On the numerical simulation of plasma flows with mixing

Céline Baranger

in collaboration with G. Carré - D. Paillard - R. Sentis

CEA/DIF

Bruyères Le Châtel France

Numerical Flow Models for Controlled Fusion,
16-20 April 2007
Porquerolles, France

Problem setting

- In the context of Inertial Confinement Fusion (ICF), plasma collision may occur
- Its modeling by mono-fluid hydrodynamic leads to unphysical densities (shocks).
- Full modeling would require kinetic description of the plasma which is not realistic for multi-dimensional flows.
- So it affects the interpretation of experiments of ICF on laser by using an ICF hydro code.

Our aim is to present a model for the description of such phenomena, to describe the numerical method and the implementation made in the ICF code used, and to show some numerical results.

Different models

- Classical model = the 2 ionized fluids can be described by a system of six equations which correspond to the conservation of mass, momentum and energy for each fluid.
- 5-equation model = by the use of a closure, we get a five-equation model: the conservation equation of mass, momentum and energy for the average fluid coupled with an equation of concentration and relative velocity.
- Reduced model = the 5-equation model is then simplified by using assumptions on the friction coefficient and on the relative velocity (diffusion approximation).

The initial model (6 equations)

Two fluids ($q = g$ and l) are assumed to fill possibly the same volume

$$\frac{\partial}{\partial t}\rho_q + \nabla\left(\rho_q \overrightarrow{u_q}\right) = 0,$$

$$\frac{\partial}{\partial t}\left(\rho_q \overrightarrow{u_q}\right) + \nabla\left(\rho_q \overrightarrow{u_q}\overrightarrow{u_q} + P_q\right) = \overrightarrow{\Xi_q},$$

$$\frac{\partial}{\partial t}(\rho_q E_q) + \nabla\left(\rho_q \overrightarrow{u_q} E_q + \overrightarrow{u_q} P_q\right) = \Omega_q + C_{e,q}.$$

Knowing that the friction or drag term may reads as

$$\overrightarrow{\Xi}_g = -\overrightarrow{\Xi_l} = \nu^0 \rho_g \rho_l (\overrightarrow{u_l} - \overrightarrow{u_g}),$$

one has to add an equation on electronic energy (two temperature Euler equations).

The 5-equations model: Notations

The mean quantities

$$\rho = \rho_g + \rho_l, \qquad \overrightarrow{u} = (\rho_g \overrightarrow{u_g} + \rho \overrightarrow{u_l})/\rho, \qquad E = (\rho_g E_g + \rho_l E_l)/\rho,$$

concentration of material g and relative velocity:

$$c = \frac{\rho_g}{\rho}, \qquad \overrightarrow{V} = \overrightarrow{u_g} - \overrightarrow{u_l},$$

the total material pressure and the mixing pressure:

$$P_* = P_g + P_l, \qquad \overline{\overline{P_b}} = \rho \overrightarrow{V}\overrightarrow{V} c(1-c).$$

We set $\sigma = \rho\nu^0$. We can see after standard calculus that

$$\varepsilon \equiv c\varepsilon_g + (1-c)\varepsilon_l = E - \frac{1}{2}|\overrightarrow{u}|^2 - \frac{1}{2}c(1-c)|\overrightarrow{V}|^2.$$

The 5-equations model:

We get after standard calculus

$$\rho D_t \rho^{-1} - \nabla \overrightarrow{u} = 0,$$
$$\rho D_t \overrightarrow{u} + \nabla P_* + \nabla . \overline{\overline{P_b}} = 0,$$
$$\rho D_t c + \nabla \left(\rho c (1-c) \overrightarrow{V} \right) = 0,$$
$$\rho D_t (\rho^{-1} \overrightarrow{V}) - \nabla . (|\overrightarrow{V}|^2 \frac{2c-1}{2}) + \mathcal{G} = -\sigma \overrightarrow{V},$$
$$\rho D_t E + \nabla \left(P_* \overrightarrow{u} + \overline{\overline{P_b}} . \overrightarrow{u} \right) + ... = C_{e,i},$$

where

- where $D_t = \frac{\partial}{\partial t} + \overrightarrow{u} \nabla$ is the lagrangian derivative.
- $\mathcal{G} = \frac{1}{\rho c} \nabla P_g - \frac{1}{\rho (1-c)} \nabla P_l$
- $C_{e,i}$ corresponds to the coupling between the ions and the electrons.

