Ghost Fluid Method for Interfaces flow computations

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Interfaces flow

Interfaces Flow

For interfaces flow, it is assumed that a characteristic volume is always occupied by a pure phase fluid.

Numerical strategies

- “Explicit” interface tracking (Front tracking)
- “Implicit” interface tracking (Level Set)
- Interface reconstruction (VOF)
- Diffusive Interface

Outline

1. **Interfaces flow Model**
   - Level Set for Interface tracking
   - Governing equations
2. **Ghost Fluid Methods**
   - Finite volume
   - Applications: 1D Cases
3. **Extension to unstructured meshes**
   - Finite Volume
   - 2D Applications
4. **Conclusions, Coming and Future work**
   - DG approach for the level set equation
   - Finite Element

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Interfaces flow model

Interface Model: bi-fluid case

The interface $\mathcal{S}$ is the zero of a single level set function ϕ.

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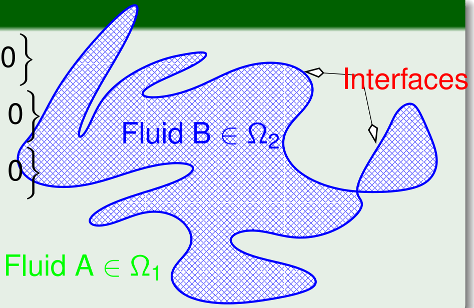
The interface $\mathcal{S}$ is the zero of a single level set function ϕ.

Example

$$\mathcal{S}(t) = \left\{ x \text{ such that } \phi(t,x) = 0 \right\}$$

$$\Omega_1(t) = \left\{ x \text{ such that } \phi(t,x) < 0 \right\}$$

$$\Omega_2(t) = \left\{ x \text{ such that } \phi(t,x) > 0 \right\}$$



Dynamic of the Level set function

Transport Equation formulation

$$\partial_t \phi + \boldsymbol{u}(\phi) \cdot \nabla \phi = 0$$

where ϕ is any regular function such that $\phi(t, x) = 0$ for $x \in \mathcal{S}(t)$

$\boldsymbol{u}(\phi)$, the velocity field, function of the interface motion.

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Hamilton-Jacobi formulation

$$\partial_t \phi + F(\nabla \phi)|\nabla \phi| = 0, \quad \text{where} \quad F(\nabla \phi) = \frac{\boldsymbol{u}(\phi) \cdot \nabla \phi}{|\nabla \phi|}$$

In this context, ϕ is usually a signed distance function.

General Case : $\Omega = \Omega_1(t) \cup \Omega_2(t) \cup \mathcal{S}(t)$.

Mathematical Model

$$\mathcal{L}_1 \omega_1 = 0 \qquad \text{for } \phi(t,x) < 0 \tag{1}$$

$$\mathcal{L}_2 \omega_2 = 0 \qquad \text{for } \phi(t,x) > 0 \tag{2}$$

$$\mathcal{G}_1 \omega_1 - \mathcal{G}_2 \omega_2 = \Sigma(\phi, \omega_1, \omega_2) \qquad \text{for } \phi(t,x) = 0 \tag{3}$$

$$\partial_t \phi + \boldsymbol{u}(\phi, \omega_1, \omega_2) \cdot \nabla \phi = 0 \qquad \text{for } (t,x) \in [0,T] \times \Omega \tag{4}$$

Definitions : ( )

1. $\mathcal{L}_k$ and ω_k are differential operator and the set of the unknown, relevant for the flow description in the region Ω_k
2. $\mathcal{G}_k$, $\boldsymbol{u}(\phi, \omega_1, \omega_2)$ and $\Sigma(\phi, \omega_1, \omega_2)$ are associated to jump conditions and waves transmission at interfaces.

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Compressible/Compressible Interfaces

Assumptions

We assume that pure fluids are inviscid, compressible and flow described anywhere by the conservative Euler Equations.

Definitions : (States, Fluid Model,)

1. $\omega_k = \omega = (\rho, \rho \mathbf{u}, \rho e)^T$
2. $\mathcal{L}_k \omega_k = \partial_t \omega + \nabla \cdot \mathbf{f}(\omega, p_k)$;
   Pressures are given by equations of state $p_k = p_k(\omega)$
3. [illegible]
4. [illegible] $= 0$ *in absence of tension forces, chemical reaction and phase transition.*
5. *How are defined* [illegible] *and* [illegible] *? Shock or CD?*

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3. $\mathcal{G}_k(\phi)\omega_k = (\boldsymbol{u}|_{\mathcal{S}} \cdot \nabla\phi)\boldsymbol{\omega} + \nabla\phi \cdot \boldsymbol{f}(\boldsymbol{\omega}, p|_{\mathcal{S}})$
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Equation of State (EOS)

Mie-Gruneisen family of equation of state

$$\Gamma_k(\rho)p_k + \pi_k(\rho) = \rho\varepsilon \quad \text{with} \quad \varepsilon = e - \boldsymbol{u}\cdot\boldsymbol{u}/2,$$

where $\Gamma(\rho)$ and $\pi_k(\rho,\varepsilon)$ are given functions.

Example (Perfect Gas EOS)

$$\Gamma(\rho) = \frac{1}{\gamma-1}, \quad \pi(\rho) = 0, \quad c^2 = \frac{\gamma p}{\rho}$$

Example (Modified Tait's EOS)

$$\Gamma(\rho) = \frac{1}{m-1}, \quad \pi(\rho) = \frac{m(\pi_* - \pi_0)}{m-1},$$

Water : $m = 7.15, \quad \pi_* = 3.3110^8 Pa, \quad \pi_0 = 10^5 Pa$

Outline

Finite volume formulation

Explicit scheme

- $a_i \omega_i^{n+1} = a_i \omega_i^n - \sum_{j \in \nu(i)} \Phi\left(\boldsymbol{n}_{ij}, \omega_i^n, \omega_j^n\right) - \sum_{j \in \kappa(i)} \Phi_{\mathcal{S}}\left(\boldsymbol{n}_{ij}, \omega_i^n, \omega_j^n\right)$
- $a_i \phi_i^{n+1} = a_i \phi_i^n - \mathcal{R}_i\left(\phi^n, \omega^{n+1}\right)$

1. $j \in \nu(i)$ is a neighbor cell of i such that $\phi_i^n \phi_j^n > 0$. The flux Φ is a classical one (Roe, HLL, HLLC).
2. $j \in \kappa(i)$ is a neighbor cell of i such that $\phi_i^n \phi_j^n < 0$. The flux $\Phi_{\mathcal{S}}$ have to be consistent with jump conditions and wave transmission at the interface.

