The Vofire (Reconstructed Finite Volume) method for the tracking of interfaces on unstructured meshes

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Outline

- Motivations
- Geometric formulation for pure advection
- Application in the ALE framework
- Results in 3D
- Conclusions

Motivations

- Physical constraints (for instance for the hohlraum calculation)
  - Several materials and phases
    (for instance four for an ICF target and four for the surrounding cavity)
  - Very different properties for the materials
    (equation of states, opacity, reaction,...)

- Numerical features
  - Millions of cells in 3D
  - Million(s) of mixed cells
  - Unstructured meshes
  - Parallel Calculation

- Need : a robust, quick and efficient reconstruction method

Subjective review of interface tracking

	locally conservative in mass	interface geometry	cheap	robust	parallel
Front-T. [a]	- -	++	-	- -	-
Level-Set [b]	- -	+	+	+	+
VOF [c]	+ [d]	+	-	+	+

[a] S.O. Unverdi - G. Tryggvasen (1992) J. Comput. Phys.

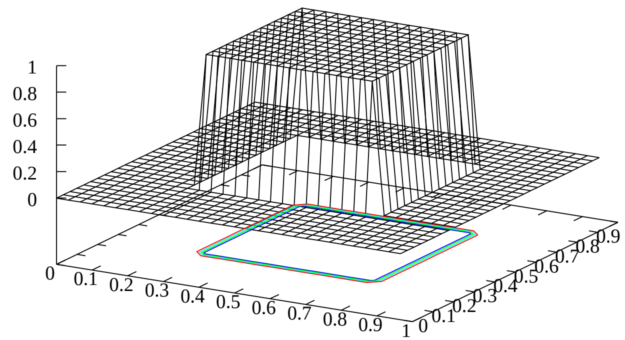
[b] C.W. Hirt - B.D. Nichols (1981) J. Comput. Phys.

[c] B.J. Parker - D.L. Youngs (1992) UK Atomic Weapon Establishment report

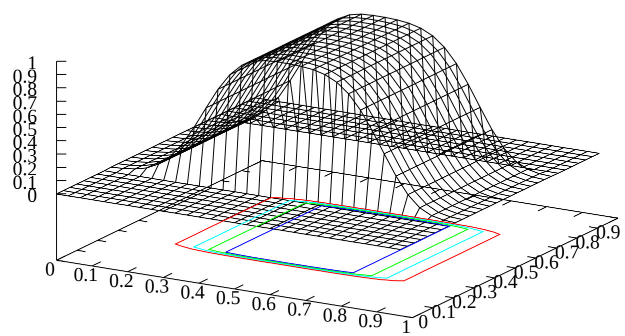
[d] Controled by the Newton convergency

Initial observation

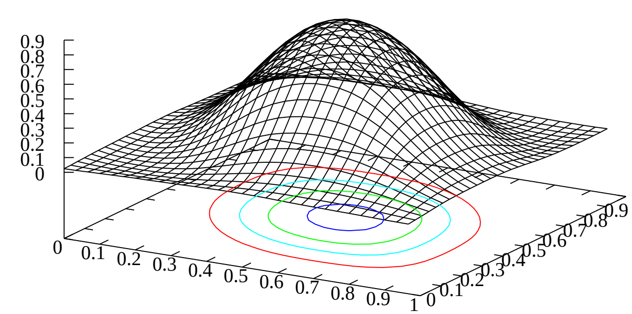
We observe the numerical diffusion in elementary advection test cases is made up of two distinct phemomena in the case of a transport velocity unaligned with the mesh main directions.



initial value



aligned mesh



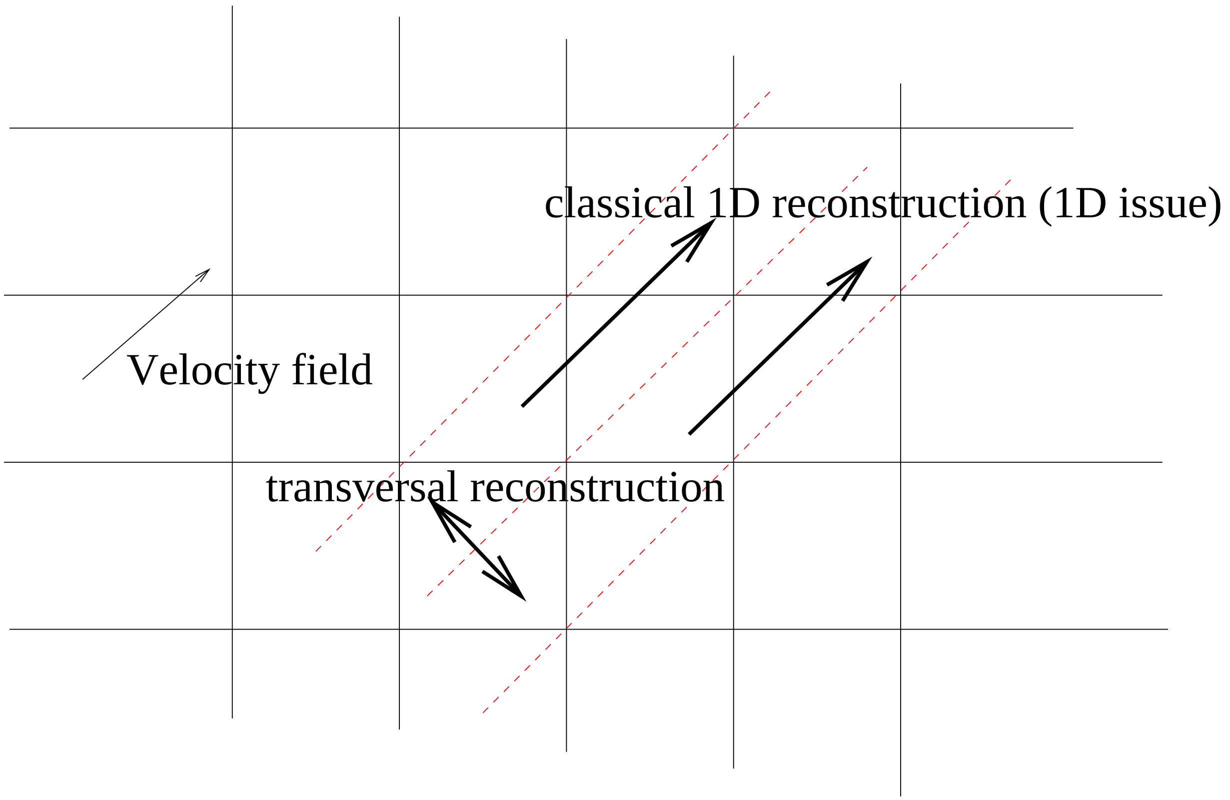
unaligned mesh

Therefore, we must distinguish between the diffusion parallel to the transport direction (1D), and the transversal diffusion (multi-D).

