Magneto-Hydro-Dynamic Instabilities in Tokamak Plasmas

Guido Huysmans

Association Euratom-CEA
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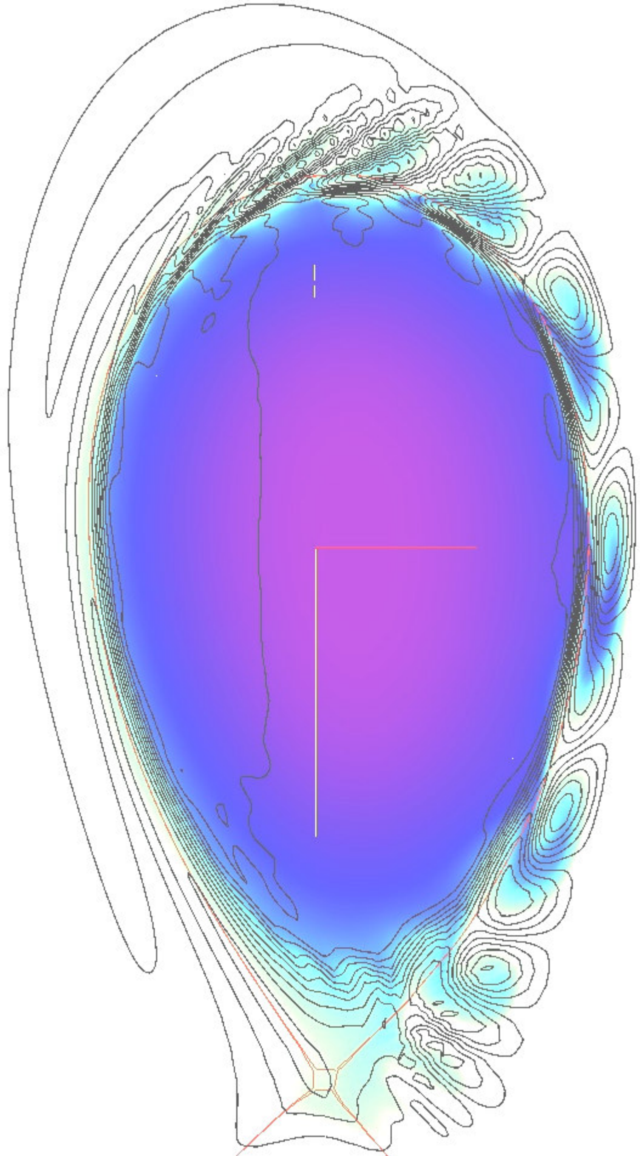
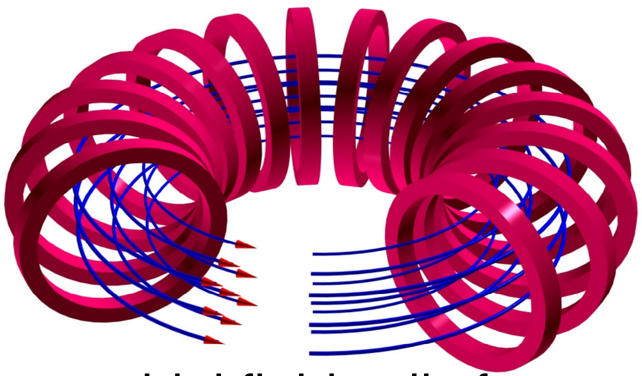


Contributions:
H. Lutjens, P. Maget, A. Kirk, W. Zwingmann

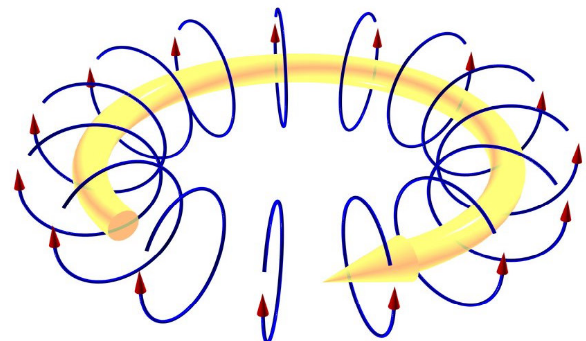
Outline

- Introduction
  - Tokamaks
- Magneto-Hydro-Dynamics
  - Equilibrium
  - MHD Instabilities
- Non-linear MHD simulations
  - XTOR : double tearing modes in Tore Supra
  - JOREK : Edge Localised Modes
- Open questions/Conclusion

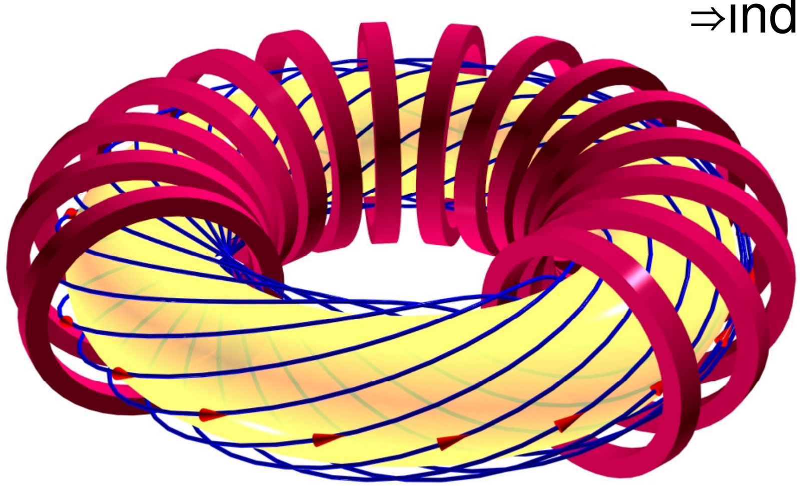
Tokamak



toroidal field coils for
main magnetic field



plasma is secondary ring
of a transformer
⇒induces toroidal current



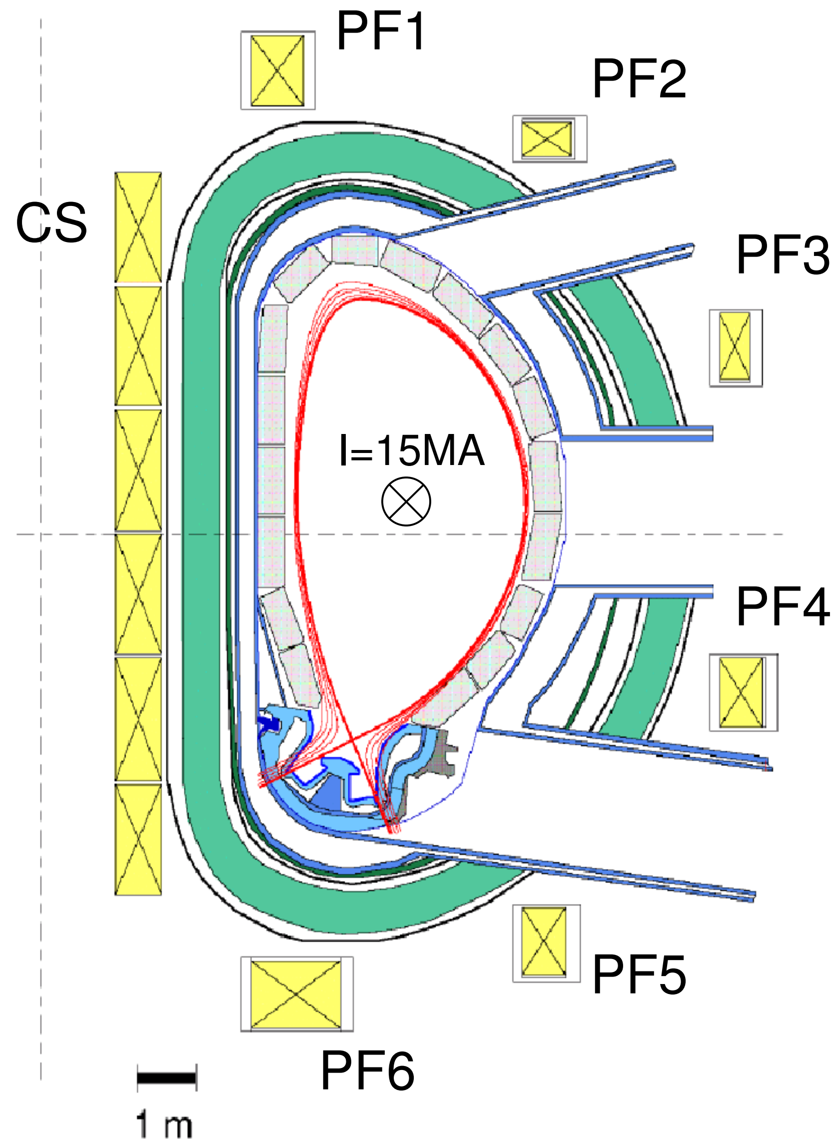
total helical field

winding number ~ 1/q

ITER

- Toroidal field coils (18):
  - B=5.3T (plasma centre)
- Poloidal field coils
  - plasma shaping, X-point
- Central Solenoid
  - Induction of the plasma current (15MA)

Plasma Major/Minor Radius	6.2m / 2.0m
Plasma Volume	840m^3
Plasma Current	15.0MA
Fusion Power	500MW
Burn Flat Top	>400s
Power Amplification	>10
Density	10^{20} m^{-3}
Pressure	2.8x10^5 Pa
$\langle\beta\rangle=2\mu_0\langle P\rangle/B^2$	2.5%



Magneto-Hydro-Dynamics (MHD)

- Plasma model as a conducting fluid in a magnetic field:
  - (mass)Density conservation: $$\frac{\partial \rho}{\partial t} = -\nabla \bullet (\rho \mathbf{v})$$
  - Momentum conservation: $$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho \mathbf{v} \bullet \nabla \mathbf{v} - \nabla p + \frac{1}{\mu_0} \mathbf{J} \times \mathbf{B}$$
  - Energy conservation: $$\frac{\partial p}{\partial t} = -\mathbf{v} \bullet \nabla p - \gamma p \nabla \bullet \mathbf{v}$$
  - Faraday: $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$
  - Ohm's Law: $$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{J}$$

$$\nabla \bullet \mathbf{B} = 0$$

Typical time scale/speed

- Normalisation momentum equation:

$$v = v_0 \tilde{v} = \frac{a}{t_0}\tilde{v} \qquad t = t_0 \tilde{t} \qquad \rho = \rho_0 \tilde{\rho} \quad B = B_0 \tilde{B} \quad p = \frac{B_0^2}{\mu_0}\tilde{p} \quad J = \frac{B_0}{a\mu_0}\tilde{J}$$

$$\frac{\rho_0 a}{t_0^2}\tilde{\rho}\frac{d\tilde{\mathbf{v}}}{d\tilde{t}} = -\frac{B_0^2}{\mu_0 a}\left(\tilde{\nabla}\tilde{p} + \frac{1}{\mu_0}\tilde{\mathbf{J}}\times\tilde{\mathbf{B}}\right) \qquad t_0 = \frac{a\sqrt{\mu_0\rho_0}}{B_0}$$