Ghost Fluid Method Principle

Use a classical Flux with ghost states

$$\Phi_{\mathcal{S}}\left(\boldsymbol{n}_{ij},\omega_i^n,\omega_j^n\right) \simeq \Phi\left(\boldsymbol{n}_{ij},\tilde{\omega}_i^n,\tilde{\omega}_{ij}^n\right) \text{ where } \tilde{\omega} = \tilde{\omega}\left(\tilde{\rho},\tilde{\boldsymbol{u}},\tilde{p}\right)$$

Properties of ghost states

The ghost states should be such as the flux $\Phi\left(\boldsymbol{n}_{ij},\tilde{\omega}_i^n,\tilde{\omega}_{ij}^n\right)$ be consistent with

- the jump conditions (static constraint),
- the wave transmission (dynamic constraint).

Original Ghost Fluid Method (Fedkiw et al. 99)

Strategy based on static constraint: jump conditions

$$\tilde{\omega}_i = \omega_i, \quad \text{and} \quad \tilde{\omega}_{ij} = \tilde{\omega}_{ij}\left(\rho_{ij}, \mathbf{u}_j, p_j\right)$$

where ρ_{ij} is given by $\quad s(\rho_i, p_i) = s'(\rho_{ij}, p_j)$

ρ_{ij} is an evaluation of the density close to the interface.

Implicit assumptions

- Equation with entropy functions s and s', is invertible. OK for perfect gas and some Mie-Grunieson EOS.
- Entropy can be extrapolated “near” the interface.

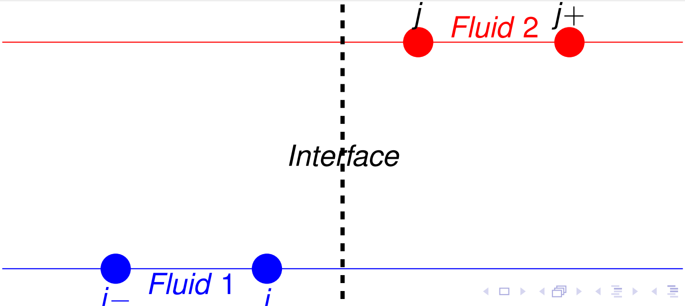
Isobaric fix Ghost Fluid Method (Fedkiw et al. 99)

Isobaric fix (Fedkiw et al. JCP, 1999)

In order to prevent overheating errors in the GFM method

$$\tilde{\omega}_i = \tilde{\omega}_i\left(\rho_{i-}, \boldsymbol{u}_i, p_i\right), \quad \tilde{\omega}_{ij} = \tilde{\omega}_{ij}\left(\rho_{ij}, \boldsymbol{u}_j, p_j\right), \quad s(\rho_{i-}, p_i) = s'(\rho_{ij}, p_j)$$

Neighbor cells in 1D Case : $i- = i-1$, $j = i+1$, $j+ = j+1$.



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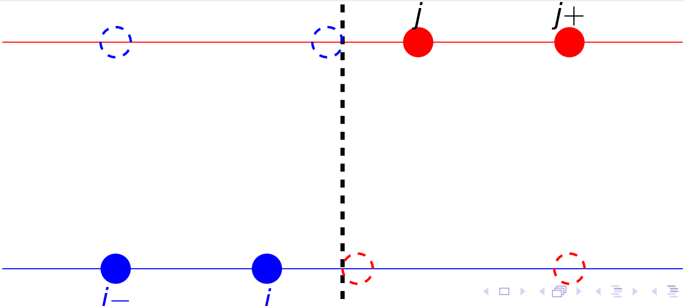

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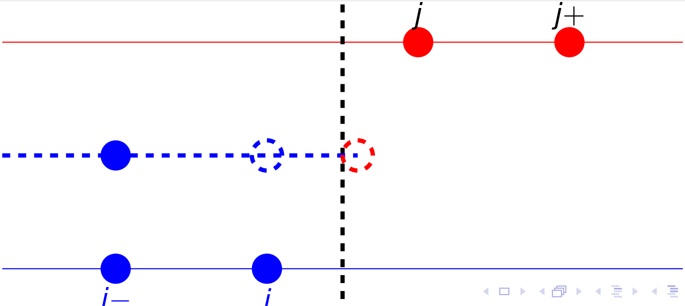

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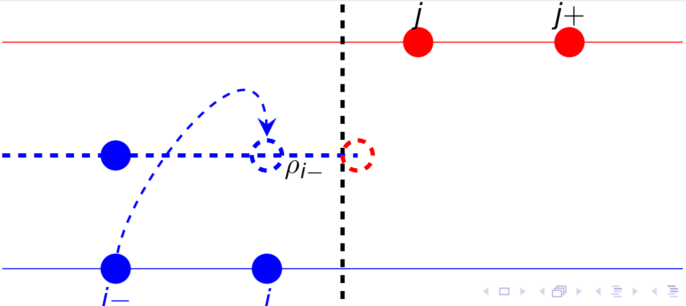

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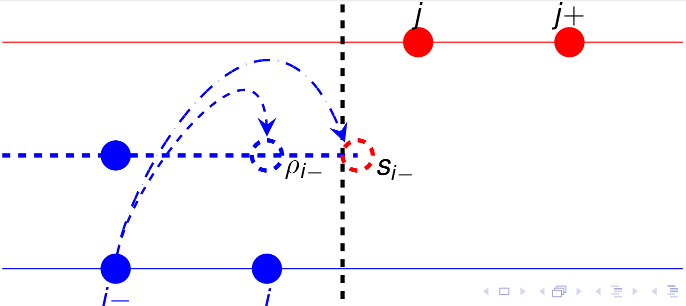

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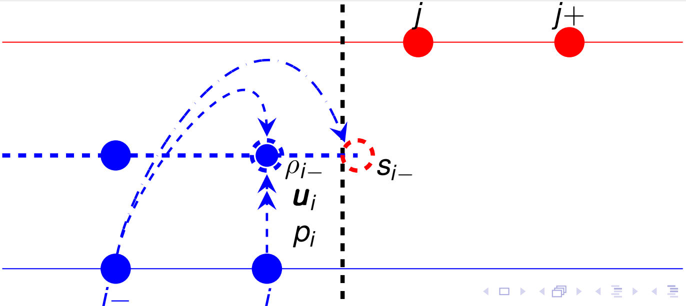