How to deal with this issue ?

An anti-diffusive process must act

- in the direction normal to the velocity field,
- then in the direction of the velocity field.

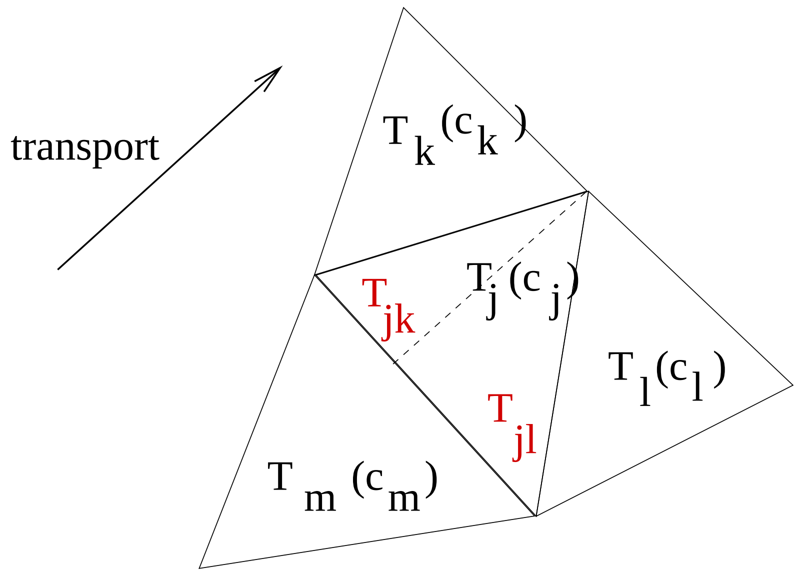


sharp interface reconstruction in each cell

Geometric formulation (1/3)

We focus on the pure advection equation : $\partial_t c + a.\nabla c = 0$.

First stage : the transversal reconstruction



Notations

We call s_j the surface of T_j. T_j is geometrically split into two subgrid triangles T_{jk} and T_{jl} of surface s_{jk} and s_{jl}, with $s_{jk} + s_{jl} = s_j$.

How to reconstruct c_{jk} and c_{jl} (subgrid values of c in T_{jk} and T_{jl}) ?

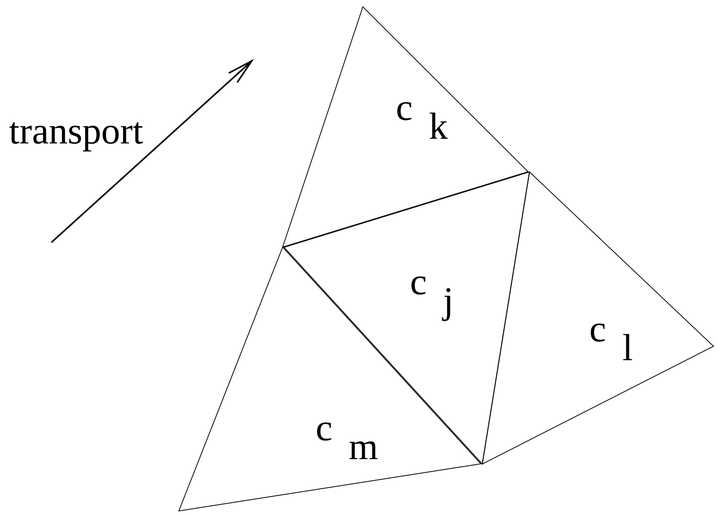
Goal : c_{jk} has to be as close as possible of c_k, c_{jl} has to be as close as possible of c_l.

Constraints : conservative reconstruction, maximum principle preservation.

Geometric formulation (2/3)

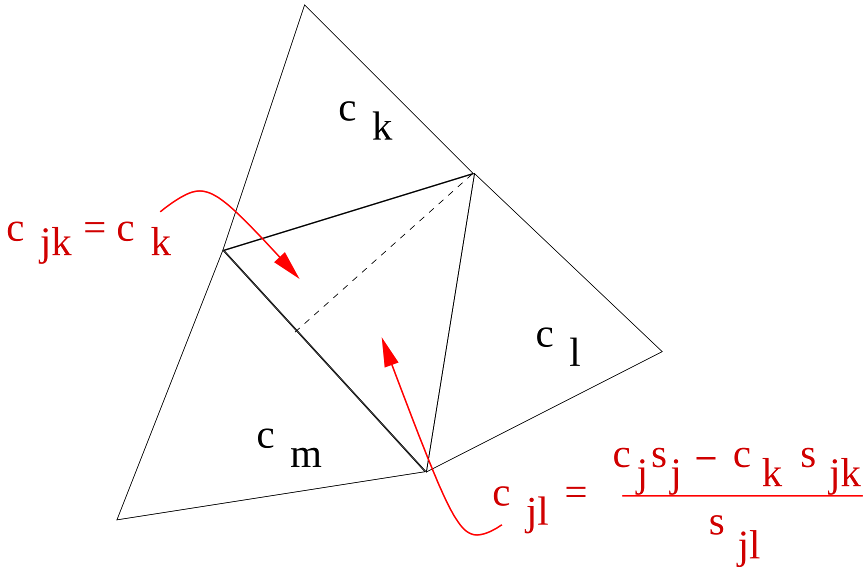
Assuming for instance $c_k \leq c_l$, 4 configurations can occur :

a)