The reduced model

If one assumes that the flow of each fluid is isentropic, the following closure holds:

$$\mathcal{G} \rightrightarrows \nabla(\Psi(c)\varepsilon)$$

with

$$\Psi(0) = \gamma_g, \qquad \Psi(1) = -\gamma_l.$$

So we get

$$\rho D_t(\rho^{-1}\overrightarrow{V}) + \nabla(\Psi\varepsilon) - \nabla.(|\overrightarrow{V}|^2\frac{2c-1}{2}) = -\sigma\overrightarrow{V},$$

$$\rho D_t E + \nabla(\overrightarrow{u}P_* + \rho c(1-c)\overrightarrow{V}\Psi\varepsilon) + \nabla(\overline{\overline{P_b}}\overrightarrow{u} - \frac{2c-1}{2}\rho c(1-c)\overrightarrow{V}|\overrightarrow{V}|^2) = 0.$$

This leads to an hyperbolic system (for $|\overrightarrow{V}|$ small enough).

Remark

By denoting

$$K = \frac{c(1-c)|\overrightarrow{V}|^2}{2},$$

this mixing kinetic energy satisfies

$$\rho D_t K + 2\rho K \nabla . \overrightarrow{u} + \rho c(1-c)\overrightarrow{V}.\nabla(\Psi\varepsilon) + 2\sigma\rho K - \nabla\left((2c-1)\rho K \overrightarrow{V}\right) = 0.$$

Then, we get the following equation for the internal energy ε:

$$\rho D_t \varepsilon + P_* \nabla . \overrightarrow{u} + \Psi\varepsilon\nabla(\rho c(1-c)\overrightarrow{V}) = 2\sigma\rho K.$$

Diffusion approximation

This consist on assuming that:

$$\nabla(\Psi\varepsilon) + \sigma\overrightarrow{V} = 0.$$

We take

$$\overrightarrow{V} \simeq -\frac{1}{\sigma}\Psi'\varepsilon\nabla c \equiv D\nabla c.$$

Then

$$\rho D_t c - \nabla.\left(\rho c(1-c)D\nabla c\right) = 0,$$

$$\rho D_t E + \nabla.\left(P_*\overrightarrow{u} + 2\rho K\overrightarrow{u}\right) - \nabla.\left((c(1-c)\Psi\varepsilon - (2c-1)K)\rho D\nabla c\right) = 0.$$

For the internal energy, equation is similar to the classical Euler but with different right hand side:

$$\rho D_t\varepsilon + P_*\overrightarrow{\nabla}.\overrightarrow{u} = \Psi\varepsilon\overrightarrow{\nabla}.\left(\rho c(1-c)D\overrightarrow{\nabla}c\right) + 2\sigma\rho K.$$

Final system

We could resume the whole model in this system

$$\rho D_t \rho^{-1} - \nabla.\vec{u} = 0,$$

$$\rho D_t \vec{u} + \nabla P_* + \nabla.\bar{\bar{P}}_b = 0,$$

$$\rho D_t c - \nabla.(c(1-c)D\nabla c) = 0,$$

$$\rho D_t \varepsilon + P_* \nabla.\vec{u} = \Psi\varepsilon\nabla.(\rho c(1-c)D\nabla c) + 2\sigma\rho K,$$

$$\rho D_t K + 2\rho K \nabla.\overrightarrow{u} + \sigma\rho(2K - c(1-c)|D|^2|\nabla c|^2) - \nabla\left((2c-1)\rho K D\nabla c\right) = 0.$$