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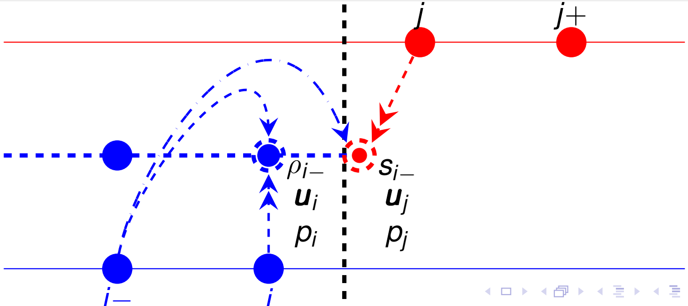

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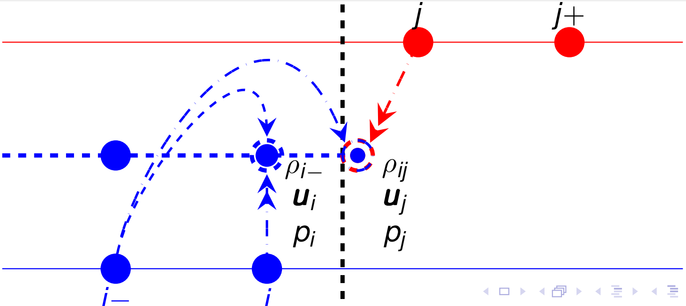

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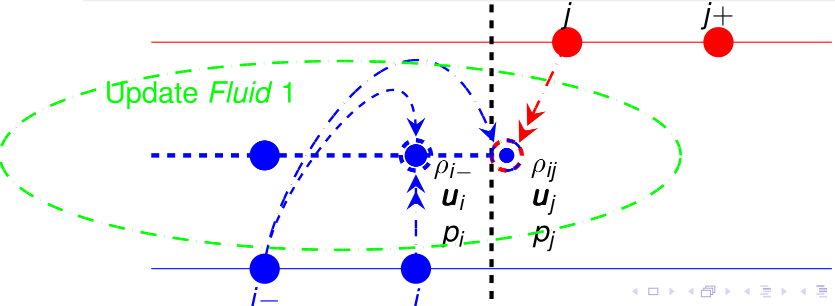


Modified Ghost Fluid Method (Liu et al. 03)

Strategy mixing static and dynamic constraints.

Wave transmission: two-shock bi-fluid solver

$$dp + \tilde{\rho}_+ \tilde{c}_+ du = 0 \qquad along \quad \frac{dx}{dt} = \tilde{u} + \tilde{c}_+ \tag{5}$$

$$dp - \tilde{\rho}_- \tilde{c}_- du = 0 \qquad along \quad \frac{dx}{dt} = \tilde{u} - \tilde{c}_- \tag{6}$$

$u = \boldsymbol{u} \cdot \nabla\phi / |\nabla\phi|$, is the normal velocity.

Jump conditions on material interfaces

$$p_{ij} = p_{ji} = \tilde{p}, \quad u_{ij} = u_{ji} = \tilde{u}$$

Modified Ghost Fluid Method (Liu et al. 03)

Jump conditions in a pure fluid gives $\tilde{\rho}$ as a function of $\tilde{p}$

$$\left[\frac{(1+\Gamma_k(\rho))\,p_k+\pi_k(\rho)}{\rho}+p_k\right]=0 \longrightarrow \begin{cases}\tilde{\rho}_i\left(\tilde{p},\omega_{i-}\right), \\ \tilde{\rho}_j\left(\tilde{p},\omega_{j+}\right)\end{cases}$$

Nonlinear system defining $\tilde{p}$ and $\tilde{u}$

$$\tilde{u}-u_{i-}=\int_{p_{i-}}^{\tilde{p}}\frac{dp}{\tilde{a}_-(p)} \quad \text{and} \quad \tilde{u}-u_{j+}=-\int_{p_{j+}}^{\tilde{p}}\frac{dp}{\tilde{a}_+(p)}.$$

$\tilde{a}_-$ and $\tilde{a}_+$ are approximated acoustic impedances.

Modified Ghost Fluid Method (Liu et al. 03)

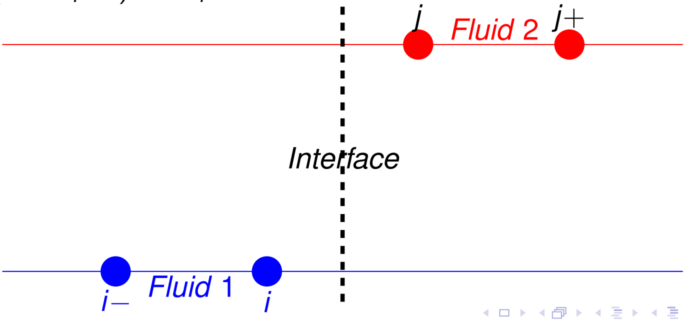
M-GFM

$$\omega_i \equiv \tilde{\omega}_i = \tilde{\omega}_i\left(\tilde{\rho}_i,\ \tilde{\boldsymbol{u}}_i,\ \tilde{p}\right)$$
$$\tilde{\omega}_{ij} = \tilde{\omega}_{ij}\left(\tilde{\rho}_j,\ \tilde{\boldsymbol{u}}_j,\ \tilde{p}\right)$$

H-GFM

$$\tilde{\omega}_i = \tilde{\omega}_i\left(\overline{\rho}_i,\ \tilde{\boldsymbol{u}}_i,\ \tilde{p}\right)$$
$$\tilde{\omega}_{ij} = \tilde{\omega}_{ij}\left(\tilde{\rho}_i,\ \tilde{\boldsymbol{u}}_j,\ \tilde{p}\right)$$
$$s(\overline{\rho}_i, \tilde{p}) = s(\rho_i, p_i)$$

$$\tilde{\boldsymbol{u}}_i = (\tilde{u} - \boldsymbol{u}_i \cdot \boldsymbol{n})\boldsymbol{n} + \boldsymbol{u}_i$$



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$$s(\bar{\rho}_i, \tilde{p}) = s(\rho_i, p_i)$$