$c_j \notin]c_k, c_l[$
no transversal reconstruction

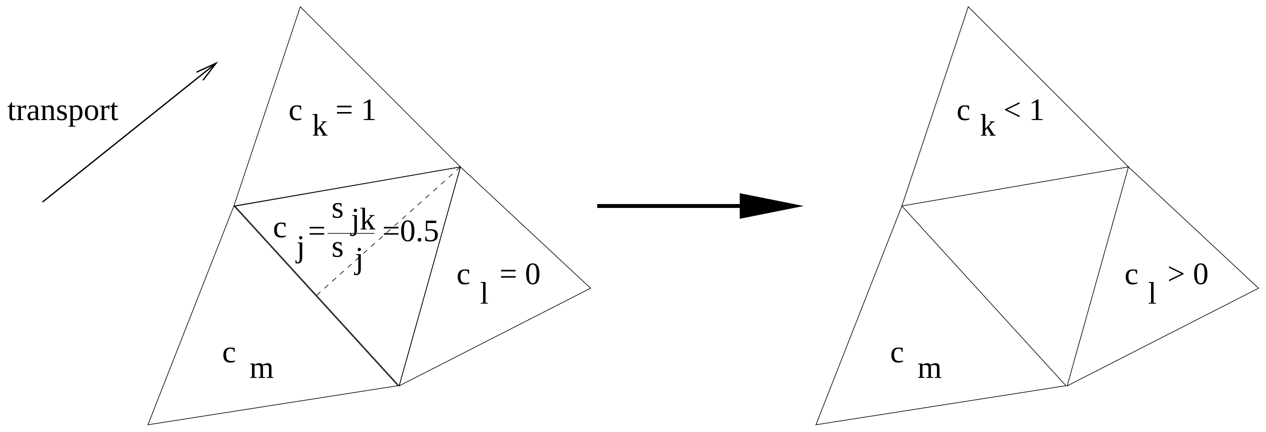
b)



$c_j \in]c_k, c_l[$
$|c_k - c_j| s_{jk} \leq |c_l - c_j|$
sharp, conservative reconstruction

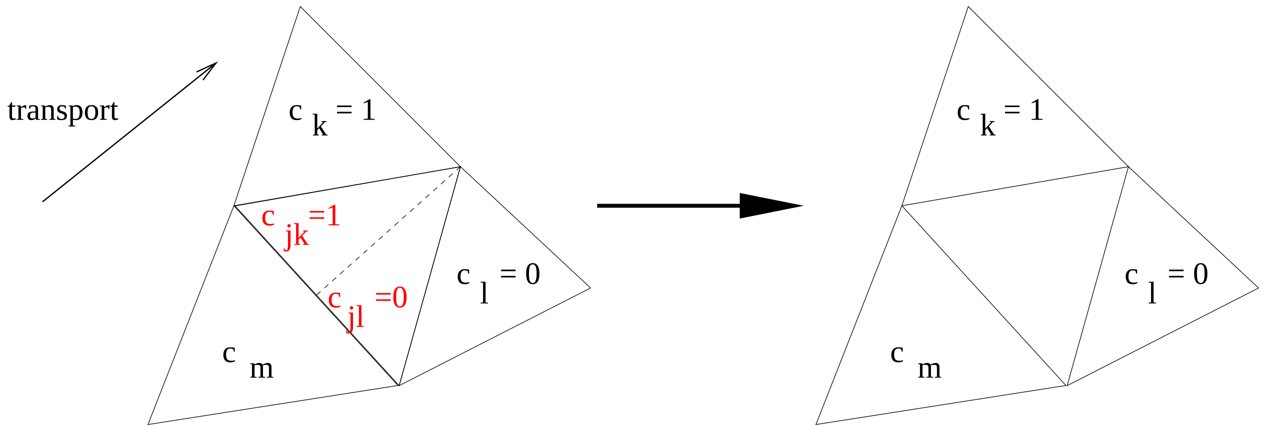
c) The other cases are similar.

Example of reconstruction (1/2)



Diffusion occurs

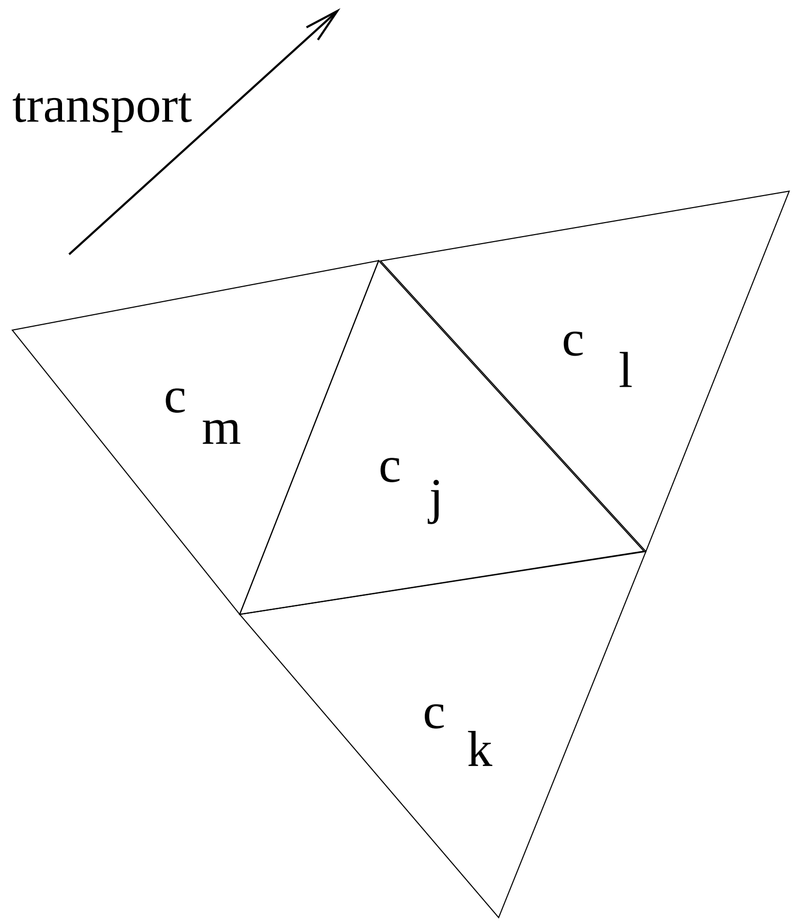
No reconstruction : the advection phase is diffusive,



No diffusion

With transversal reconstruction : no more diffusion.