Final system

We could resume the whole model in this system

$$\rho D_t \rho^{-1} - \nabla.\vec{u} = 0,$$

$$\rho D_t \vec{u} + \nabla P_* + \nabla.\bar{\bar{P}}_b = 0,$$

$$\rho D_t c - \nabla.(c(1-c)D\nabla c) = 0,$$

$$\rho D_t \varepsilon + P_* \nabla.\vec{u} = \Psi\varepsilon\nabla.(\rho c(1-c)D\nabla c) + 2\sigma\rho K,$$

$$\rho D_t K + 2\rho K \nabla.\overrightarrow{u} + \sigma\rho(2K - c(1-c)|D|^2|\nabla c|^2) - \nabla\left((2c-1)\rho K D\nabla c\right) = 0.$$

Numerical method

Brief description of our simulations

- Arbitrary-Lagrangian-Eulerian hydro code.
- Electronic thermal conduction is taken into account by using the classical Spitzer-Härm formula. Ionic thermal conduction is neglected.
- Equation for the concentration:
  - non-linearity -> implicit scheme with fixed point method,
  - Diffusion scheme: 5 points scheme on quadrilaterals.

Numerical method

- We have to model the diffusion coefficient D: it has the following form (for a given function Ψ)

$$D(c) \simeq \frac{\varepsilon}{\beta_0}\frac{\Psi'(c)}{\rho}\left((\frac{9\pi}{2\alpha_0^2})^{\frac{1}{3}}(\gamma-1)(2+(1-2c)\frac{\Psi(c)}{\gamma})\varepsilon + V_r^2\right)^{\frac{3}{2}}$$

A possible choice for the corrector term V_r is

$$V_r{}^2 = C_1^2\varepsilon^5/(L\rho\beta_0)^2$$

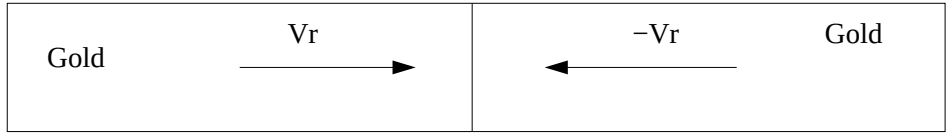
where L is a constant which is a characteristic value of the width of the mixing zone (it depends on the actual cases).

Some numerical experiments were made in order to see the influence of the size of the mesh (with respect to the mean free path) and of the value of the characteristic length L.

Numerical results

An example in a 1D configuration

Initially, two pure fluids with opposite velocity (Gold/Gold collision)

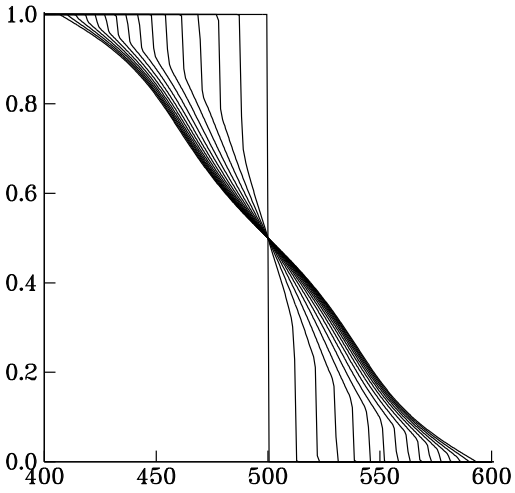


Here we take $\sigma_0 = 2.5\ 10^{36},\ V_r = 2.\ 10^8$ (in CGS units).

Limit condition: vaccum.

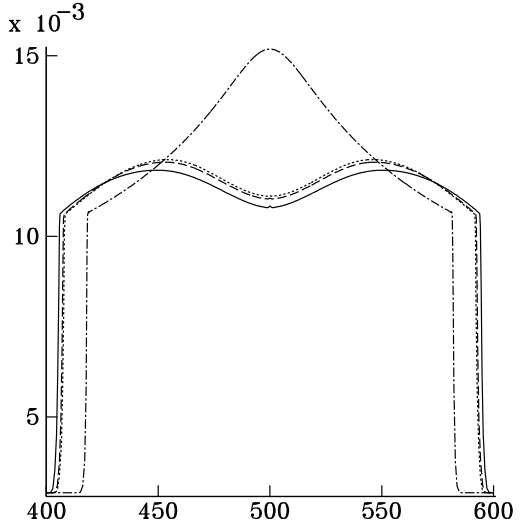
Numerical results

concentration versus position
(from 0 to 300 ps)