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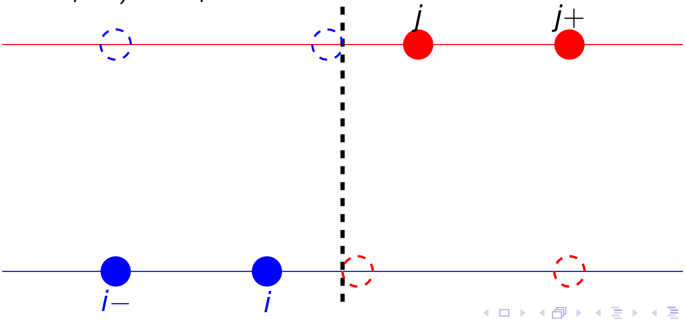

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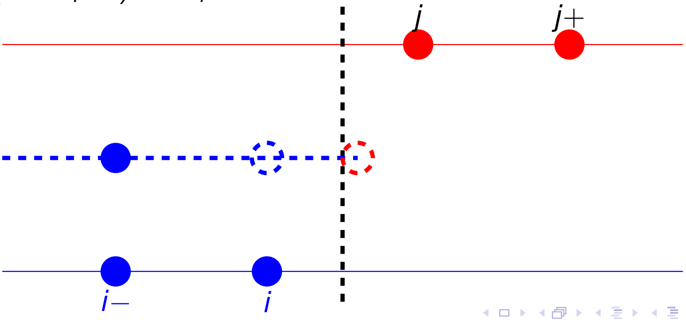

Modified Ghost Fluid Method (Liu et al. 03)

M-GFM

$$\omega_i \equiv \tilde{\omega}_i = \tilde{\omega}_i\left(\tilde{\rho}_i,\ \tilde{\boldsymbol{u}}_i,\ \tilde{p}\right)$$
$$\tilde{\omega}_{ij} = \tilde{\omega}_{ij}\left(\tilde{\rho}_j,\ \tilde{\boldsymbol{u}}_j,\ \tilde{p}\right)$$

H-GFM

$$\tilde{\omega}_i = \tilde{\omega}_i\left(\bar{\rho}_i,\ \tilde{\boldsymbol{u}}_i,\ \tilde{p}\right)$$
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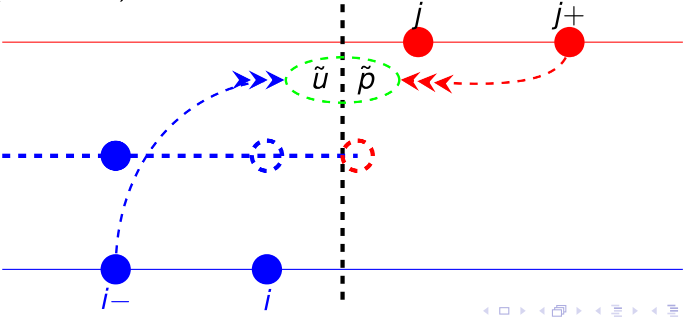

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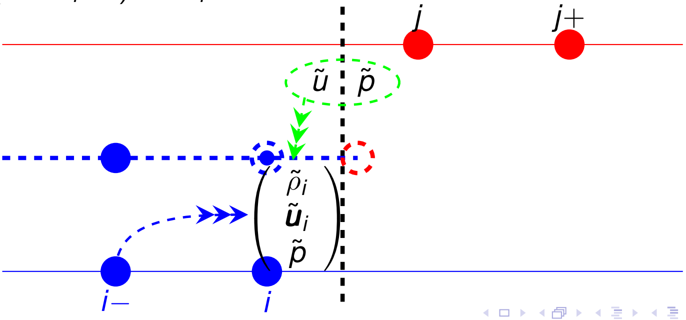

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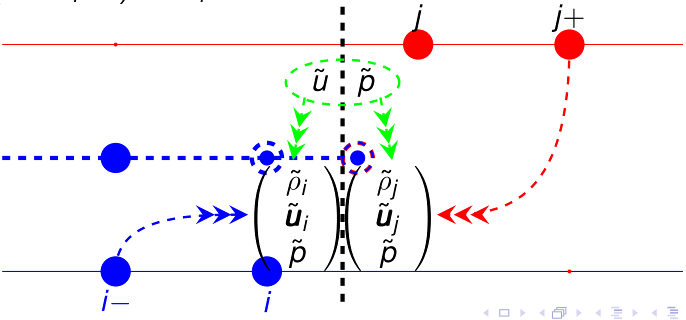

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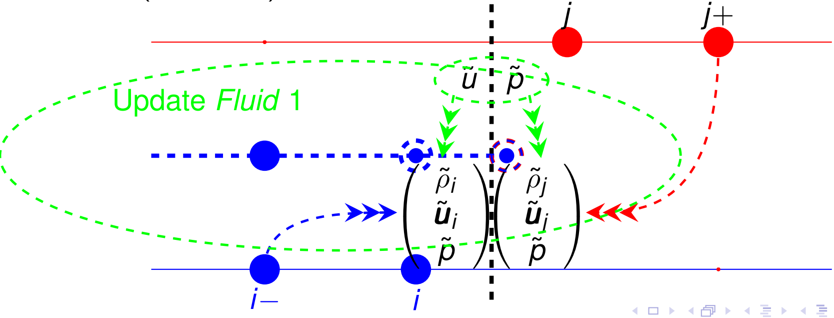

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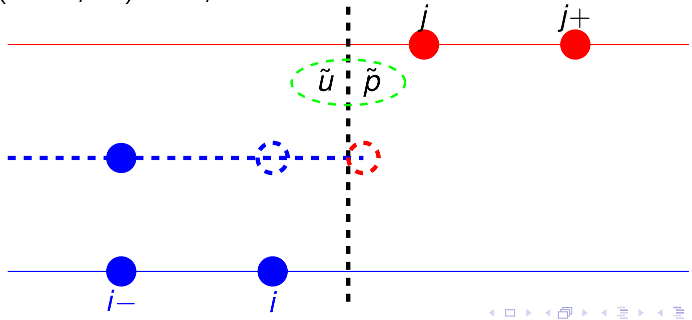

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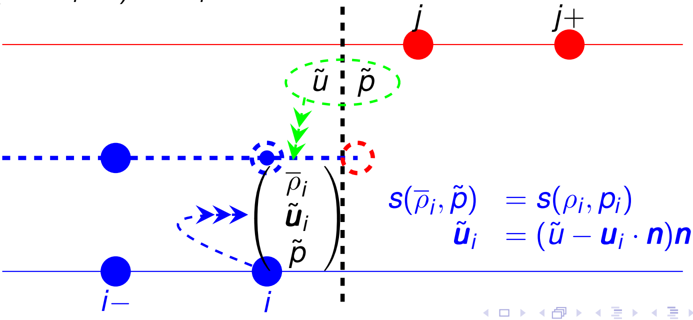

$$\begin{aligned} s(\overline{\rho}_i, \tilde{p}) &= s(\rho_i, p_i) \\ \tilde{\boldsymbol{u}}_i &= (\tilde{u} - \boldsymbol{u}_i \cdot \boldsymbol{n})\boldsymbol{n} + \boldsymbol{u}_i \end{aligned}$$