Alfvén time

$$B_0 = 5.3 \text{ T}$$
$$\rho_0 = 10^{20}\times\left(1.67\times10^{-27}\right)\times 2 \text{ kg/m}^3$$
$$a = 2 \text{ m}$$

$$t_A = 2.4\times10^{-7} \text{ s}$$
$$V_A = 8.2\times10^{6} \text{ m/s}$$

- Ohms Law: $\mathbf{E} = -\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}$

$$E = \left(V_A B_0\right)\tilde{E} \qquad \eta = \left(\mu_0 a V_A\right)\tilde{\eta} \qquad \tilde{\eta} = \frac{1}{S}$$

$\eta = 10^{-9}$ Ohm m (at T=10keV)

magnetic Reynolds number : $S = 2\times10^{10}$

Equilibrium

- Static equilibrium: $\frac{\partial}{\partial t}=0 \quad \mathbf{v}=0 \quad \frac{\partial}{\partial \varphi}=0$

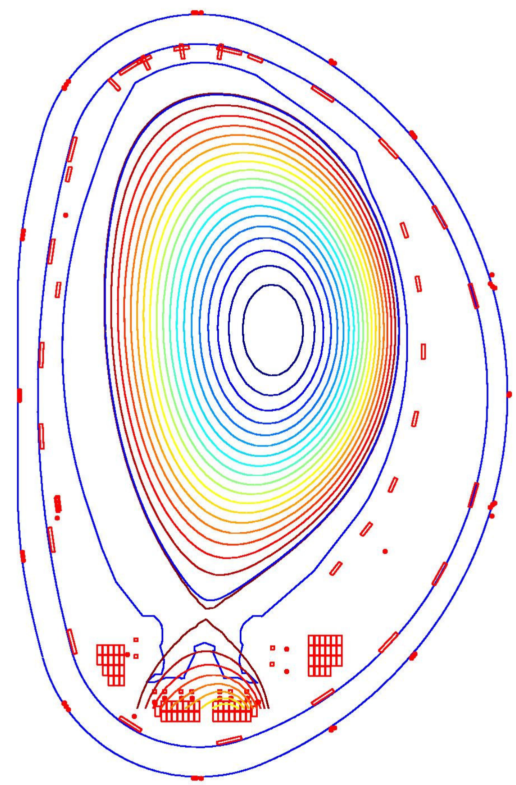
- Force balance between pressure gradient and the Lorentz force:

$$\nabla p = \frac{1}{\mu_0}\mathbf{J}\times\mathbf{B} \qquad \nabla\bullet\mathbf{B}=0$$

- Poloidal flux ψ: $\mathbf{B}=\frac{\mathrm{F}(\psi)}{\mathrm{R}}\mathbf{e}_\varphi+\frac{1}{\mathrm{R}}\nabla\psi\times\mathbf{e}_\varphi$

- Grad-Shafranov equation describes axisymmetric equilibrium:

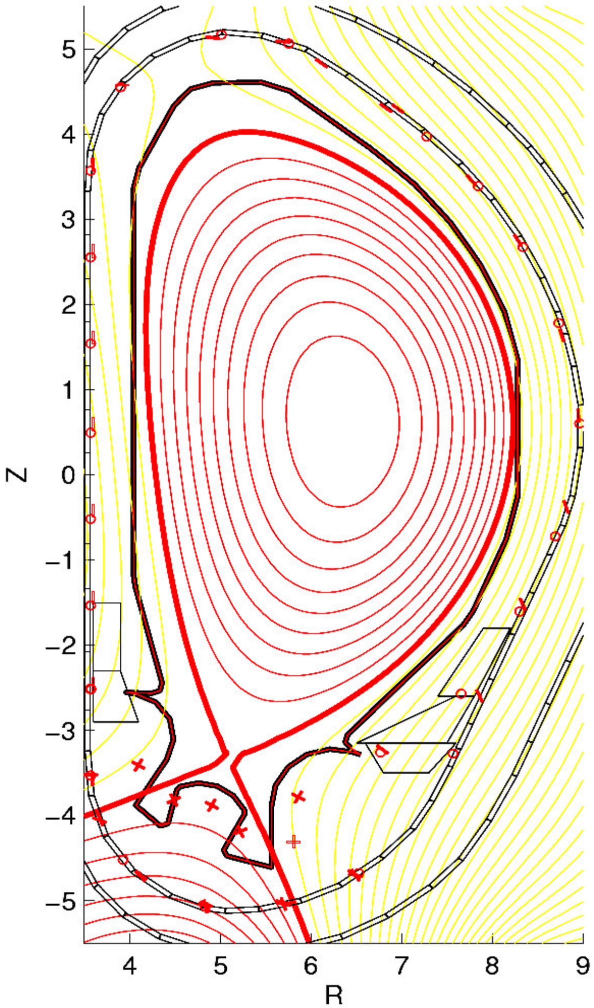
$$R^2\nabla\left(\frac{1}{R^2}\nabla\psi\right)=-RJ_\varphi=-\mu_0R^2p'(\psi)-F(\psi)F'(\psi)$$



JET equilibrium

Equilibrium Reconstruction

- Equilibria can be reconstructed from measurements of magnetic field outside the plasma.
  - minimisation measurements with values from numerical solution
  - Internal measurements of pressure and magnetic field can also be used



ITER equilibrium reconstruction using EFIT (W. Zwingmann)

See J. Blum, tuesday

Ideal MHD Instabilities

- Driving forces for ideal (no dissipation) MHD instabilities:
  - Parallel current density
  - Pressure gradient

Compression magnetic field
fast waves

Bending of magnetic field lines
Alfven waves

Compression of pressure,
sound (slow) waves

$$\delta W = \tfrac{1}{2}\int dV \left(\left|B_{1,\perp}\right|^2 + B_0^2 \left|\nabla \cdot \xi_\perp + 2\xi_\perp \cdot \kappa\right|^2 + \lambda p_0 \left|\nabla \cdot \xi\right|^2 \right)$$

$$- \int dV \left(2(\xi_\perp \cdot \nabla p_0)(\kappa \cdot \xi_\perp) + J_{0,\parallel}(\xi_\perp \times B_0 / B_0) \cdot B_{1,\perp} \right)$$

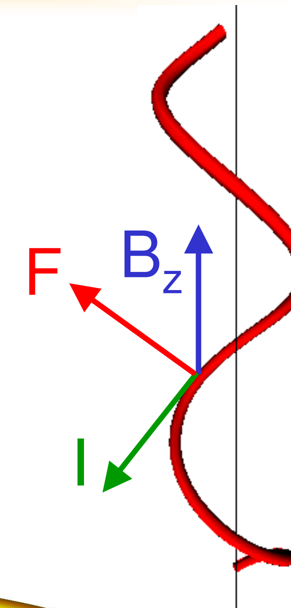
Pressure gradient
Curvature(K)
Ballooning instability

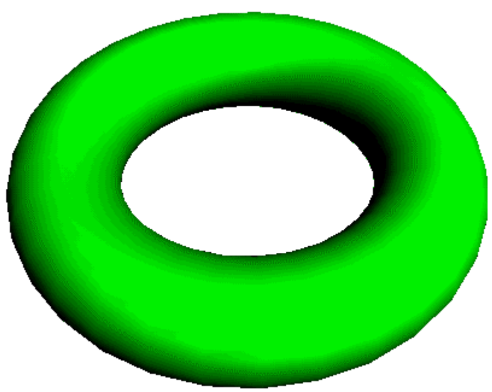
Parallel current drive
kink instability

External Kink modes

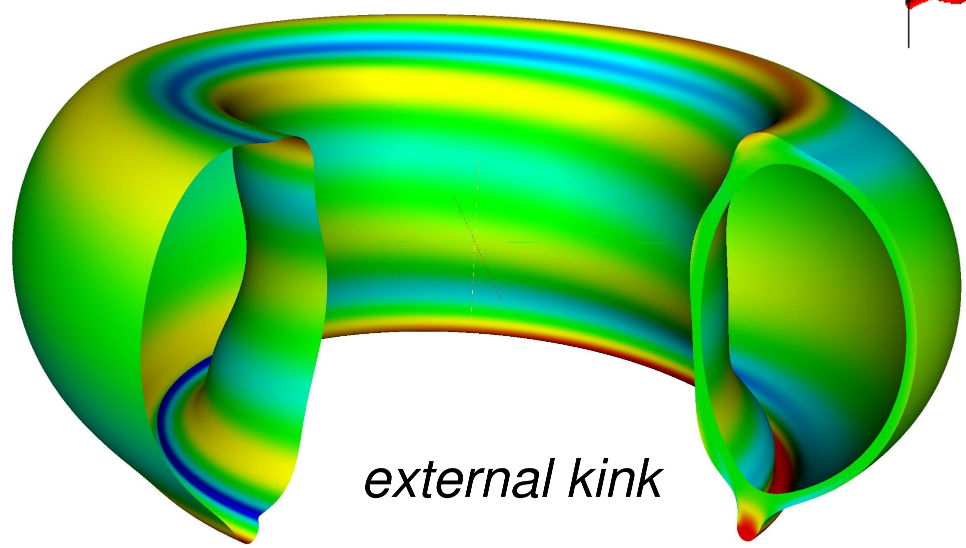
- Simplest model for current driven kink modes is a current carrying wire in a parallel magnetic field
  - unstable to helical deformation
- Ideal MHD kink mode deforms surface
  - driven by parallel current
  - requires a rational q surface just outside plasma
  - Magnetic topology remains the same in ideal MHD

$$\lambda = \sqrt{I_0 B_z k / \rho}$$





Cartoon of n=1/m=3 kink mode



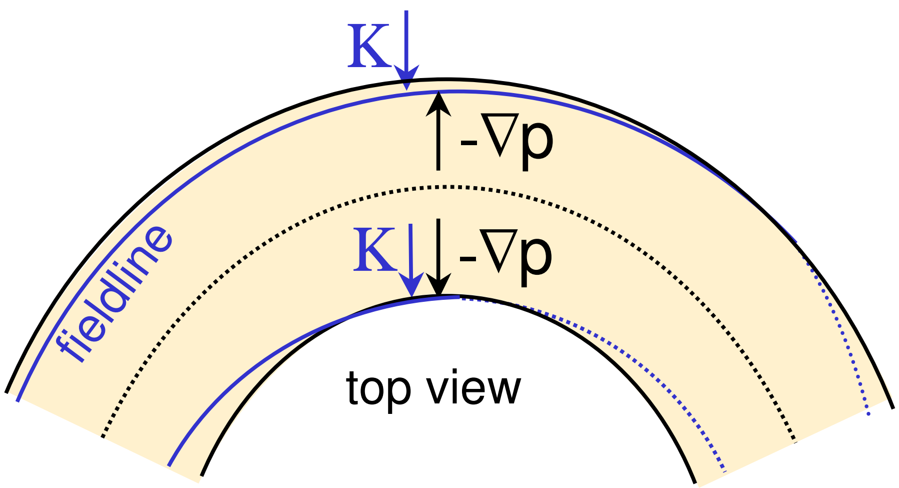
external kink instability in JET tokamak