Example of reconstruction (2/2)



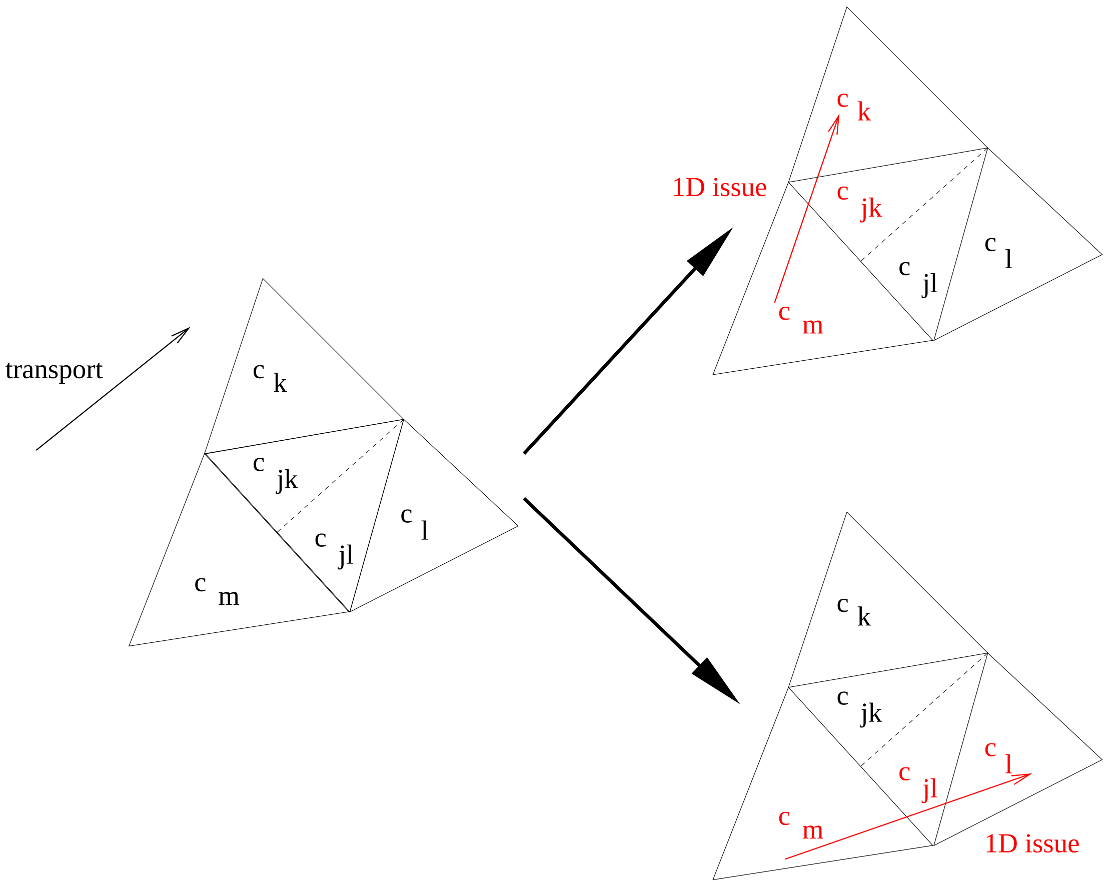
We can not reconstruct (in cell j)

No transversal reconstruction possible, but no “transversal” diffusion anyway, because there is only one downwind cell.

Geometric formulation (3/3)

Second stage : transport direction

After the transversal reconstruction, 1D problems remain.

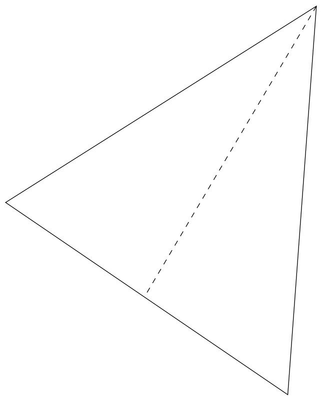


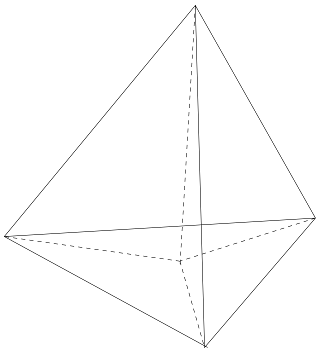
construction of two 1D problems

To be as anti-diffusive as possible, these problems are solved using the downwind scheme under stability constraint.

Other cell shape?

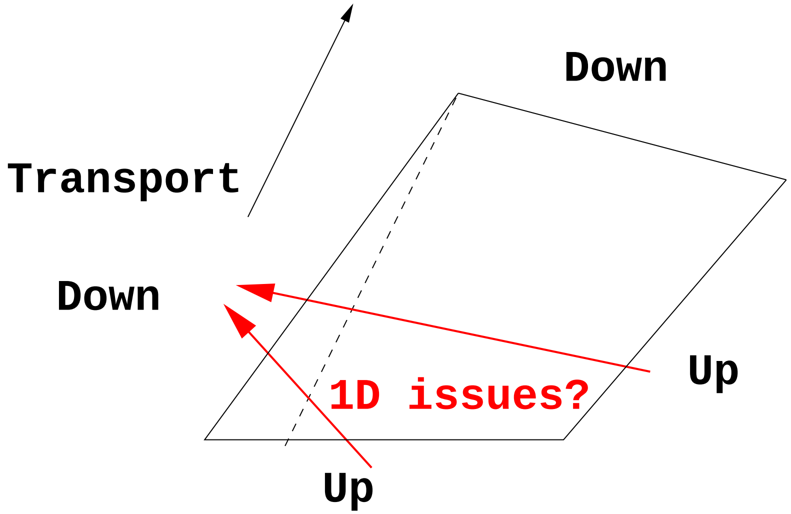
a)

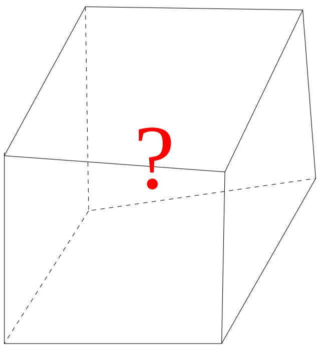




geometric formulation adapted

b)





geometric formulation not adapted

This geometrical formulation can be recast as an algebraic formulation on 3D and on any cells, in minimizing fonctionals (not presented here).
For the case a), the algebraic formulation is equivalent to the geometric formulation.