Modified Ghost Fluid Method (Liu et al. 03)

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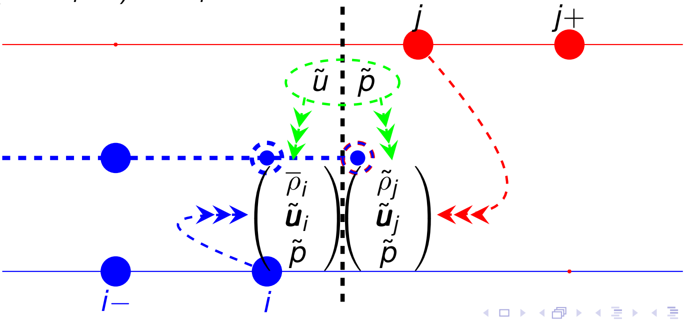

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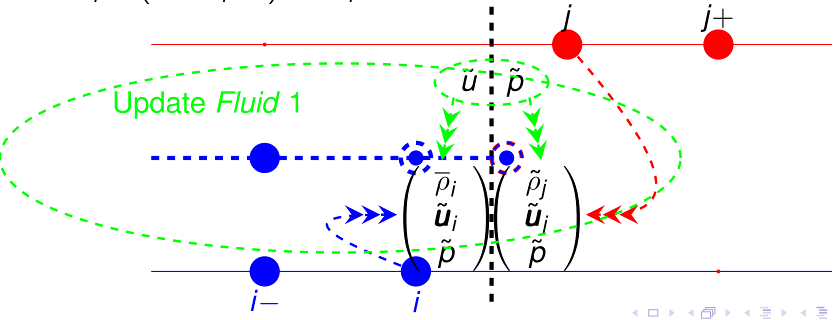
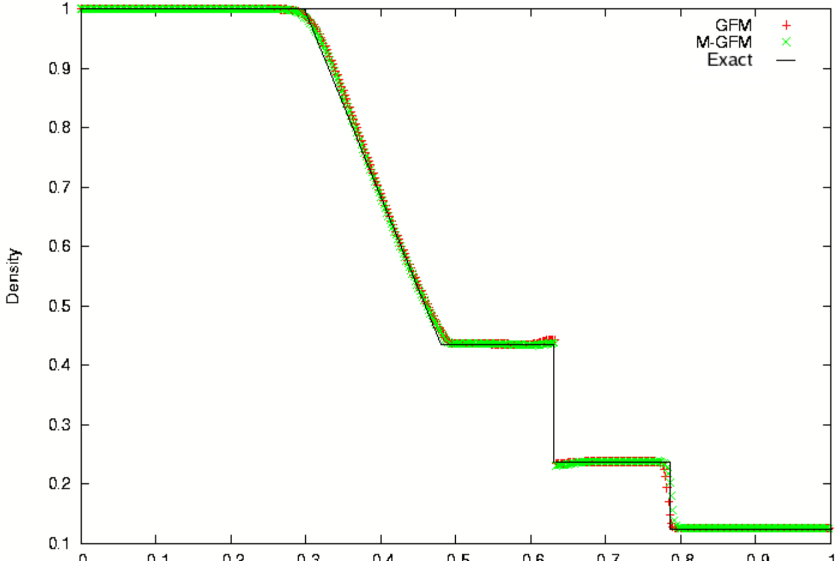
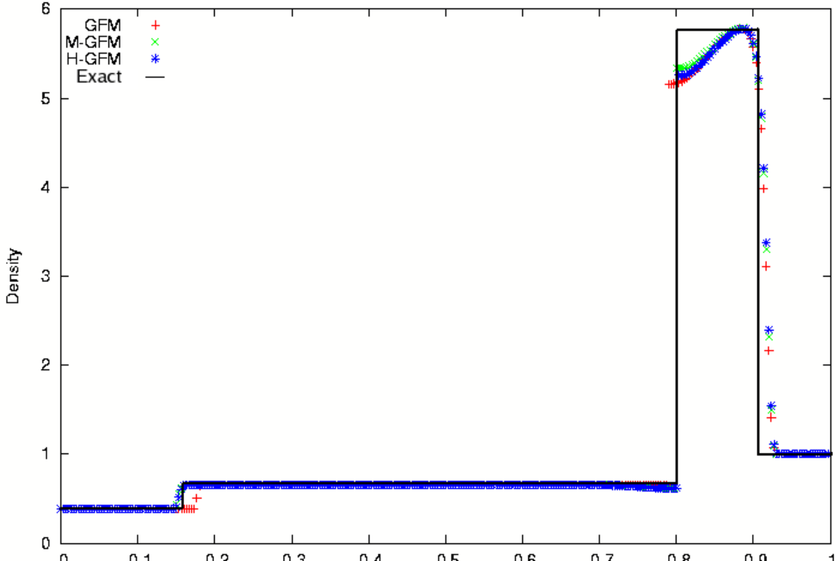


