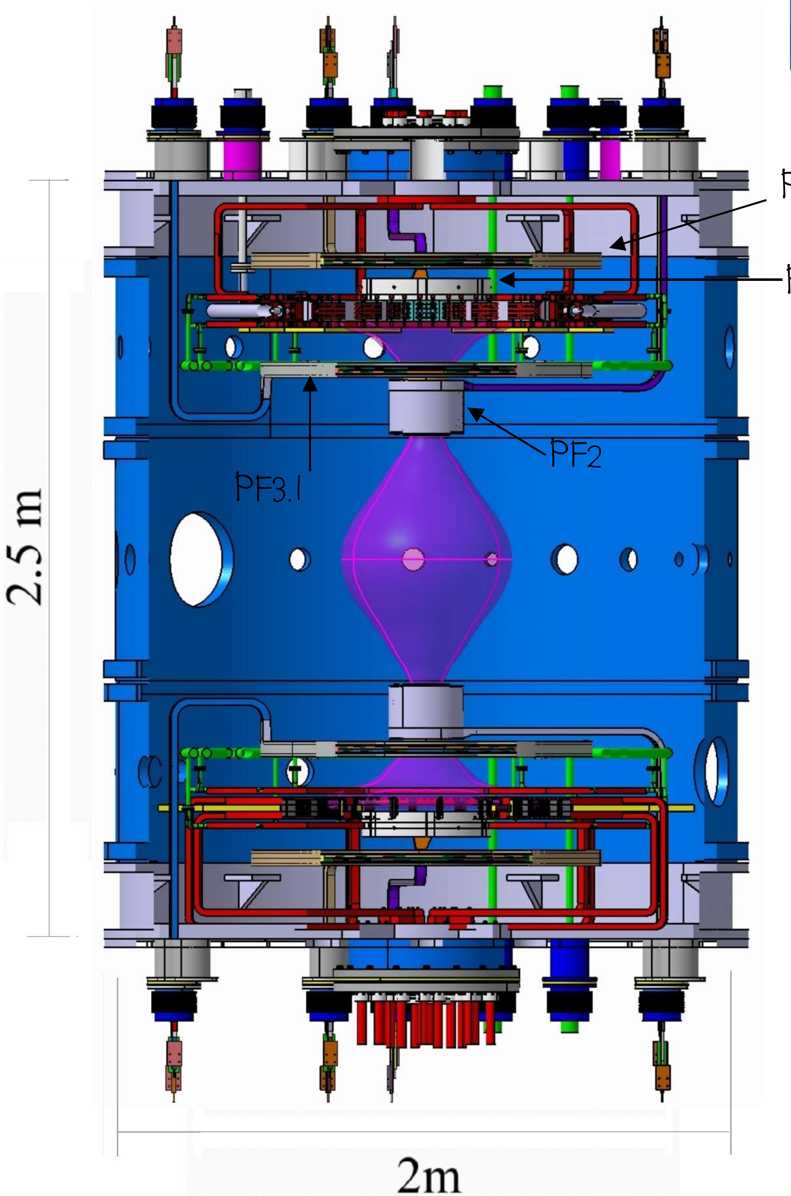
PROTO-SPHERA obtained later, adding PF compression coils and their power supplies,

and implementing the PCC power supply



[3] N. Amemiya, et al., **JPSJ 63**, 1552 (1993)

Multi-Pinch Experiment



Aims:
stable PROTO-SPHERA Screw Pinch plasma with full dimension, disk shape, and current I_e=8.5 kA

Philosophy:

almost all parts should be reutilized in PROTO-SPHERA

START vacuum vessel

(in Frascati since May 2004)

- **Partial PF coils system (constant current):** **only the "Pinch shaping coils"**