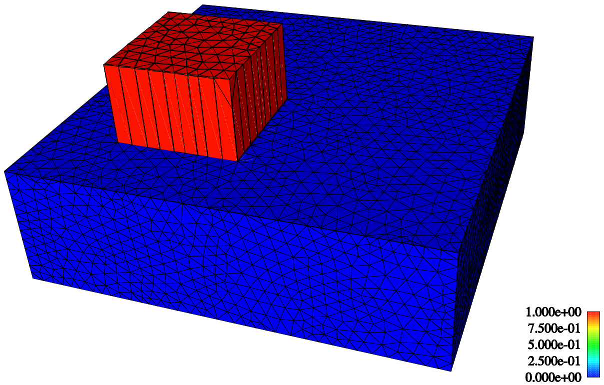
Summary

The scheme is made up of three stages:

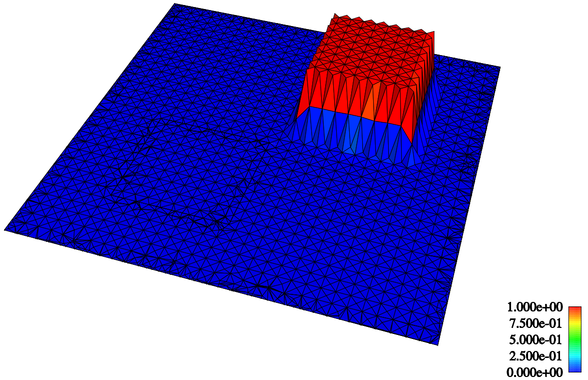
- *the transversal reconstruction stage* - The *constant by cell, discontinuous at faces* field is recast as a *discontinuous in cell, as continuous as possible at faces* field,
- *the transport direction stage* - Using the previous field, compute the fluxes with the UltraBee calculation,
- *the projection stage* - Compute the resulting *constant by cell, discontinuous at faces* field.

Advection test case

Pure advection in the diagonal direction (on tetraedra : 2D solution, 3D mesh).



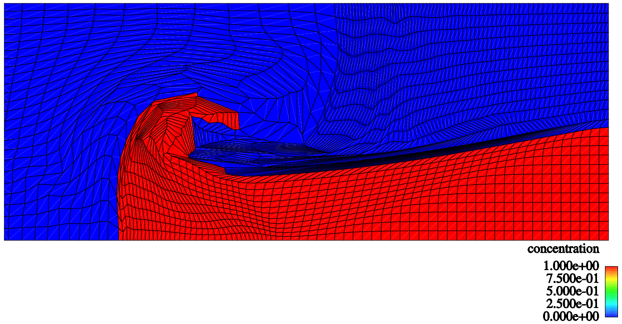
mesh & extruded initial value

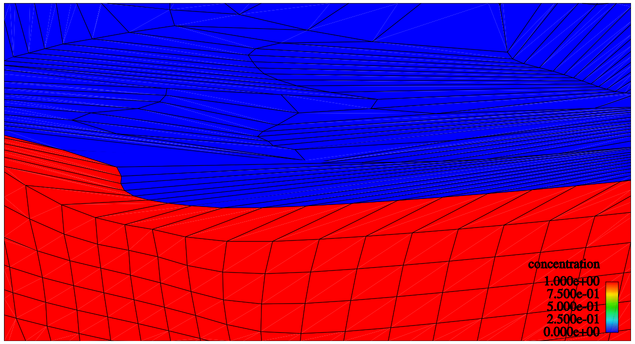


extruded solution for section $z = z_{ave}$

Issues in lagrangian framework

Lagrangian : the mesh follows the material interfaces

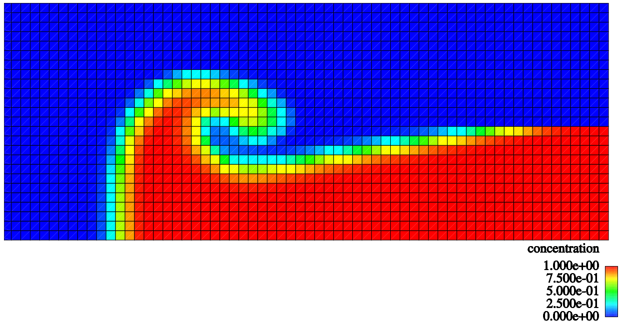


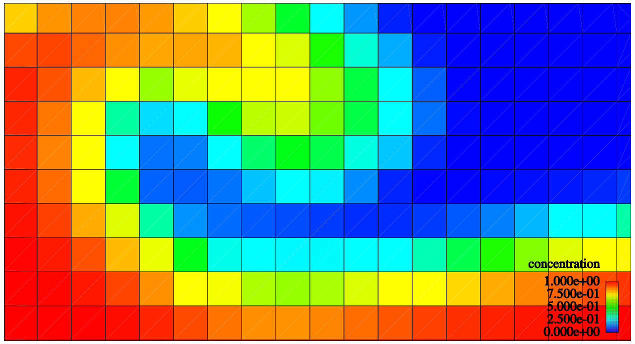


+ interfaces match the mesh

- the mesh is deformed with the interfaces

Eulerian : the mesh is fixed





+ mesh is free from the interfaces

- interfaces are diffused

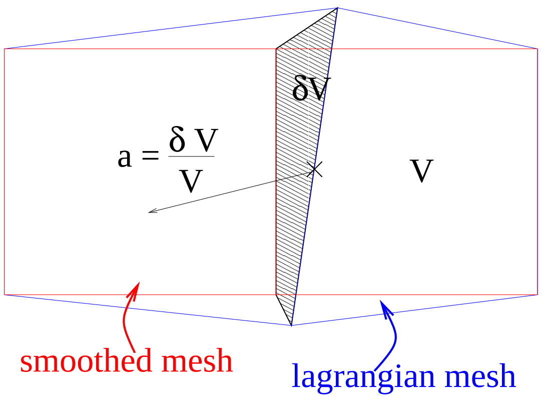
ALE method

- Lagrangian phase

$$\frac{d}{dt}\int_{V_{lag}(t)} \rho dV = 0$$
$$\frac{d}{dt}\int_{V_{lag}(t)} \rho u dV + \int_{S(t)} P dS = 0$$
$$\frac{d}{dt}\int_{V_{lag}(t)} \rho e dV + \int_{S(t)} P u.n dS = 0$$