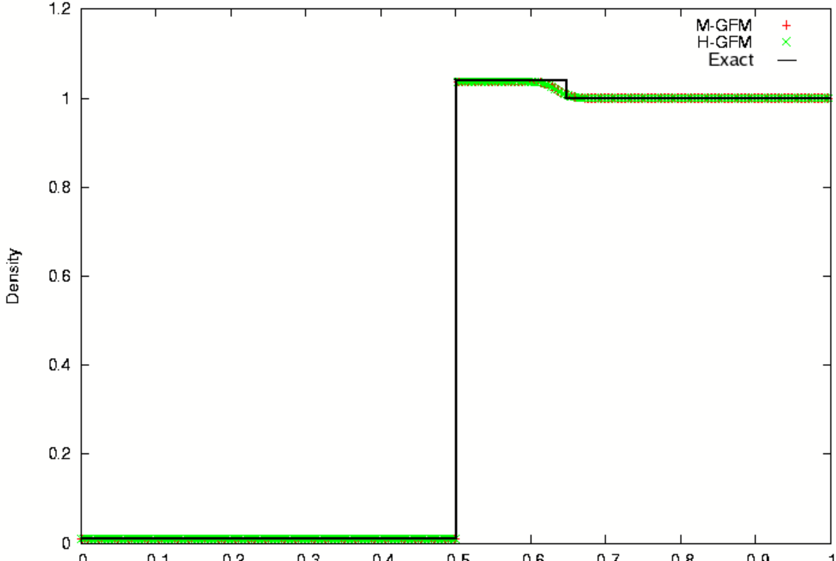
Sod shock tube



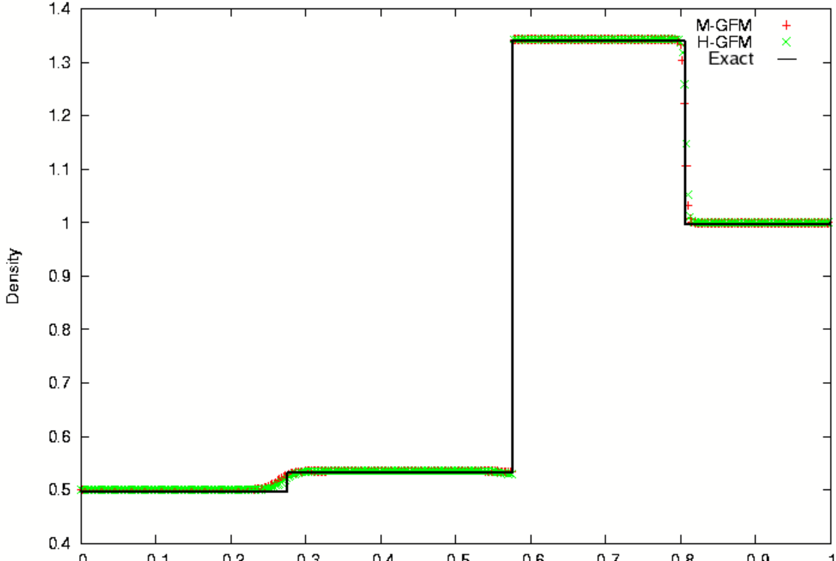
Helium-Air shock tube



Shock interaction with Water-Air interface



Stronger shock interaction with Water-Air interface



Outline

Multi-D Finite volume schemes

Directions Based Schemes

$$a_i\omega_i^{n+1} = a_i\omega_i^n - \sum_{j\in\nu(i)} \Phi\left(\boldsymbol{n}_{ij},\omega_i^n,\omega_j^n\right) - \sum_{j\in\kappa(i)} \Phi_{\mathcal{S}}\left(\boldsymbol{n}_{ij},\omega_i^n,\omega_j^n\right)$$

$$\Phi_{\mathcal{S}}\left(\boldsymbol{n}_{ij},\omega_i^n,\omega_j^n\right) = \Phi\left(\boldsymbol{n}_{ij},\tilde{\omega}_i^n,\tilde{\omega}_j^n\right)$$

$$\tilde{\omega}_i^n = \tilde{\omega}\left(\omega_i,\omega_j,\nabla\phi|_{ij},\omega_{i-},\omega_{j+}\right)$$

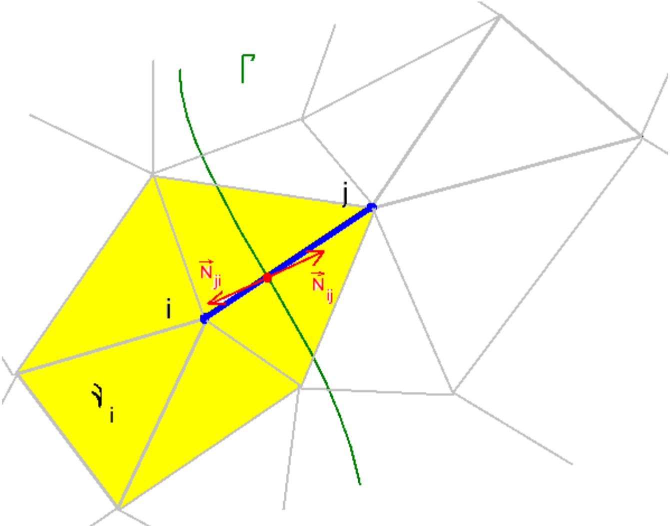
$$\tilde{\omega}_j^n = \tilde{\omega}\left(\omega_i,\omega_j,\nabla\phi|_{ij},\omega_{i-},\omega_{j+}\right)$$