non linear profile

density versus position at 300 ps
with or without mixing

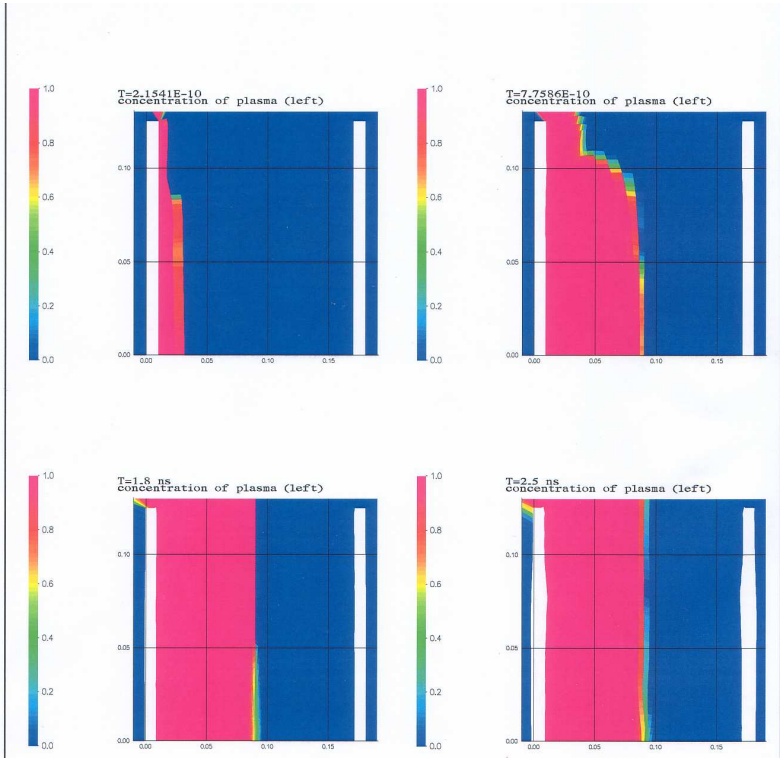


decreasing of the density

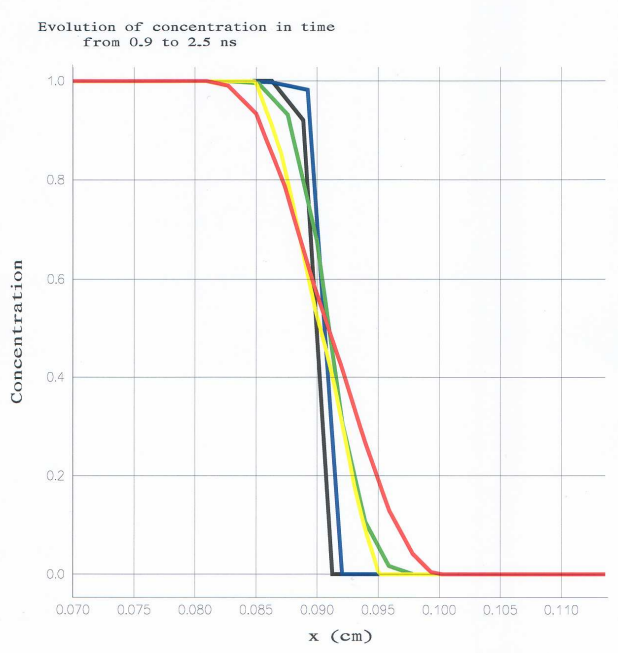
2D simulation

The simulation of the mixing of 2 plasma flows of Titanium, obtained by a laser ablation.