- EOS : $(p,T) = f(\rho, e)$
- smoothing of the resulting mesh
- Projection phase

$$\frac{D}{Dt}\int_{V_{ale}(t)} \rho_i dV + \int_{S(t)} \rho_i (u - u_g).n dS = 0$$
$$\frac{D}{Dt}\int_{V_{ale}(t)} \rho u dV + \int_{S(t)} \rho u (u - u_g).n dS = 0$$
$$\frac{D}{Dt}\int_{V_{ale}(t)} \rho e dV + \int_{S(t)} \rho e \underbrace{(u - u_g)}_{\vec{a}}.n dS = 0$$



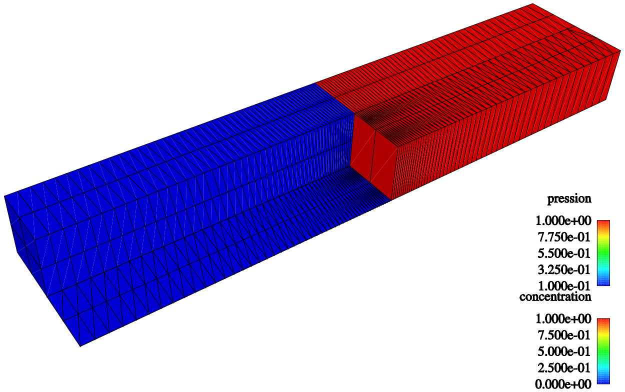
Swept used for the projection

with i the material, $\rho_i = \rho c_i$
and $\sum_i c_i = 1$

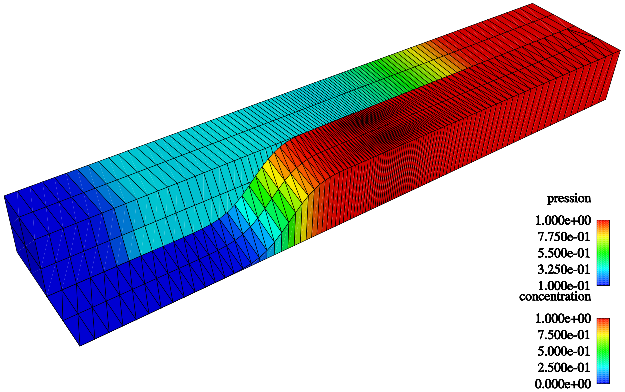
- EOS : $(p,T) = f(\rho, e)$

Results : two-phase shock tube

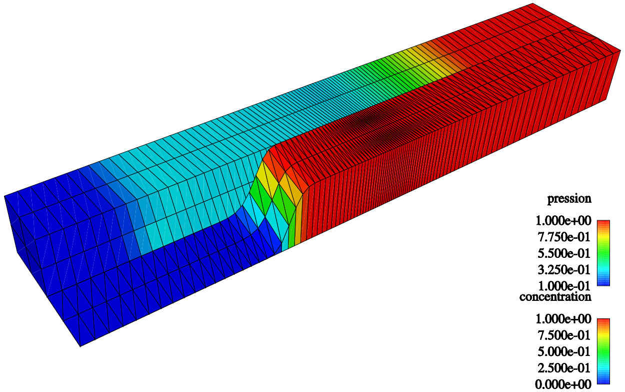
Sod's shock tube on distorted mesh: 1D solution on a 3D mesh