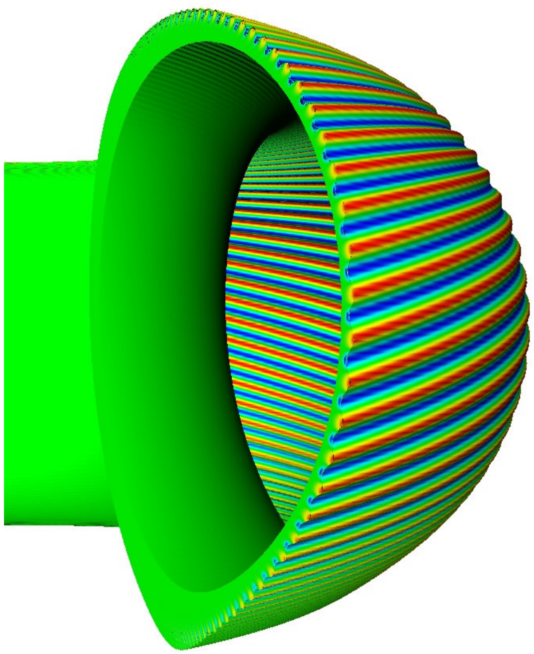
Ballooning Modes

- Instability drive: pressure gradient (∇p) against curvature (K)
  - Unstable on outside of torus, stabilising on inside

  ⇒ ballooning mode localised on low-field (outer-side) of torus

  - radially localised (in high pressure gradient region) to avoid stabilising bending of magnetic field lines
  - High toroidal mode numbers most unstable

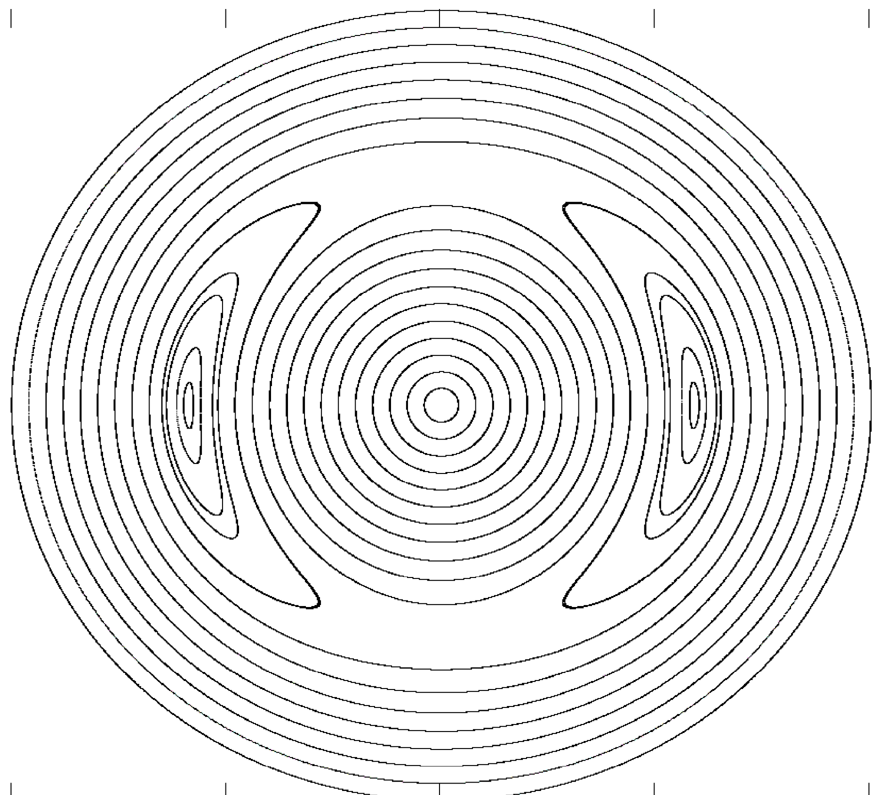




n=10
ballooning mode

Tearing modes

- Finite resistivity allows a change of topology of magnetic configuration
  - Tearing modes, driven by current gradients, lead to the formation of magnetic islands on rational q (=m/n) surfaces
    - Local flattening of current and pressure profile
  - "Neoclassical" tearing modes are driven by a local pressure gradient
    - absence of the pressure gradient (bootstrap current) inside (existing island) increases island size
    - requires a large enough initial perturbation, f.e. by another MHD mode
    - can lead to pressure limit below ideal MHD stability limits



m=2 islands

Linear MHD

- Linear ideal MHD model :
  - generalised eigenvalue problem

$$\mathbf{A}\mathbf{x} = \omega \mathbf{B}\mathbf{x}$$

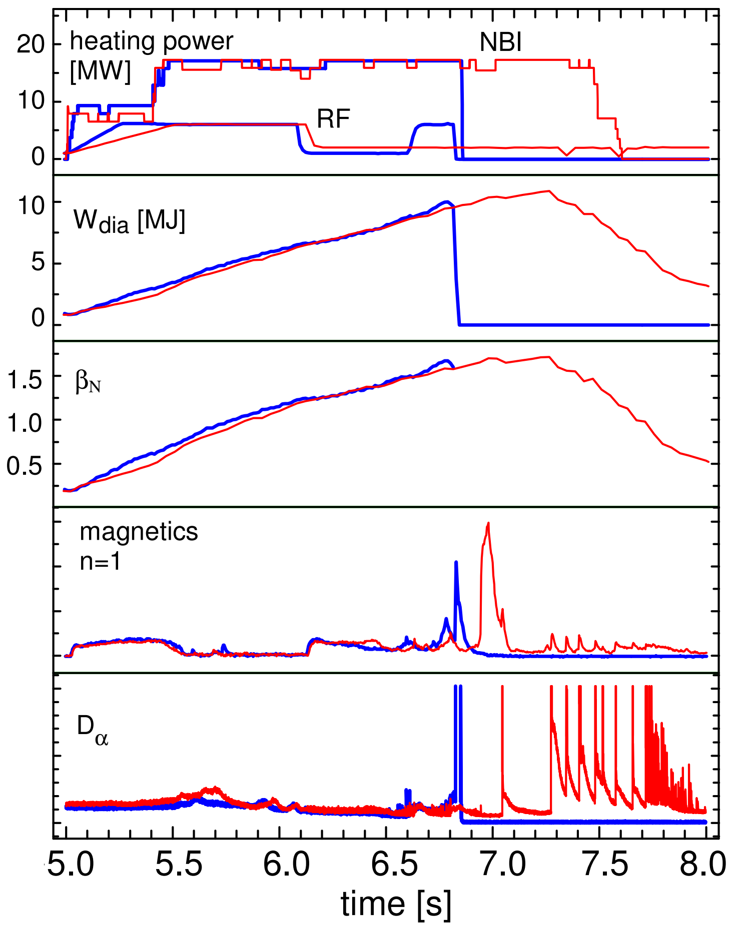
$$p(t) = p_0 + p_1 e^{i\omega t}; p_1 << p_0$$
$$\mathbf{v}(t) = \mathbf{v}_1 e^{i\omega t}; \mathbf{v}_0 = 0$$
$$\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1 e^{i\omega t}; \mathbf{B}_1 << \mathbf{B}_0$$

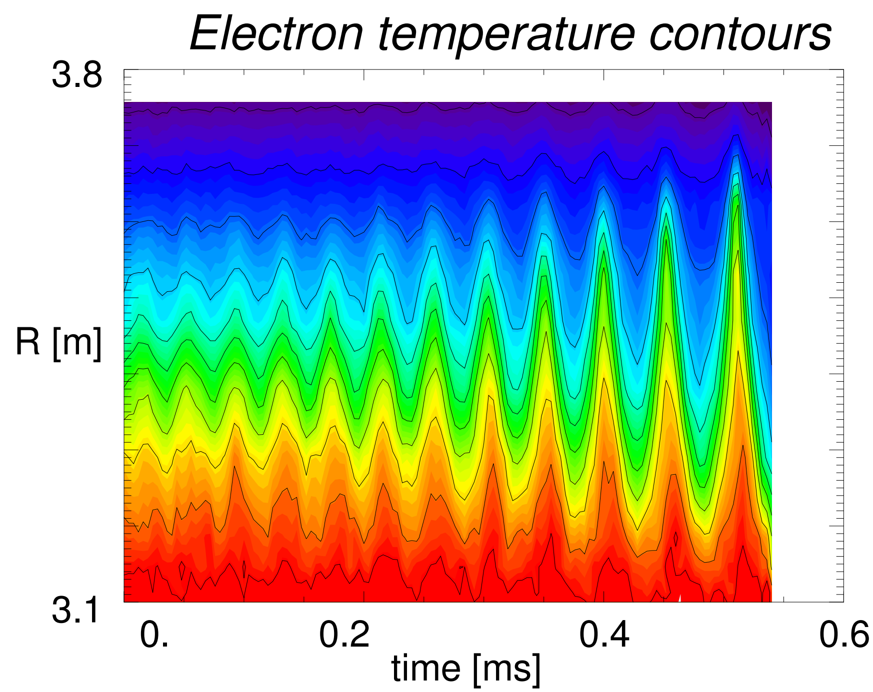
- Linear MHD codes give:
  - MHD stability limits
  - MHD mode structures
  - MHD spectrum of waves
- Codes are well established:
  - MISHKA, CASTOR, MARS, PEST, …
- Routinely used to analyse experiments

$$\omega \rho_0 \mathbf{v}_1 = -\nabla p_1 + \frac{1}{\mu_0}(\nabla \times \mathbf{B}_0) \times \mathbf{B}_1 + (\nabla \times \mathbf{B}_1) \times \mathbf{B}_0$$
$$\omega p_1 = -\mathbf{v}_1 \bullet \nabla p_0 - \gamma p_0 \nabla \bullet \mathbf{v}_1$$
$$\omega \mathbf{B}_1 = \nabla \times (\mathbf{v}_1 \times \mathbf{B}_0)$$

Example Disruption in JET Tokamak

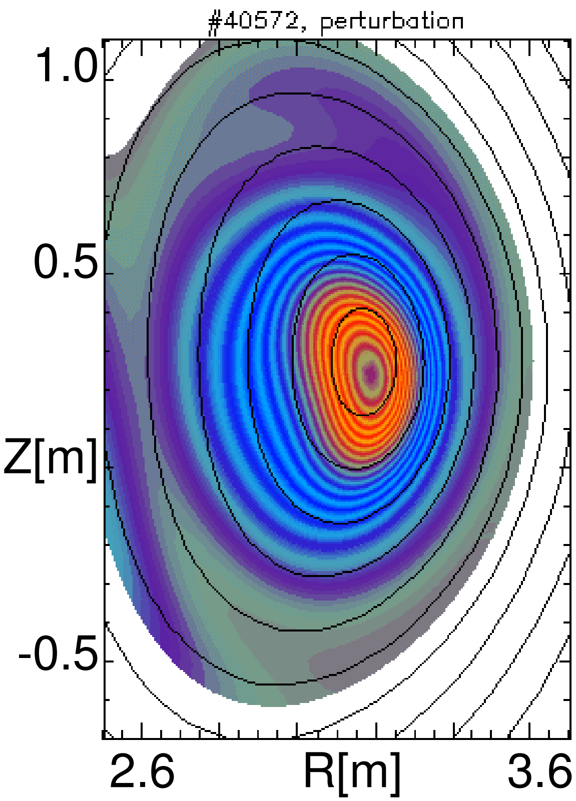
- peaked pressure profiles in ‘advanced scenarios’ can lead to sudden end of the plasma : disruption