evolution of the front of titanium
from 0.2 to 2.5 ns



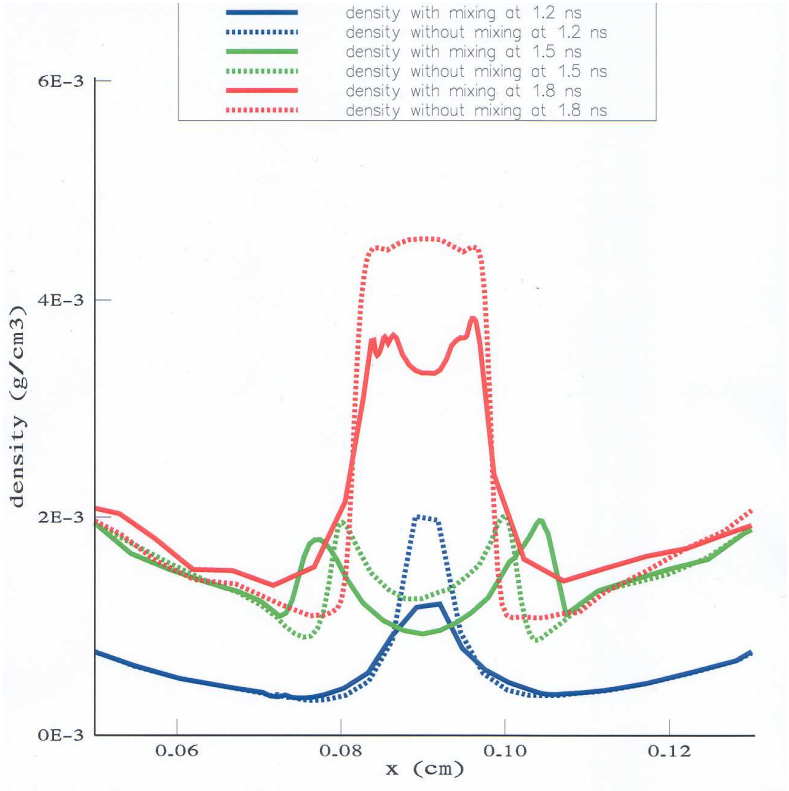
profile of concentration
from 0.9 to 2.5 ns



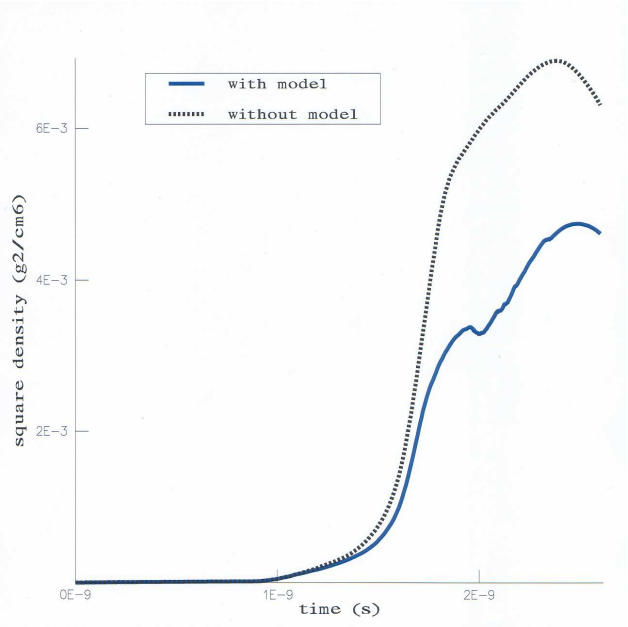
2D simulation

We observe, as request, smaller density and smaller emission of the zone of the mixing of plasma (emmision is proportionnal to ρ^2).

evolution of the density
from 1.2 to 1.8 ns



$\int \rho^2 dx$ in the mixing zone



Perspectives

- Take into account the coupling between ions and electrons at the numerical level.
- Refined the 2D case and compute the emissivity of the whole simulation, in order to made a comparison with experiments.

References:

A. Decoster, *Modeling of collisions* P.A. Raviart, ed. Masson, Paris (1997).
H.B. Stewart, B. Wendroff, *Two-phase Flows: Models and methods*, J. Comp. Phys. **56**, 363 (1984).
M. Ishii, *Thermo-Fluid dynamic theory of two phase flows*, Eyrolles, Paris (1975).