Need to be defined

- The interface normal $\nabla\phi|_{ij}$
- The states ω_{i-} and ω_{j+}

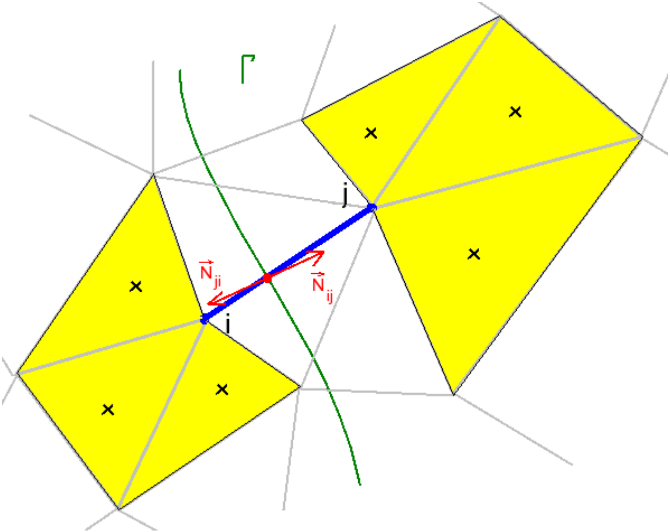
Interfaces parameters for GFM

Stencil for interface normal computation: $N_{ij} = \tilde{\nabla}\varphi_{ij}$



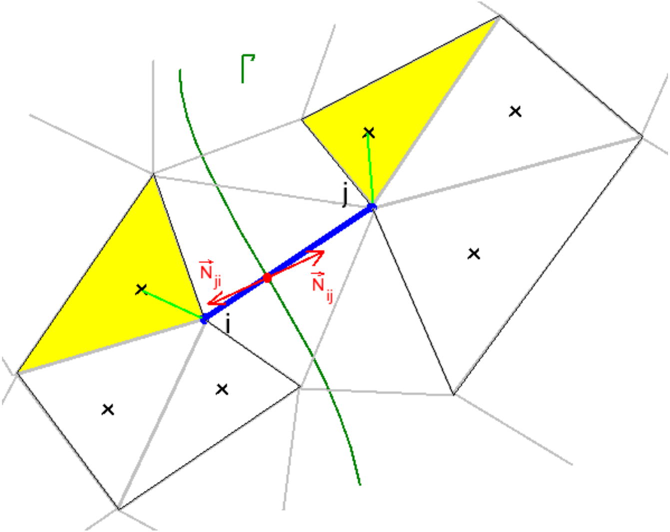
Interfaces parameters for GFM

Stencil for “isobaric 1” ω_{i-} and ω_{j+}



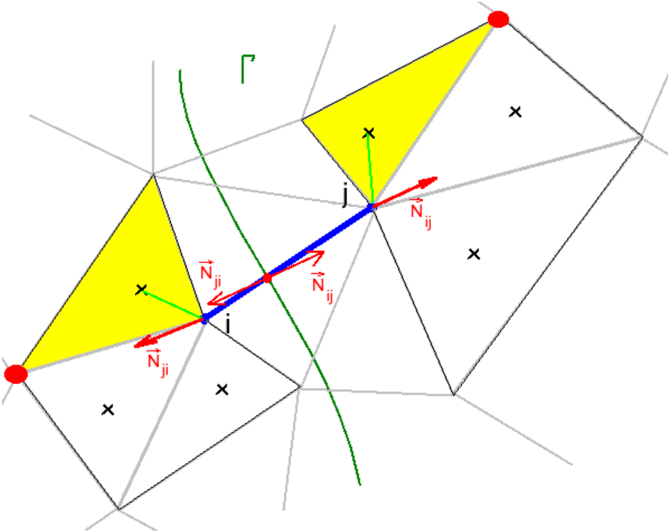
Interfaces parameters for GFM

Stencil for “isobaric 2” ω_{i-} and ω_{j+}



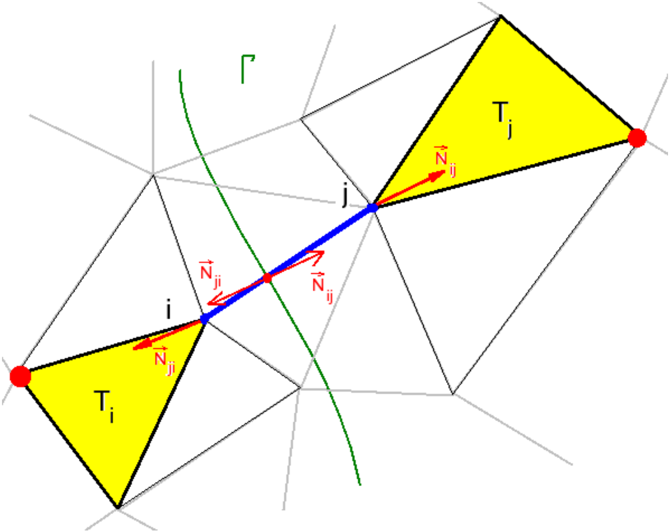
Interfaces parameters for GFM

Stencil for “isobaric 3” ω_{i-} and ω_{j+}

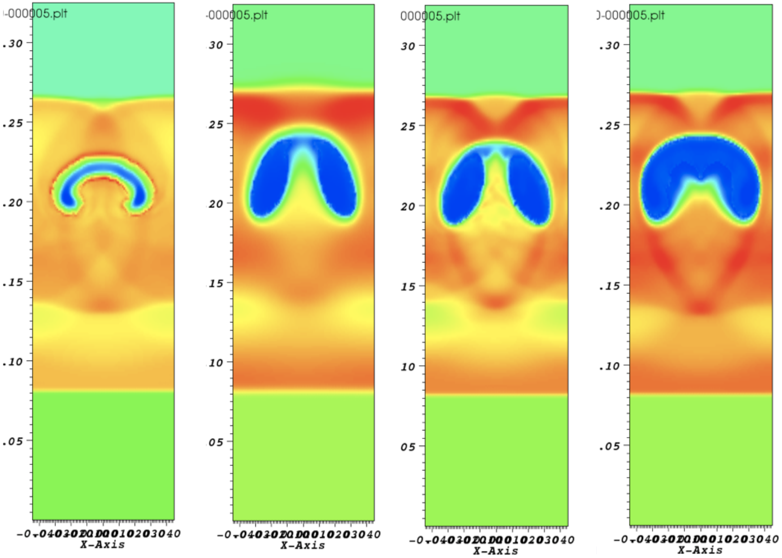


Interfaces parameters for GFM

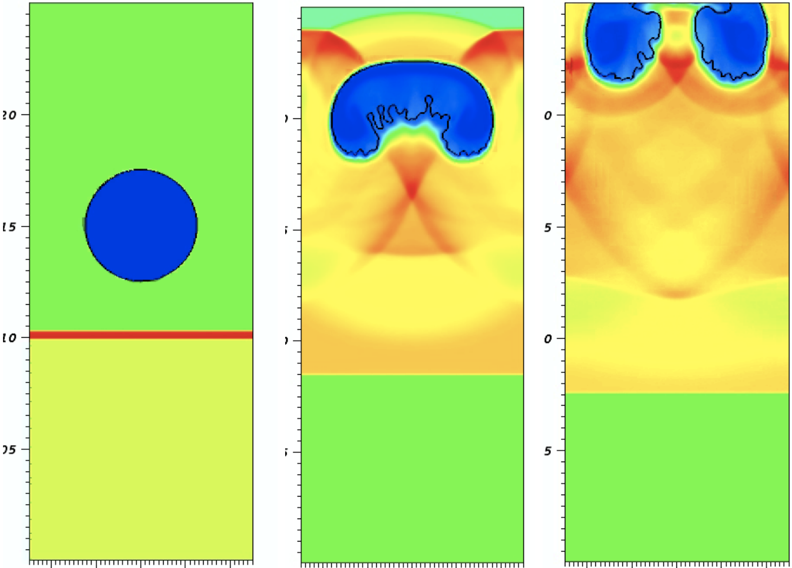
Stencil for “isobaric 4” ω_{i-} and ω_{j+}



Shock/bubble interaction: Air-Helium



Shock/bubble interaction: Air-water



Shock/bubble interaction: Air-Helium

(blabla)