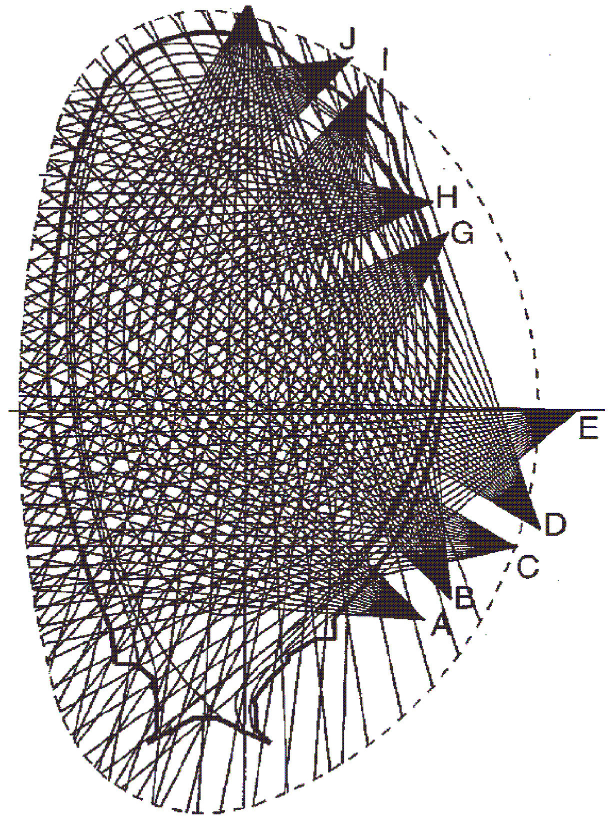


Soft X-ray Tomography

- 2D view of the plasma motion due to the MHD instability just before the disruption :



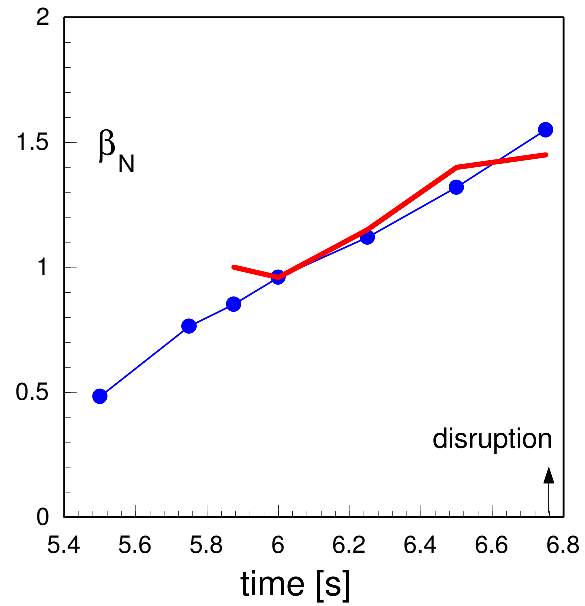
Tomographic reconstruction of X-ray emission



JET SXR cameras (1998)

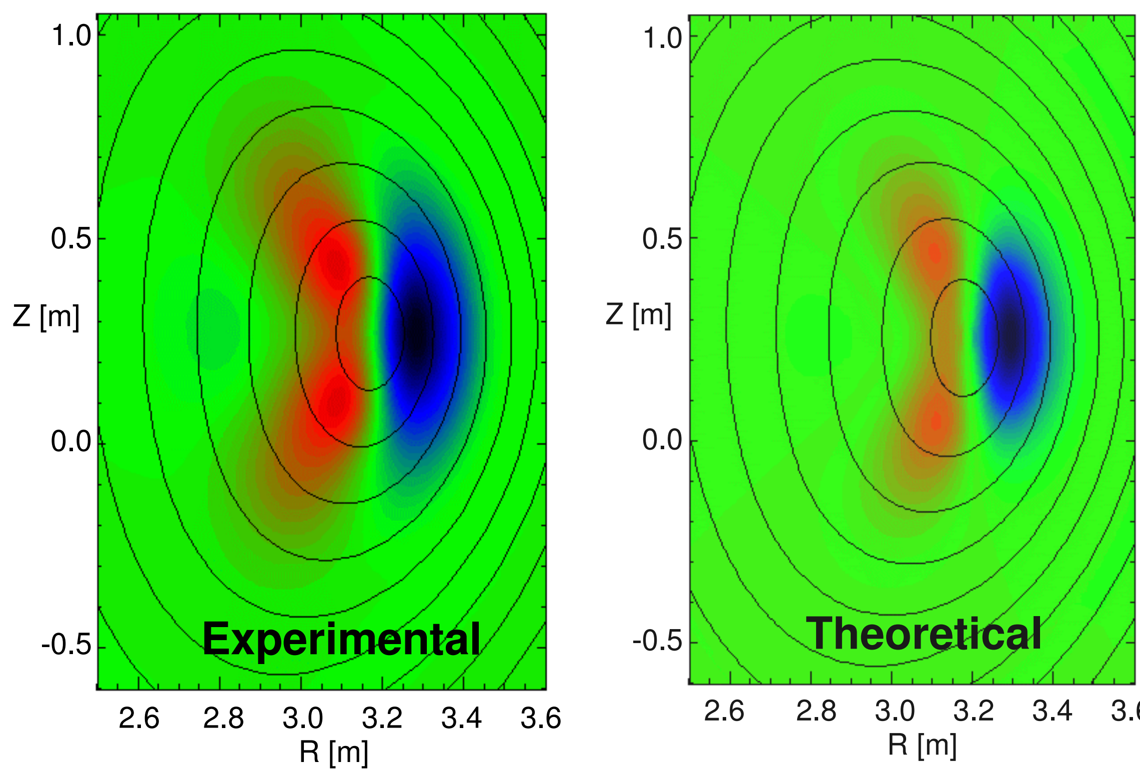
Linear MHD Stability

- The stability limit and the plasma motion in the advanced scenarios are well described by the ideal linear MHD model:



Ideal MHD limit due to pressure driven kink mode

comparison observed and calculated plasma motion (displacement)



Ideal MHD limits

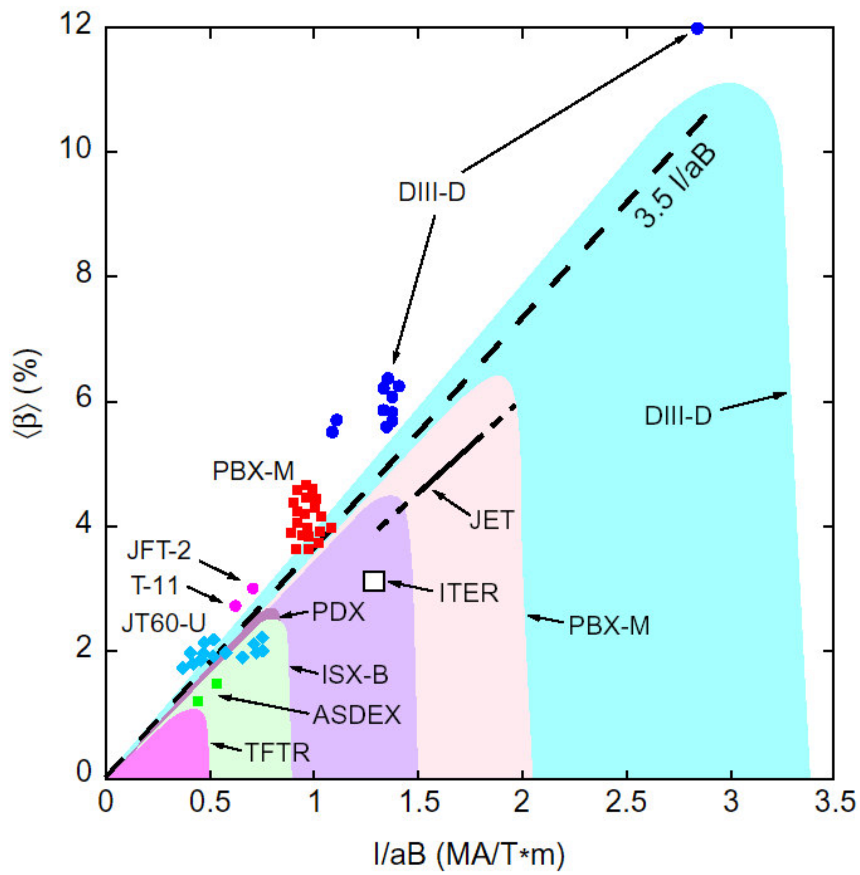
- MHD instabilities impose the ultimate limit on the plasma parameters:
  - the plasma current is limited by external kink modes ($q_{edge} \gtrsim 3$)
  - the plasma pressure is limited by ballooning modes and pressure driven kink modes

beta limit: $\beta_N < 4 l_i$

$$\beta_N = \beta[\%] \frac{a[m]B[T]}{I[MA]} \qquad \beta = \frac{2\mu_0 \langle p \rangle}{B_0^2}$$

l_i : internal inductance (~1)

  - (neo-classical tearing modes)



ITER Physic Basis, Nuclear Fusion, Vol. 39, No. 12 (1999)

Non-linear MHD simulation

- Difficulties MHD simulations :
  - Large variation of time scales:
    - Fast waves : frequency varies from order 1 Alfvén times to infinity
    - Instabilities are relatively slow 10^{-3} a 10^{-2} Alfvén times.
    - Equilibrium evolution 10^{6} - 10^{8} Alfvén times
  - Large variation in spatial scales:
    - MHD Instabilities are quasi singular
    - High (magnetic) Reynolds numbers $S \sim 10^{8} - 10^{10}$
    - Anisotropy of energy transport parallel and perpendicular to magnetic field ($\sim 10^{10}$)
  - The exact geometry of the magnetic field is essential.

XTOR

- Non-linear MHD code in toroidal tokamak geometry developed by H. Lutjens and J.F. Luciani (CPhT Ecole Polytechnique Paris)
  - Equations : viscous/resistive MHD + heat conduction

$$\rho \frac{D\vec{v}}{Dt} = \vec{J} \times \vec{B} - \nabla p + \nabla \nu \nabla \vec{v}$$

$$\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B}) - \nabla \times \eta (\vec{J} - \vec{J}_{boot})$$

$$\partial_t T = -\vec{v}.\nabla T - (\Gamma - 1) T \nabla \vec{v} + \nabla \chi_\perp \nabla T + \vec{B}.\nabla \chi_{//} \frac{\vec{B}.\nabla T}{B^2} + H$$

$$\partial_t \rho = -\vec{v}.\nabla \rho - \rho \nabla \vec{v} + \nabla D_\perp \nabla \rho$$

$$H = -\nabla \chi_\perp \nabla T_{equil} ; \eta (J_\phi - J_{\phi,boot}) = const.$$

  - Applications : internal kink modes, neo-classical tearing modes, double tearing modes (Tore Supra, JET) etc.