- **Definitive Anode & Cathode**

Progress of the Multi-Pinch construction

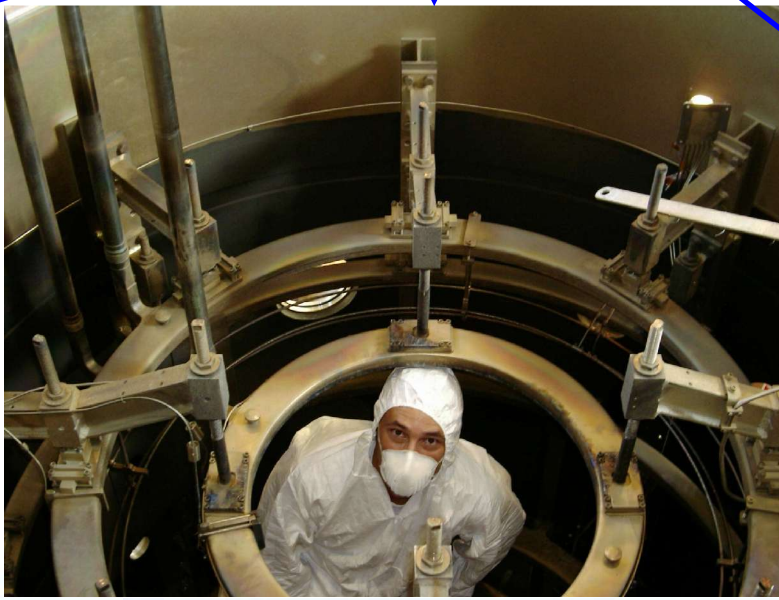
START in Frascati

Arrival

May 2004

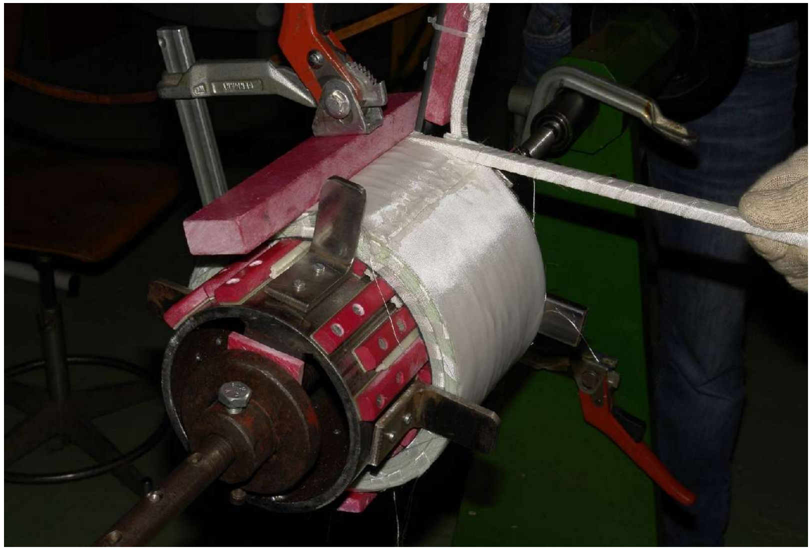
Disassembling

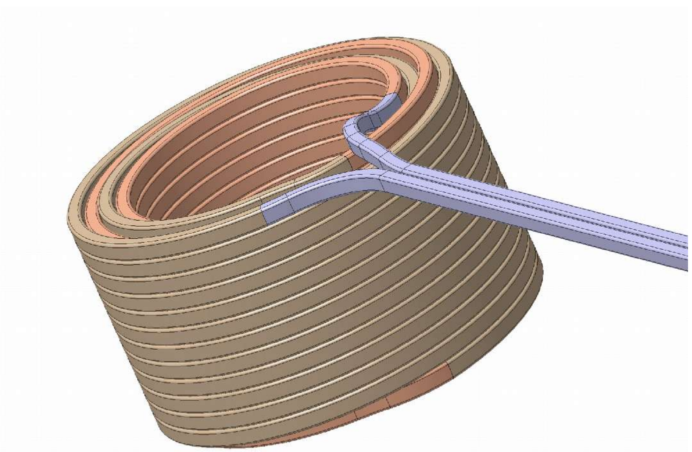
October 2004

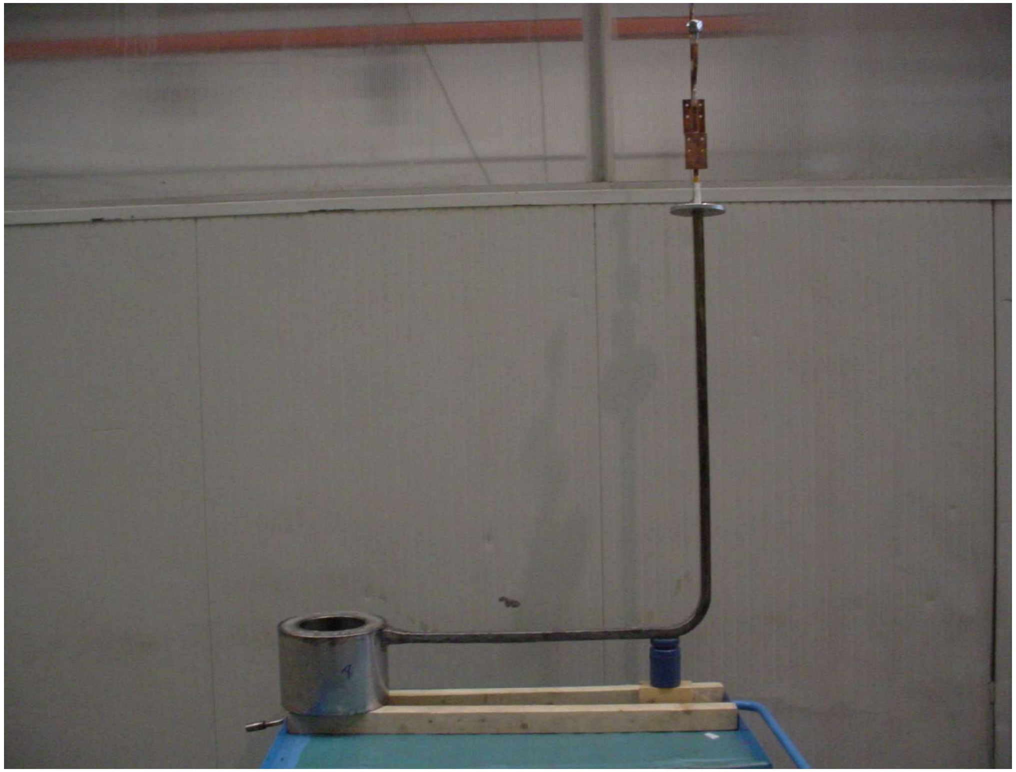