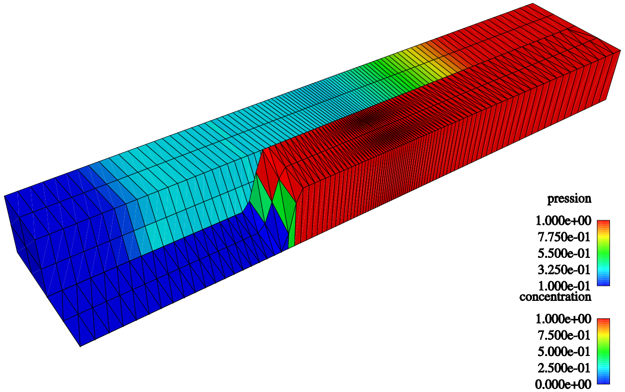
initial conditions



order 1 advection
113 mixed cells



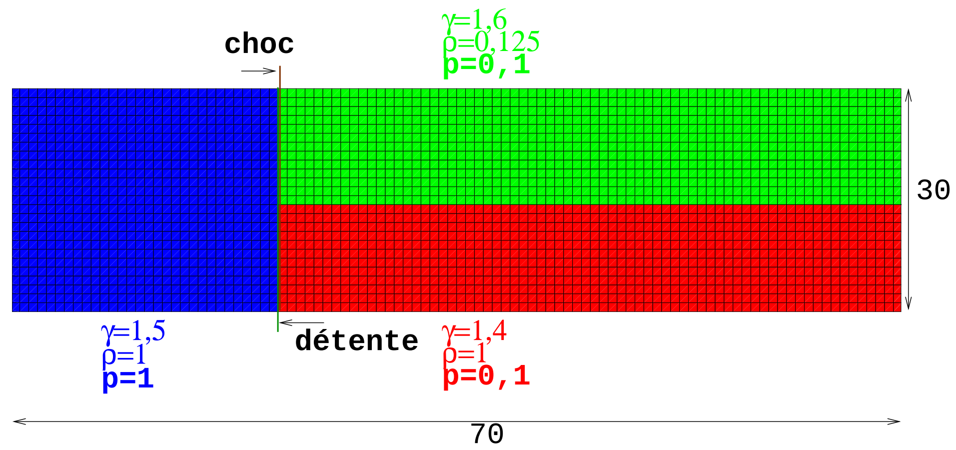
order 2 advection
47 mixed cells



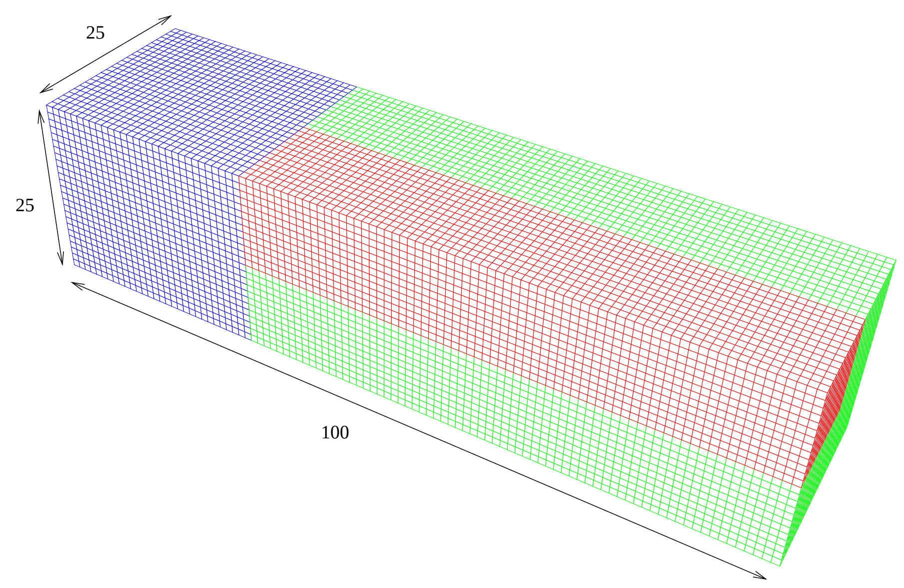
Vofire advection
6 mixed cells

Three materials instability

A shock hits a triple line : difficult benchmark for interface reconstruction (3 interfaces, 1 triple line).



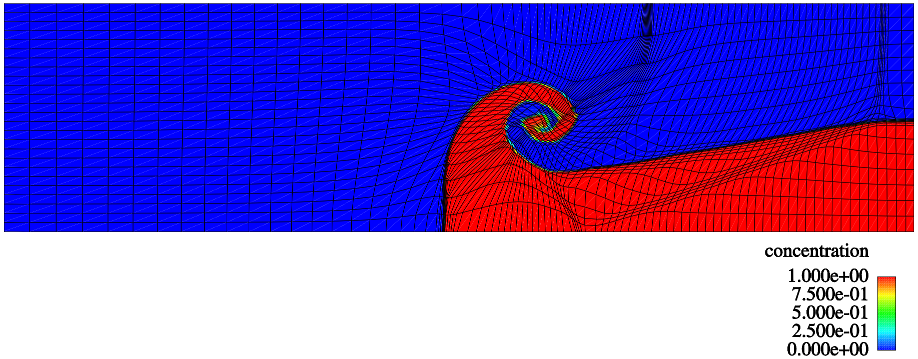
2D

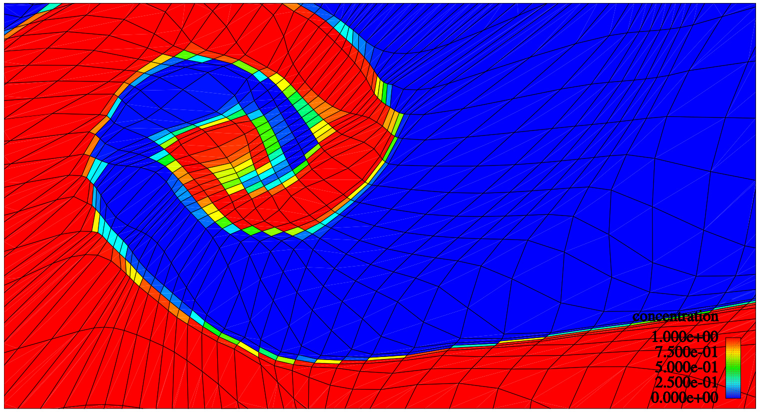


3D

Results : 3 materials instability

- Equipotential smoother (Tipton-Jun), explicit (Jacobi),
- Metrics : intersection of gradient lenghtscales (internal energy, concentrations),
- Swept advection extended at the 2nd order with MUSCL,
- Vofire reconstruction for the concentrations.