XTOR numerics

Split time stepping scheme:

- Ideal MHD part of motion and resistive part of Faraday: semi-implicit
- Linear ideal MHD fully implicit
- Predictor-corrector for other than motion ideal advances
- Scheme for ideal MHD part strongly damps fast modes

  ⇨ designed to solve shear Alfvén modes.
- Thermal transport: preconditioned fully implicit (conjugate gradients)

Boundary Conditions:

- Free slip, infinitely conducting wall

Discretisation:

- radially : Finite Differences
- poloidal and toroidal angles θ,Φ: Fourier Series

XTOR numerical scheme

- Semi-implicit schema (ideal MHD part):
  - Predictor

$$(1-L)(v^* - v_n) = \frac{\Delta t}{2} F_v(v_n, b_n, p_n)$$

$$b^* - b_n = \frac{\Delta t}{2} F_b(v^*, b_n, p_n)$$

$$p^* - p_n = \frac{\Delta t}{2} F_p(v^*, b^*, p_n)$$

$$L = L_0 \Delta t^2 + c\Delta$$

L_0 : ideal MHD operator

  - Corrector

$$(1-L)(v_{n+1} - v_n) = \Delta t F_v(v^*, b^*, p^*)$$

$$b_{n+1} - b_n = \Delta t F_b\left(\frac{v_{n+1} + v_n}{2}, b^*, p^*\right)$$

$$p_{n+1} - p_n = \Delta t F_p\left(\frac{v_{n+1} + v_n}{2}, \frac{b_{n+1} + b_n}{2}, p^*\right)$$

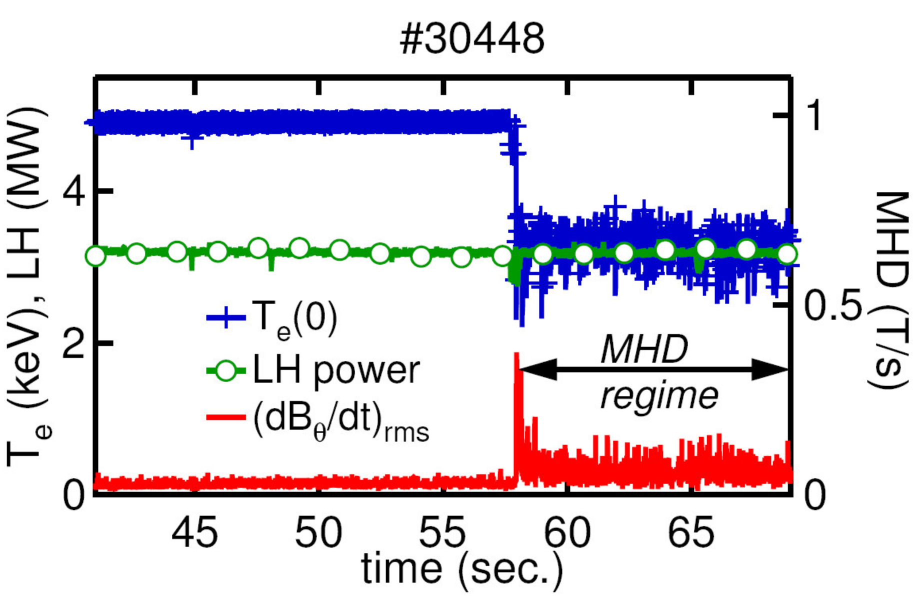
Stable for

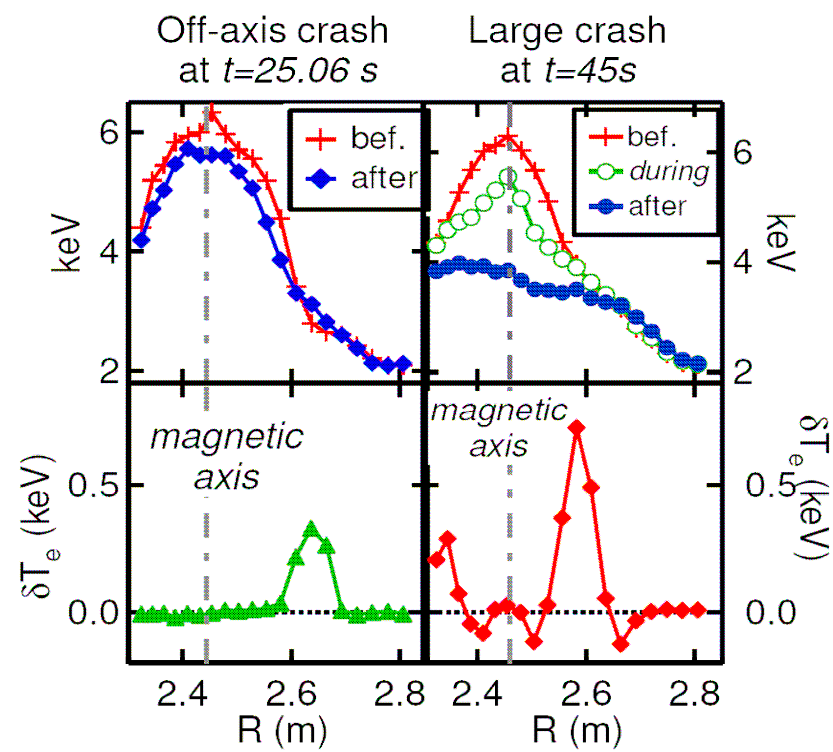
$$c > (\Delta t . \delta B)^2$$

⇒ Damping of (unwanted) high frequency (fast) modes, small damping for low-frequency modes

Tore Supra

- Steady state, long pulse discharges
  - driven by Lower-Hybrid waves leads to hollow current profiles (and non-monotonic q-profiles
  - Onset of ‘MHD regime’

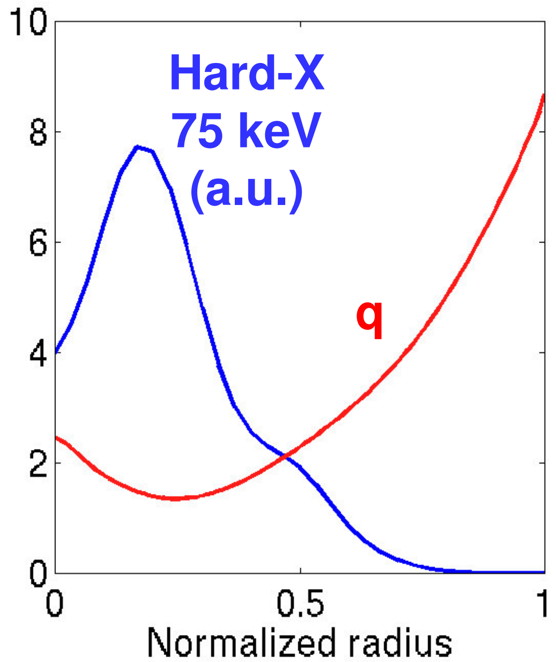


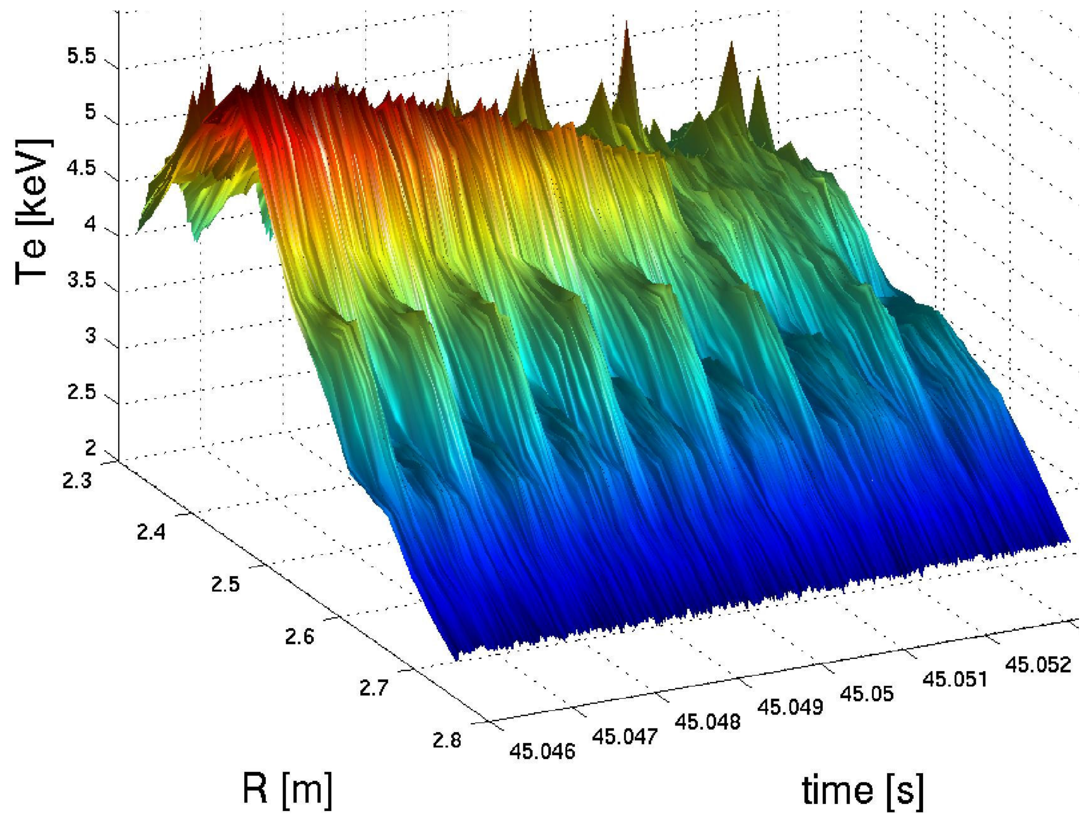


temperature perturbations due to MHD modes

Temperature Perturbation

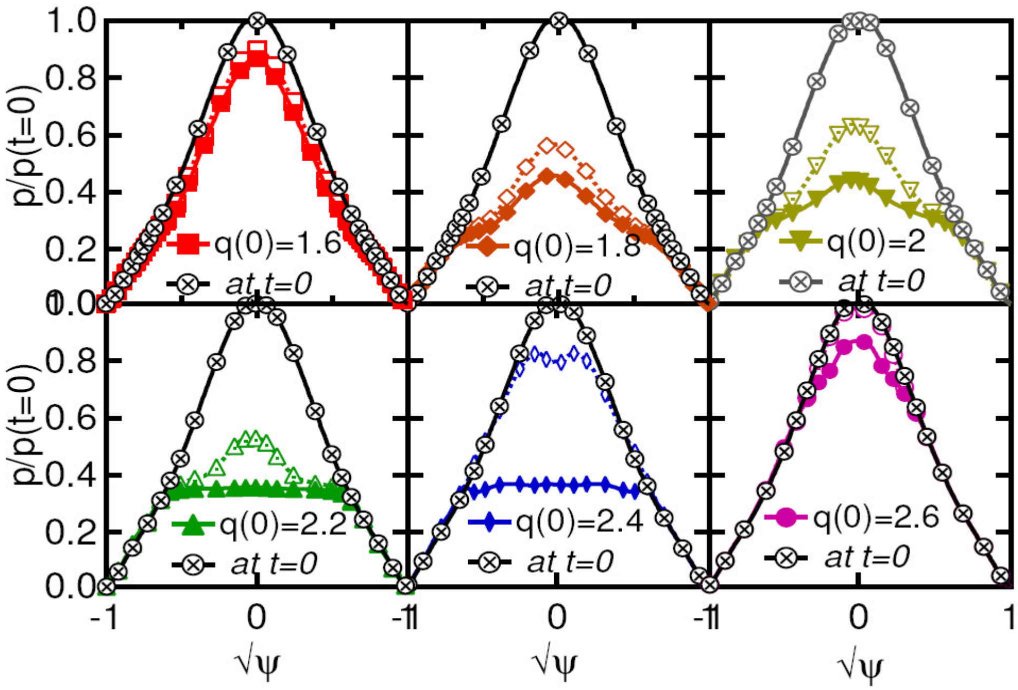
- The measured temperature perturbation indicates a double island structure with two local flattened regions
  - double tearing mode due to 2 rational surfaces (q=2)