Progress of the Multi-Pinch construction

PF coils have been built by ASG Superconductors S.p.A. (Genova, Italy):



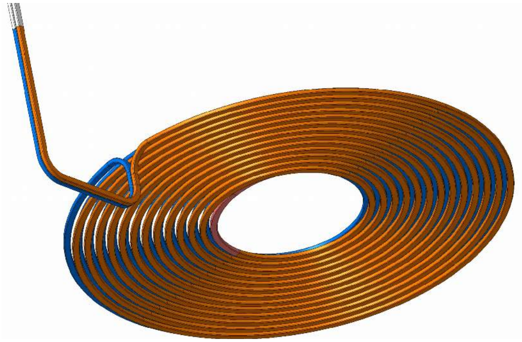


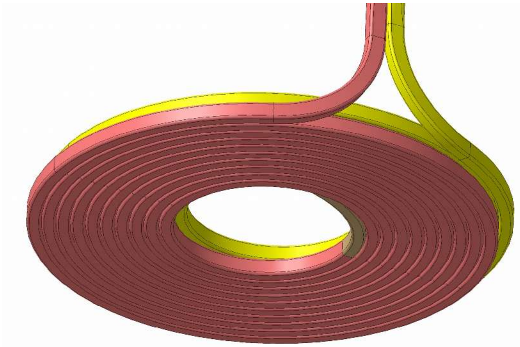


PF2: Pinch "mirror coil"

Progress of the Multi-Pinch construction

PF coils have been built by ASG Superconductors S.p.A. (Genova, Italy):