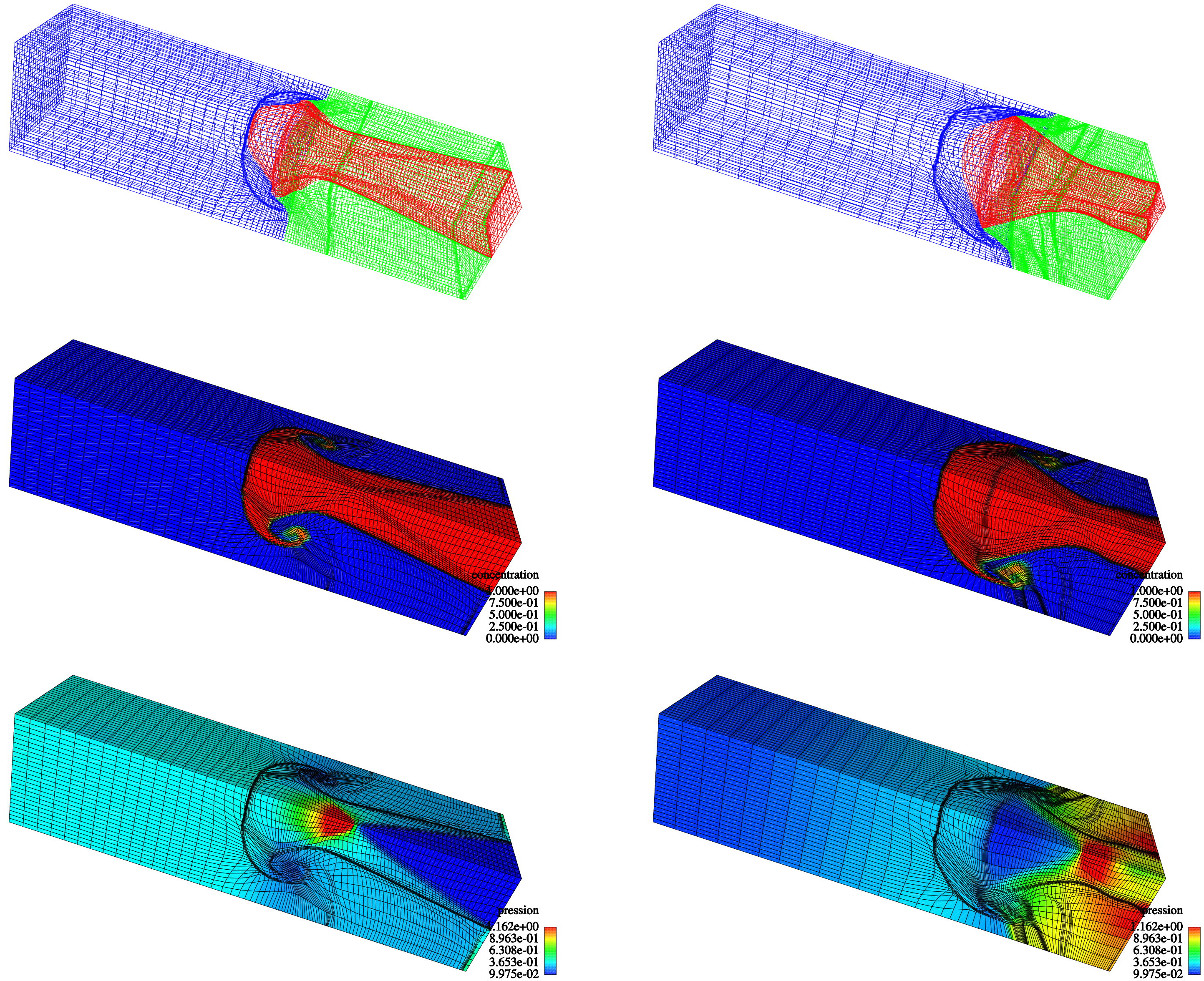


En 3D

mesh

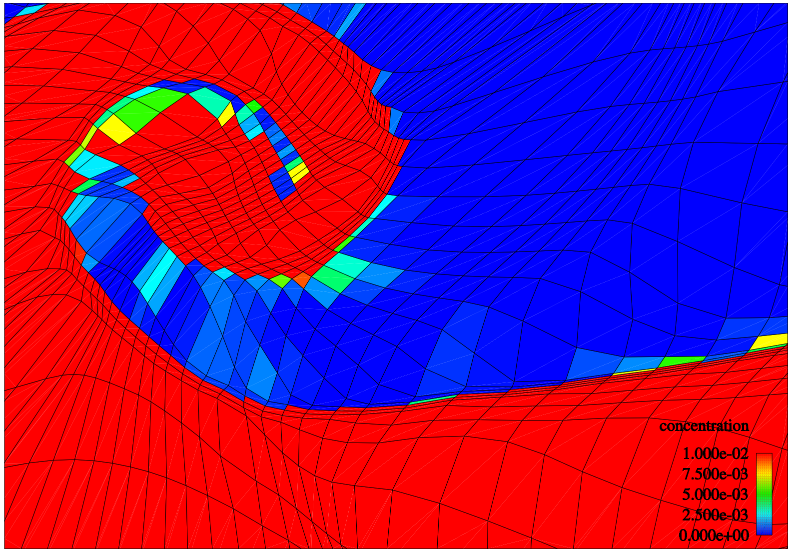
c_1

pressure

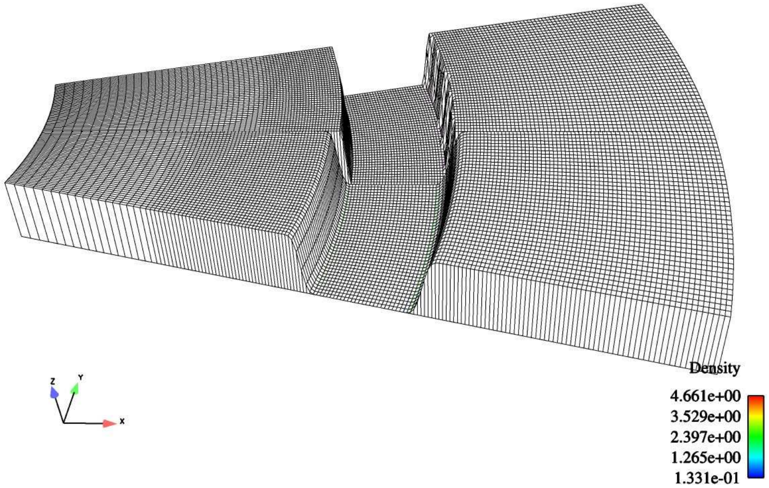


Limitations

Low level numerical diffusion remains (but the interfaces are compact),



Interface reconstruction can initiate a Rayleigh-Taylor (or Richtmyer-Meshkov) instability.



Subjective comparison in 3D

	conserv.	interface geometry	cheap	robust	parallel
Front-T.	- -	++	-	- -	-
Level-Set	- -	+	+	+	+
VOF	+	+	-	+	+
Vofire	++	-[a]	+	++	+

[a] cf. previous slide : limitations

Conclusion and futur works

- We have presented the Vofire method applied to the material interfaces in 3D configuration on unstructured meshes,
- We obtain a very favourable ratio cost/efficiency : *Vofire cost* < *0.5 advection step cost* whereas *Youngs-PLIC cost* > *10 advection step cost*,
- Discontinuities remain compact (the diffusion zone is finite),
- Parallel implementation is not difficult,
- A paper is in preparation for publication.