Shock/bubble interaction: Air-water

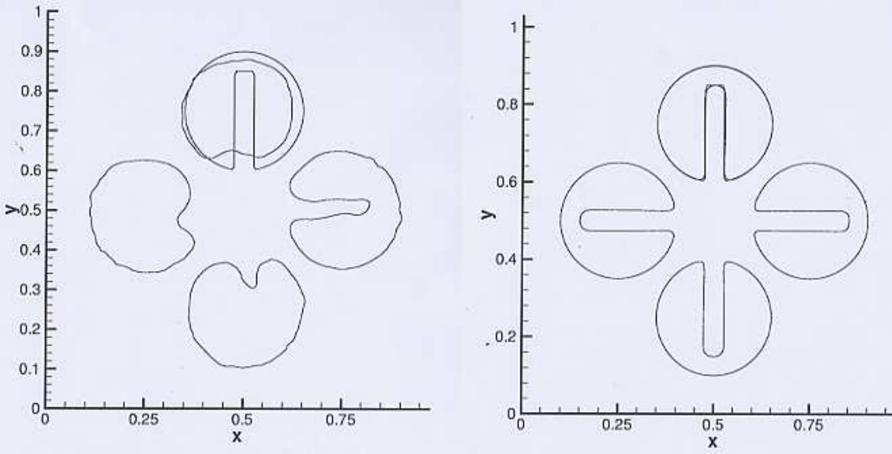
(blabla)

Shock/bubble interaction: Air-water

(blabla)

Outline

DG for the transport of the level set



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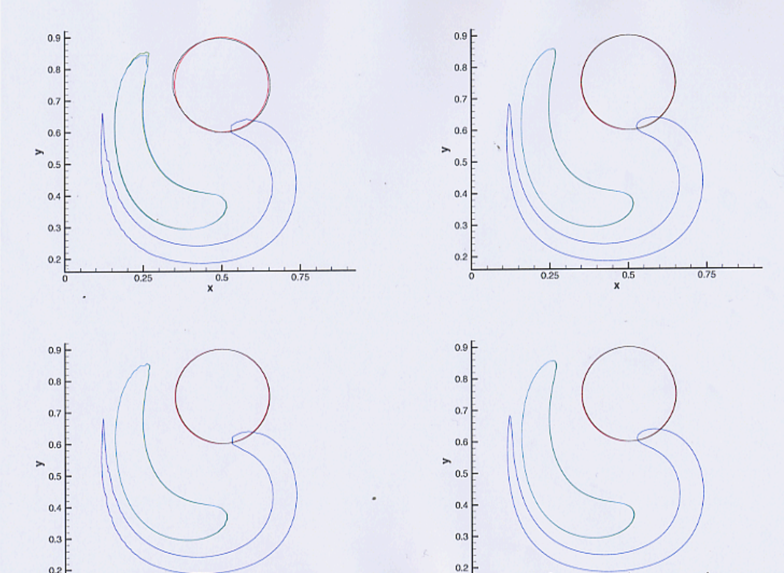
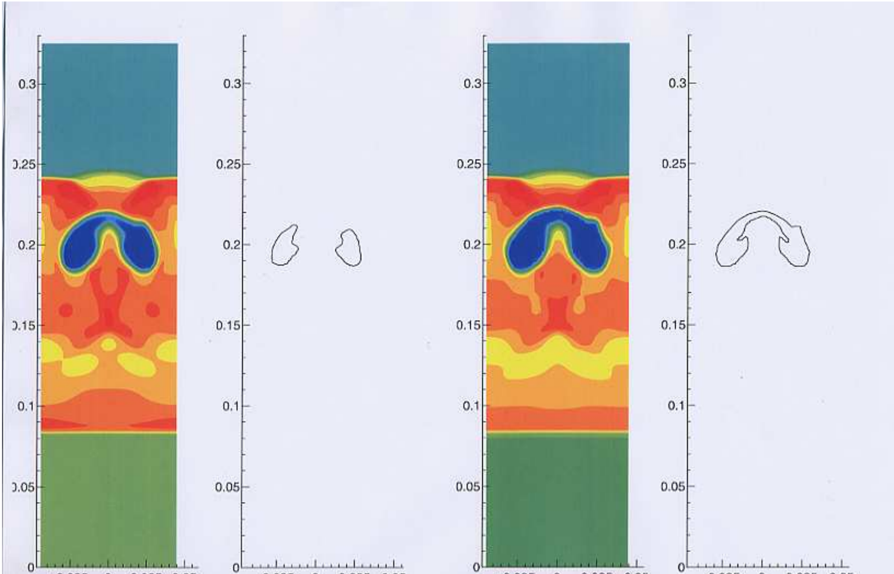


Shock/bubble interaction: Barth vs. DG (P1)



Compressible/Incompressible Interfaces

Compressible Model: $\omega_1 = (\rho, \rho \boldsymbol{u}, \rho e)^T$ Hyperbolic system.

Compressible component is defined as previously.

Incompressible Model: $\omega_2 = (\pi, \boldsymbol{u})^T$

1. $\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \frac{1}{\rho}\nabla \pi = 0$
2. $\nabla \cdot \boldsymbol{u} = 0$

Finite element formulation

Stabilized Galerkin method (with mass lumping)

- $$a_i \omega_i^{n+1} = a_i \omega_i^n - \sum_{\tau \in T(i)} \Phi\left(\omega_\tau^n\right) - \sum_{\tau \in K(i)} \Phi_{\mathcal{S}}\left(\omega_\tau^n\right)$$
- $$a_i \phi_i^{n+1} = a_i \phi_i^n - \mathcal{R}_i\left(\phi^n, \omega^{n+1}\right)$$

Stabilizations techniques

- SUPG for convection
- PSPG to handle LBB condition
- Grad-Div to enforce the incompressible constraint.

Ghost Fluid

$$\Phi_{\mathcal{S}}\left(\omega_\tau^n\right) = \Phi\left(\tilde{\omega}_\tau^n\right)$$

Other Issues

Example

- Deflagation Detonation (Fedkiw 1999)
- Eulerian Fluid/Lagrangian Solid (Fedkiw 2002, Cirak 2004)
- Thin flame and LES premixed combustion (Moureau et al. 2005)
- Phase transition (Gibou et al. 2006)
- Surface tention

Main points to deal with

1. Define the equation for the level set function.
2. Set out appropriate jump conditions at the interfaces.

Les bonnes choses ont une fin !!!!!!!!!! merci aux organisateurs !!!!!!!!!!

Thanks

and farewell!