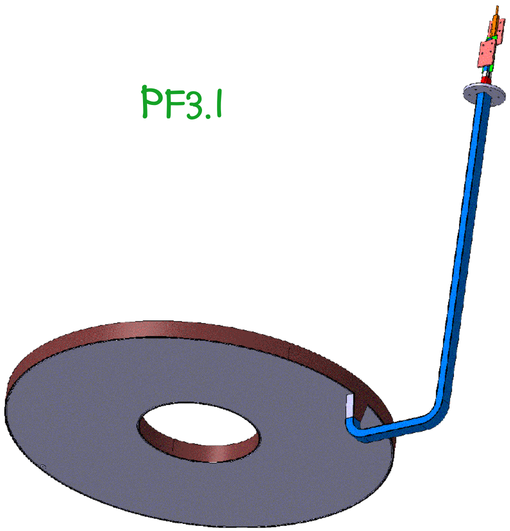


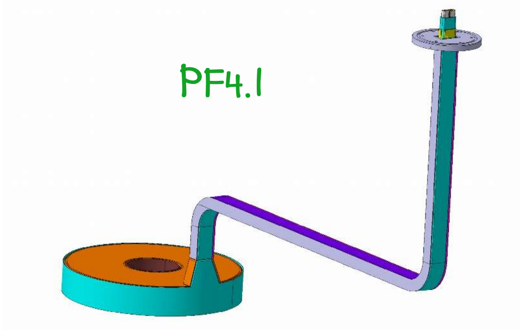


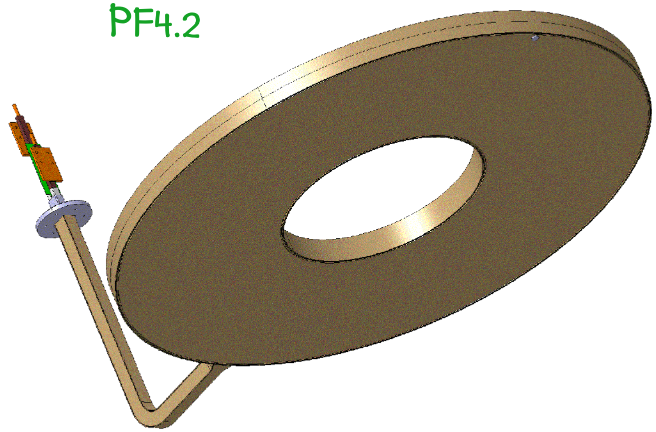
PF3.1

PF4.1

PF4.2

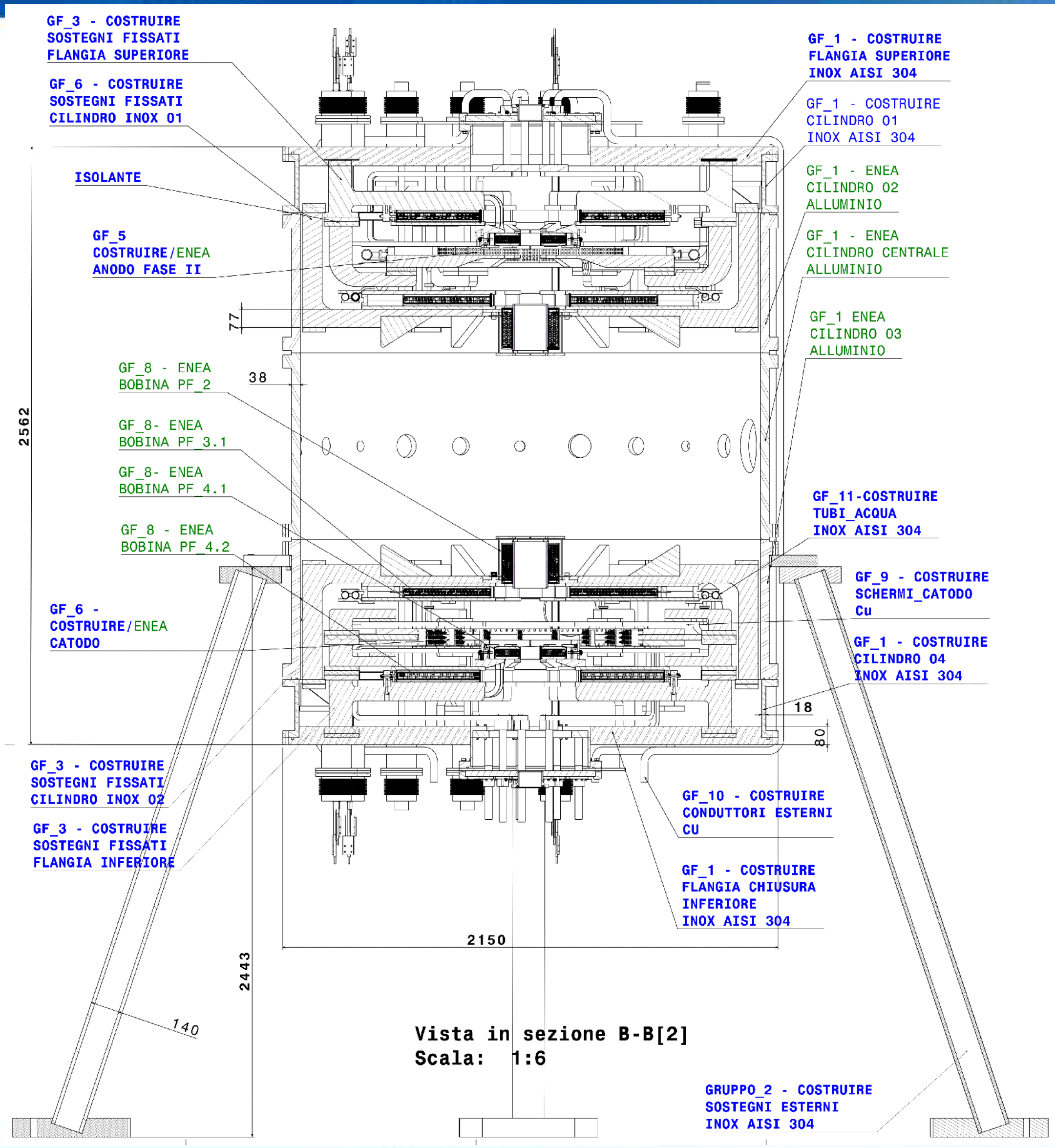






Pinch "shaping coils" (pancake kind)

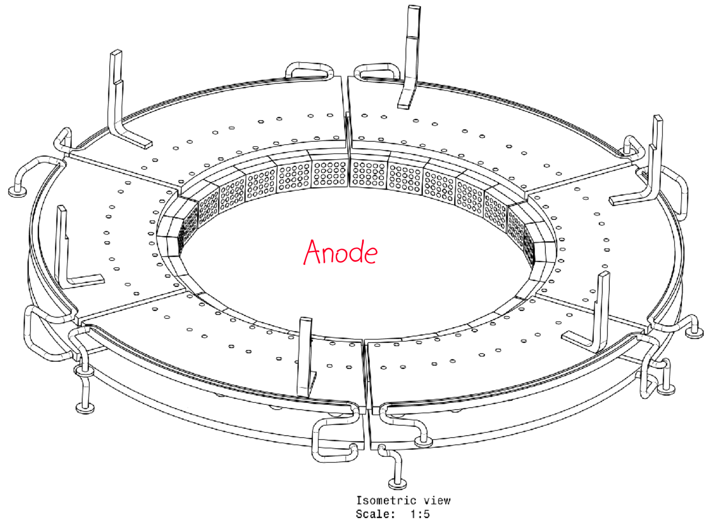
Progress of the Multi-Pinch construction

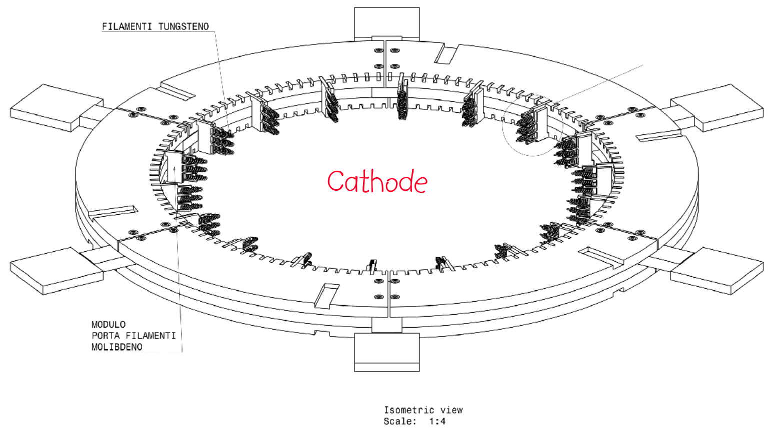


European "call for tender"

to build the Load Assembly:

starting now





Actions & Time Schedule

- **Power Supplies**: PF coils, Cathode and Screw Pinch (reduced)
- **Diagnostics**: Spectroscopy, Magnetics*, Fast Camera*
- **Vacuum System**, **Data Acquisition System** and **Control System**

Realistically the first Multi-Pinch plasma

could be obtained in the first half of 2009

* Culham collaboration?