Double Tearing modes

- XTOR simulations find same 2 behaviours as in experiment
  - but simulations are too pessimistic (too unstable)

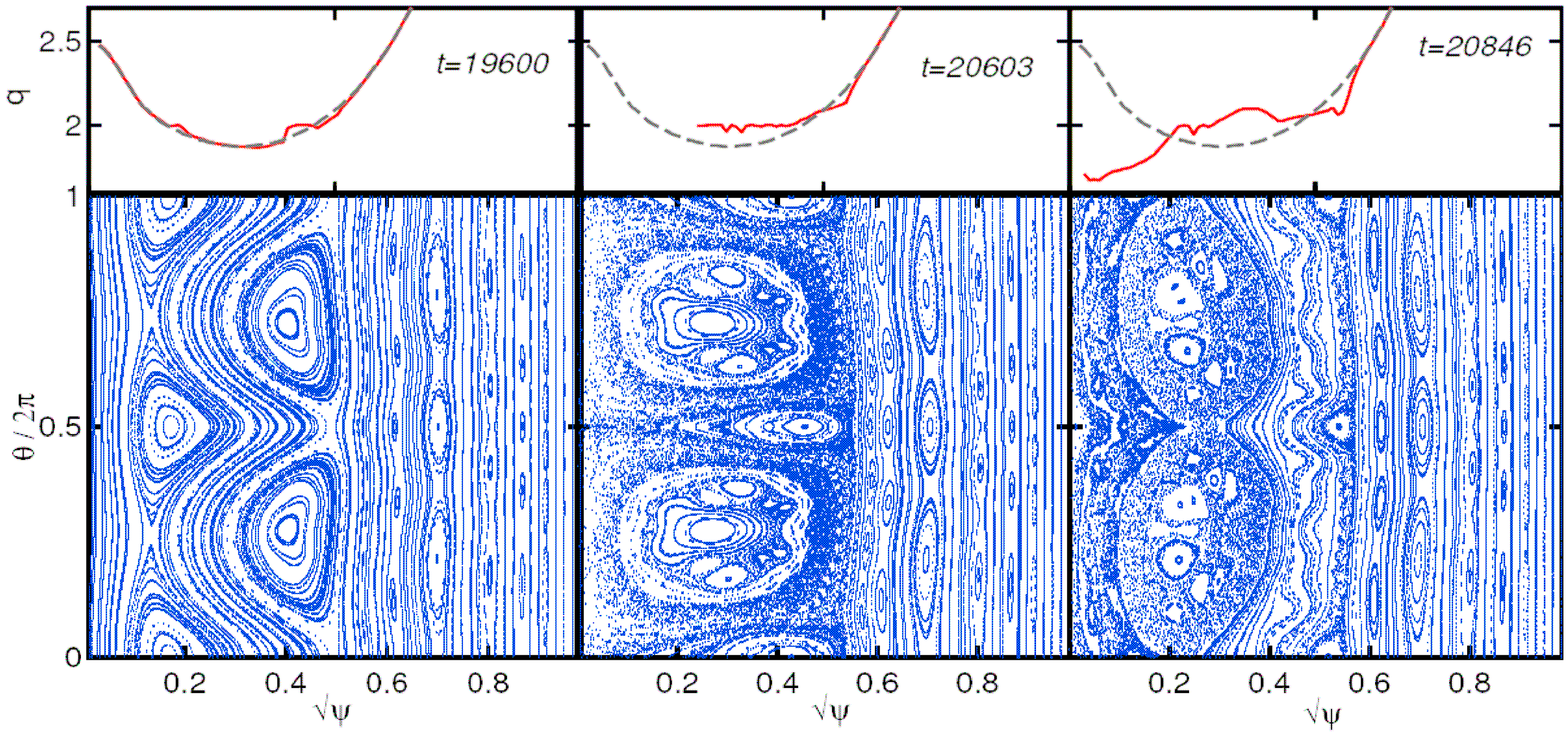


S=5x10^{6} , ν=1/S, χ(perp)=2x10^{-5} ,χ(parallel)=2x10^{+3}
resolution : N_{radial}=200(300), $M_{poloidal}$=48(64), $N_{toroidal}$=16(32)

Double Tearing Modes

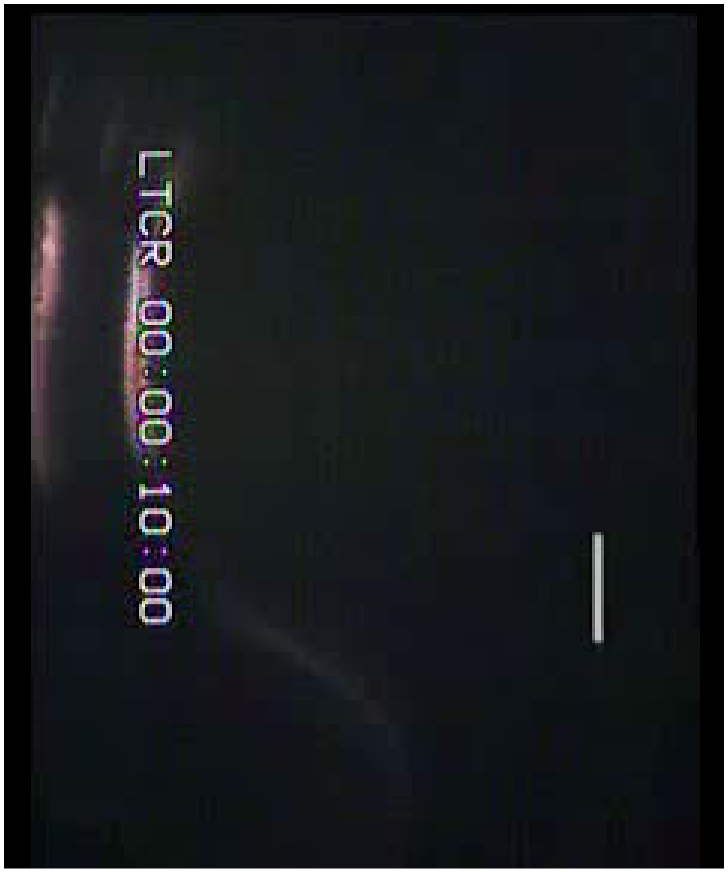
- XTOR : two islands exchange position causing a complete flattening of the q-profile (i.e current profile)

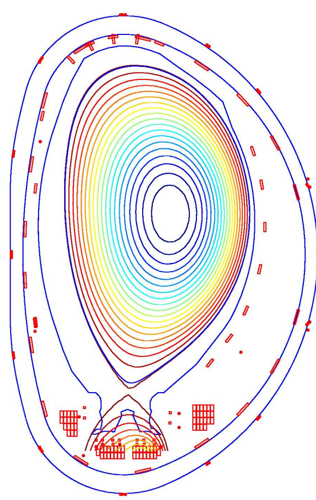
q(0)=2.5: safety factor and Poincaré plots during the reconnection process

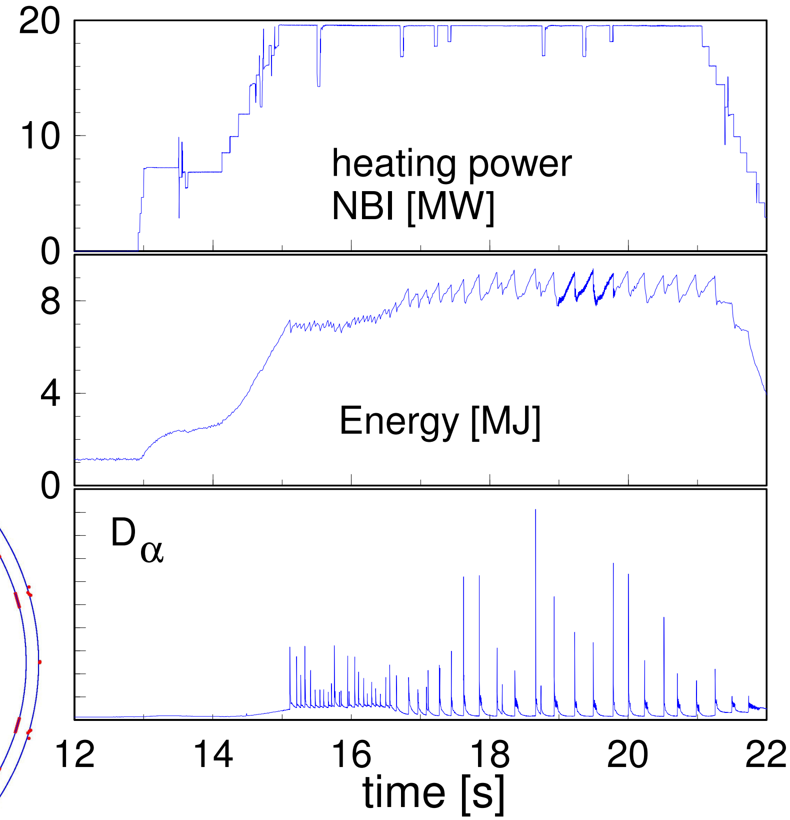


Edge Localised Modes (ELMs)

- MHD Instabilities, localised at the plasma boundary, cause large energy losses (in JET ~1 MJ) in a very short time (200 μs)
  - cause for concern in ITER (damage to the first wall)

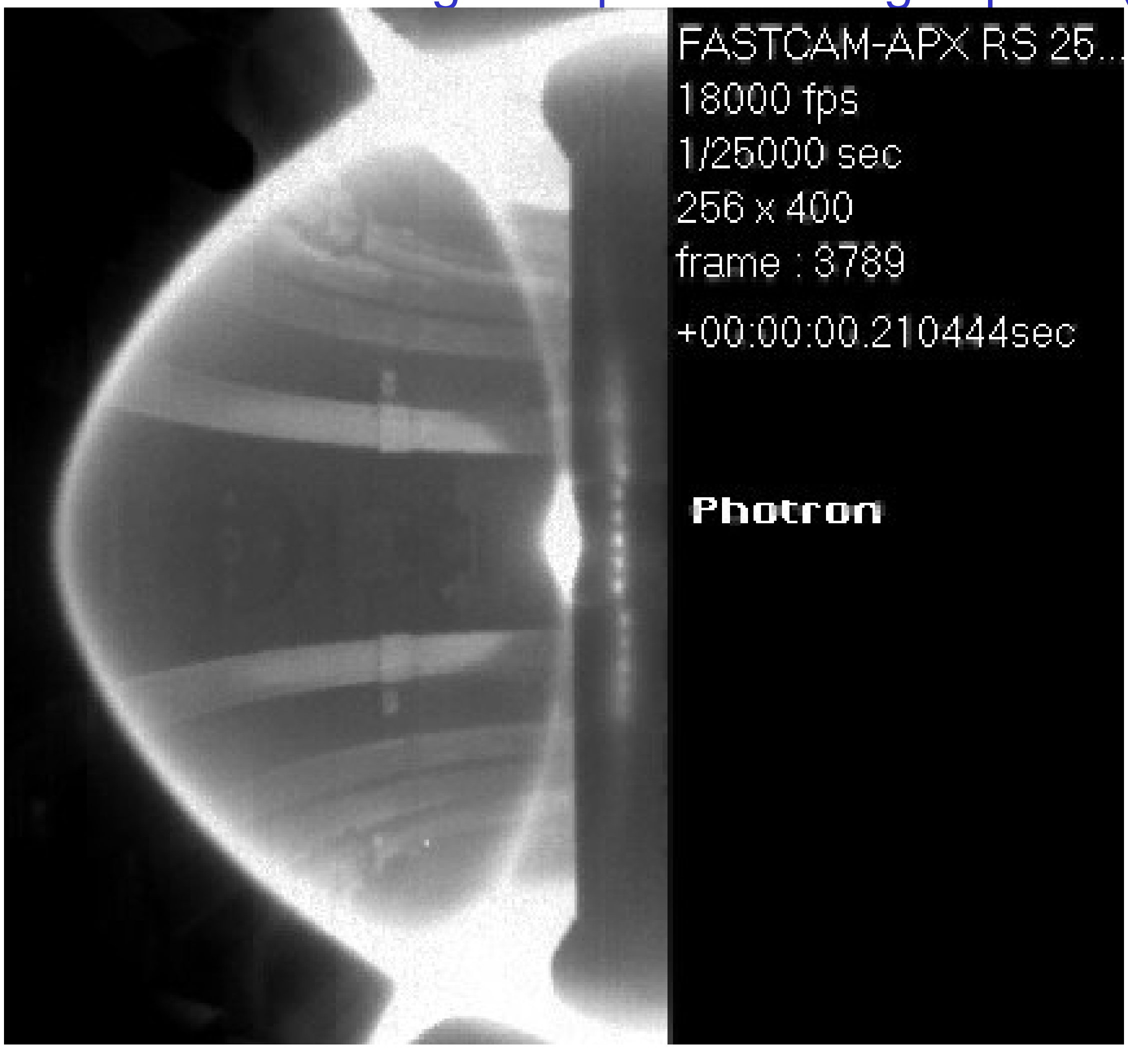


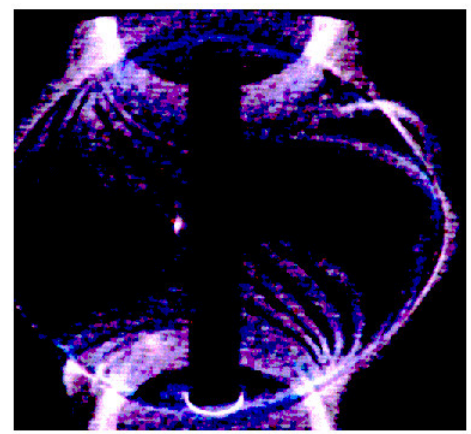




ELMs

- ELMs observed with a fast camera in MAST (A.Kirk, UKAEA):
  - Filaments detaching form plasma at high speed (~several km/s)

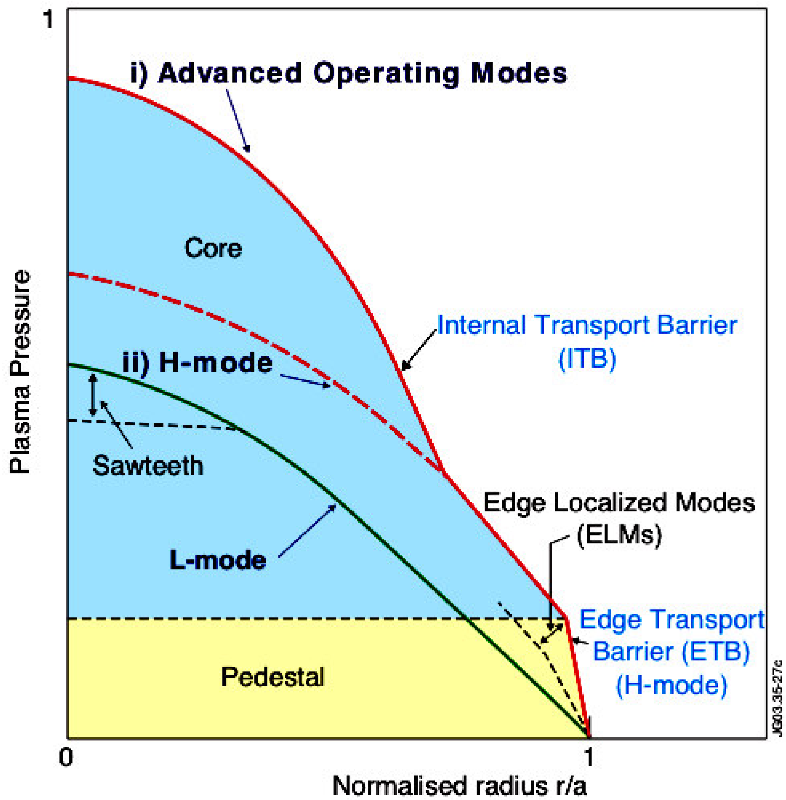


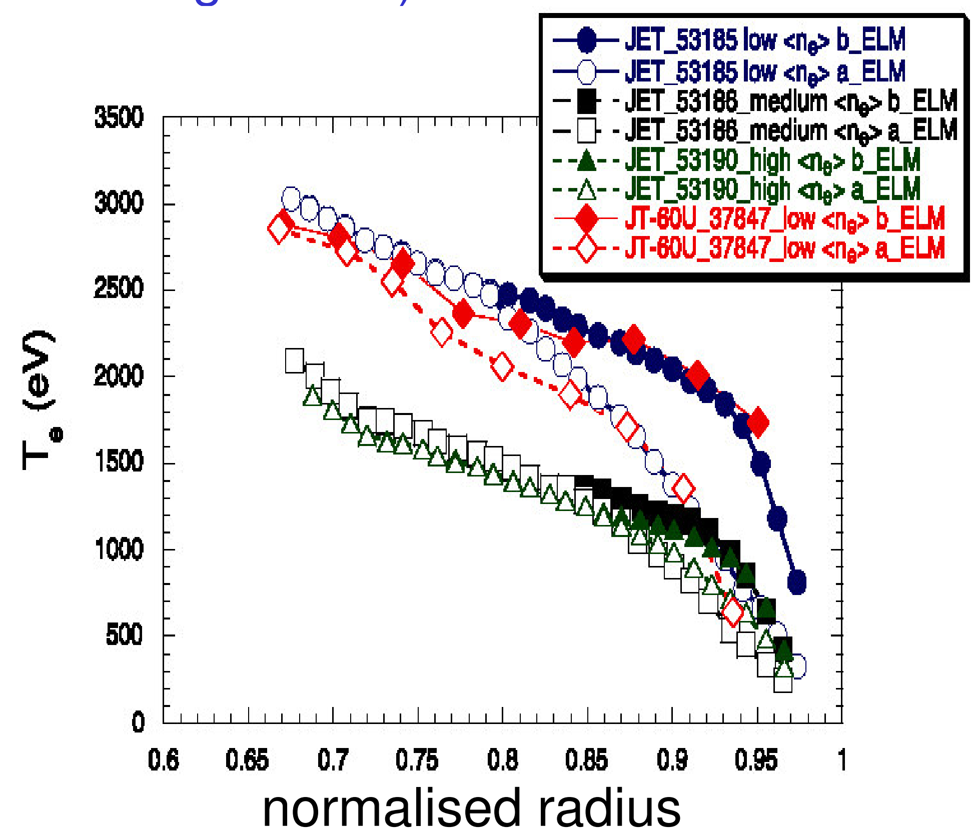


H-mode Edge Pedestal

- Improved confinement regime (H-mode) appears spontaneously when heating is large enough : formation of edge pedestal with large pressure gradient

  ⇒unstable to MHD instability (Ballooning modes)





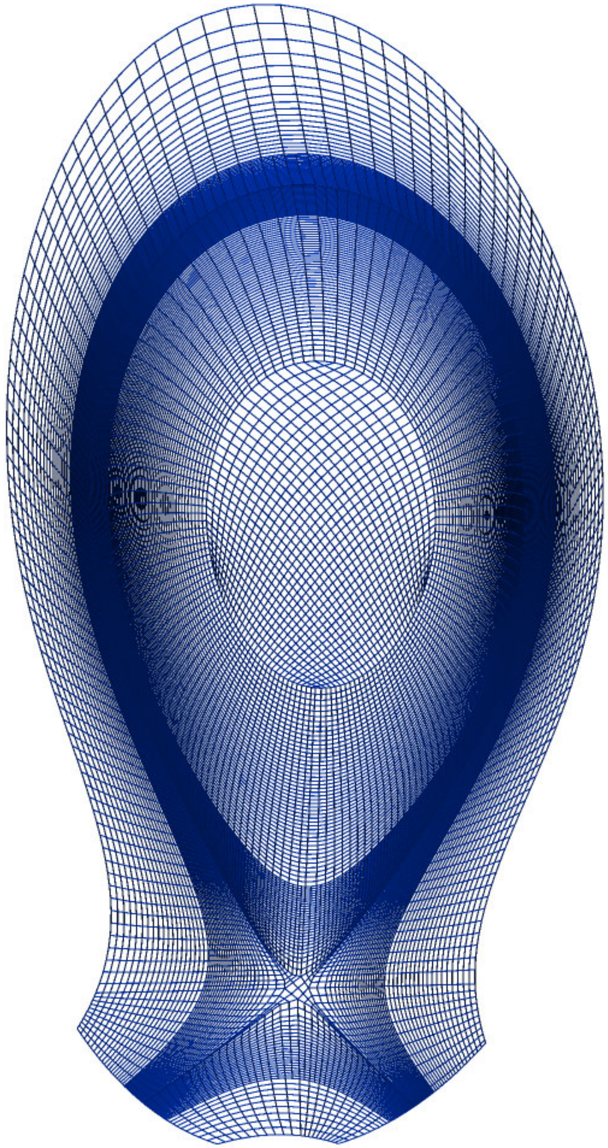
ELMs open questions

- Edge Localised Modes (ELMs) can cause large energy losses, possibly leading to damage to the first wall.
- Relevant linear MHD Stability limits are well known:
  – Ballooning modes driven by edge pressure gradient
  – External kink modes driven by edge current
- Main open question : What determines the size of an ELM?
  – How far can one cross the MHD stability boundary?
    - What is the noise level for the ideal MHD mode in stable plasma?
  – What determines the final state after the ELM?
    - Is there a correlation with the width of the linear eigenmode?
  – What is the relaxation mechanism?
    - Why not a saturated instability but a discrete event?

⇒Non-linear MHD simulations in full geometry including open and closed field lines, X-point and separatrix.

JOREK

- Non-linear MHD code JOREK under development at CEA for the simulation of ELMs
  - magnetic geometry with X-point
  - finite elements aligned on magnetic surfaces
    - refinable finite elements
  - reduced MHD model
    - 'vacuum' modelled as cold, low density plasma
  - fully implicit time evolution
  - parallelisation using MPI



Reduced MHD model

- Reduced MHD model in toroidal geometry
  - fixed large toroidal magnetic field
  - removes fast waves, easier on the numerics
  - similar to reduction Navier-Stokes to potential flow

$$\vec{B} = \frac{R_0}{R} B_0 \vec{e}_\varphi + \frac{R_0}{R} \vec{\nabla} \psi \times \vec{e}_\varphi$$

$$\vec{v} = \frac{-R}{R_0 B_0} \vec{\nabla} u \times \vec{e}_\varphi$$

  - Good for physics studies, not for detailed comparison with experiment

dimensionless form : $\left(x = (R - R_0)/a \; ; \; y = Z/a \; ; \; \varepsilon = a/R_0 \right)$

Magnetic flux:

$$\frac{\partial \psi}{\partial t} = (1+\varepsilon x)\left[\psi, u\right] + \eta \Delta^* \psi - \varepsilon \frac{\partial u}{\partial \varphi}$$

Vorticity:

$$\frac{\partial w}{\partial t} = 2\varepsilon \frac{\partial u}{\partial y} w + (1+\varepsilon x)\left[w, u\right] + \frac{1}{(1+\varepsilon x)}\left[\psi, J\right] - \frac{\varepsilon}{(1+\varepsilon x)^2} \frac{\partial J}{\partial \varphi} + \nu \nabla_\perp^2 w$$

Density:

$$\frac{\partial \rho}{\partial t} = (1+\varepsilon x)\left[\rho, u\right] + 2\varepsilon \rho \frac{\partial u}{\partial y} + \nabla \cdot \left(D_\perp \nabla_\perp \rho \right) + S_\rho$$

Temperature:

$$\rho \frac{\partial T}{\partial t} = (1+\varepsilon x)\rho\left[T, u\right] + 2\varepsilon \rho T \frac{\partial u}{\partial y} + \nabla \cdot \left(\mathrm{K}_\perp \nabla_\perp T + \mathrm{K}_\parallel \nabla_\parallel T \right) + S_T$$

$$J = \Delta^* \psi; \qquad w = \nabla \cdot \nabla_\perp u$$

JOREK time evolution

- Fully implicit time evolution to allow large time steps

- Linearised Crank Nicholson scheme:

$$\frac{\partial A(\vec{y})}{\partial t} = B(\vec{y})$$

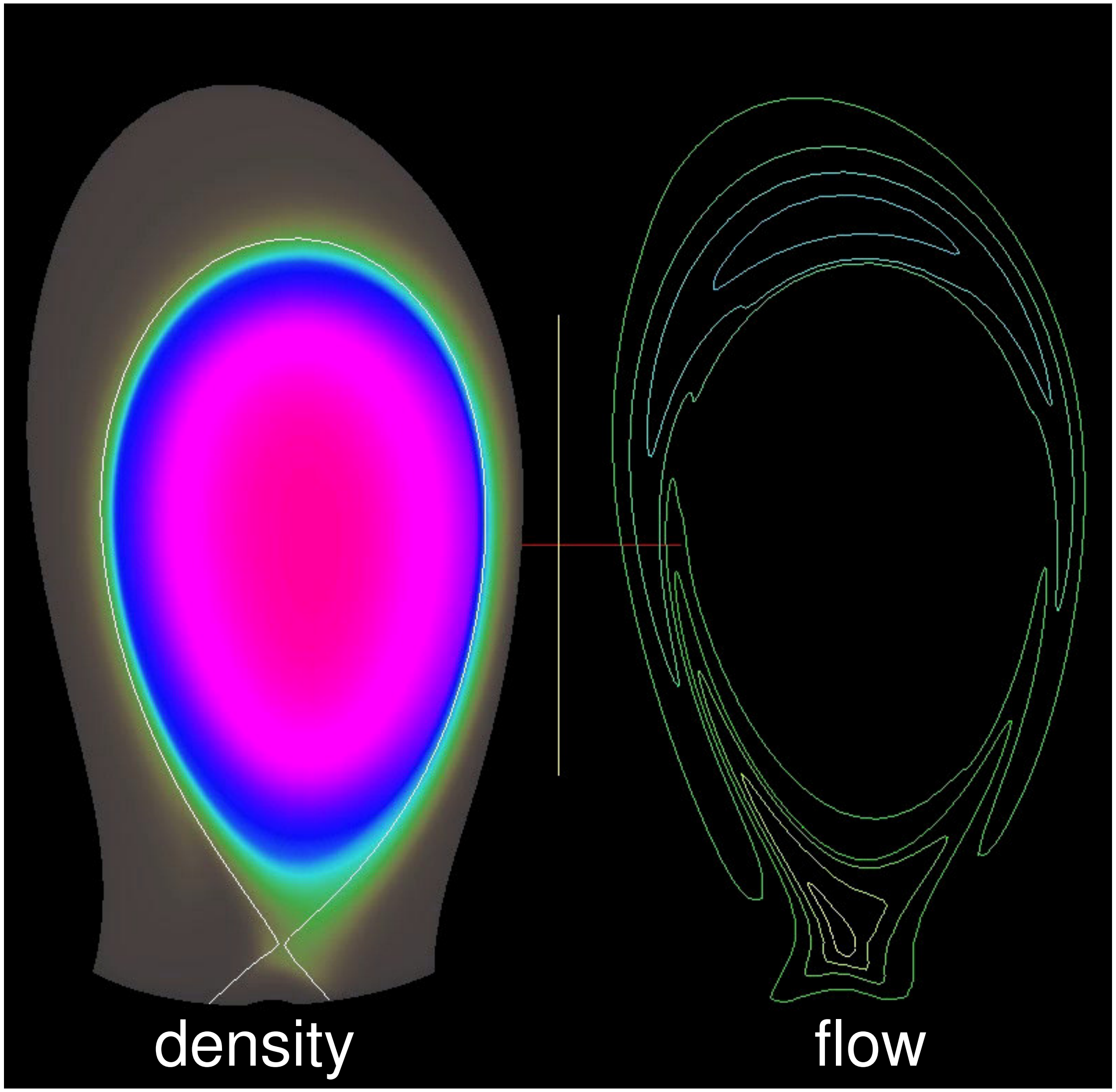
$$\frac{\partial A}{\partial y}\delta\vec{y} = \delta t B(\vec{y}_n) + \tfrac{1}{2}\delta t \frac{\partial B(\vec{y}_n)}{\partial y}\delta\vec{y}$$

$$\left(\frac{\partial A(\vec{y}_n)}{\partial y} - \tfrac{1}{2}\delta t \frac{\partial B(\vec{y}_n)}{\partial y}\right)\delta\vec{y} = B(\vec{y}_n)\delta t$$

- Large sparse system of equation solved using parallel direct sparse matrix libraries (**PASTIX**, MUMPS, WSMP)
  - Hybrid direct/indirect methods (PASTIX) under study (P. Ramet et al.)

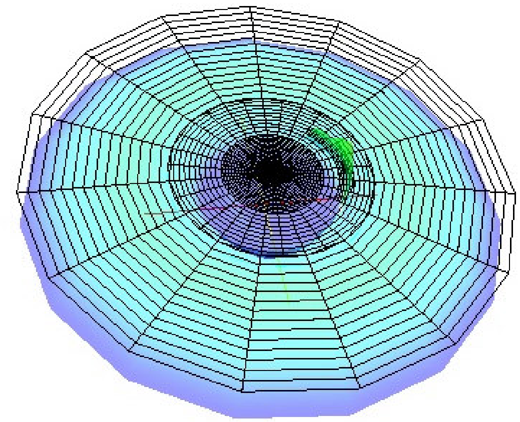
ELM simulations : ballooning mode

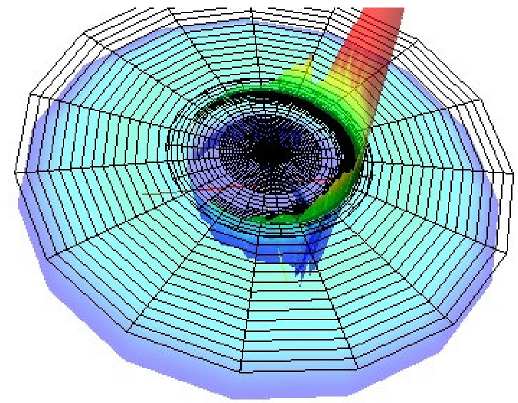
- Evolution of n=6 ballooning mode
  - Formation of multiple filaments expulsed from plasma
  - Speed ~1 km/s
  - Sheared from main plasma by induced n=0 flow
  - Filaments are cold, without magnetic structure



JOREK Developments

- Refinable bi-cubic Finite Elements (Bezier FE)
  - adaptive grid refinement (O. Czarny)
- Adaptation of PastiX sparse matrix library
  - direct/indirect parallel sparse matrix solver (P. Ramet, P. Henon, O. Coulaud)
- Optimisation of time-stepping algorithm in JOREK
  - stabilised FEM, stable residual distribution schemes (R. Abgrall, B. N'Konga)
- Full MHD model
  - Open field line boundary conditions (M. Becoulet, G. Huysmans)





project in french ANR program on
'Intensive Computing and Simulation' (ANR-CIS)

- Collaboration CEA Cadarache - INRIA Futurs-LaBRI & MAB University of Bordeaux

Open Problems

- Numerical schemes
  - High, i.e. realistic, (magnetic) Reynolds numbers
  - Resolution of ‘boundary’ layers
  - Long time integration
- Non-linear evolution of MHD modes
  - Simulation of complete ELM cycle (different ELM types)
  - Simulation of sawtooth cycle
  - Excitation of neo-classical tearing modes
- MHD + background turbulence
  - Interaction fluid turbulence with MHD instabilities
- Fast particle interaction with MHD modes
  - ITER

Summary/Conclusion

- The linear MHD model is one of the simplest and most successful models in tokamak physics
  - describes the operational limits of tokamaks (pressure and current)
  - Local pressure gradient and current density limits
  - Frequencies of (global) Alfvén waves
- Non-linear MHD is moving from theoretical studies to comparison theory-experiment
  - However, still many open basic physics questions
    - Crash of fast MHD instabilities (ELMs, sawteeth)
    - Trigger of neoclassical tearing modes
    - Interaction fast particles and MHD modes
    - Extensions to the MHD model
- Progress in numerical methods is needed
  - realistic Reynold numbers
  - extended MHD models