Two-Phase Compressible Flow Methods Based on Extended Irreversible Thermodynamic Theory.

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In the last few decades several models and associated numerical algorithms of two-phase flow have been developed [1-3], but none of these models has been universally accepted as a complete formulation for modeling two- and multi-phase models. The main challenge in the development of high-accuracy numerical methods for multiphase compressible flows is associated with the formulation of a mathematical model that satisfies important properties such as hyperbolicity, symmetric hyperbolic system in particular, fully conservative form of the governing equations and compatibility and consistency of the mathematical model with the thermodynamic laws. These properties provide a solid mathematical framework for a theory of different initial-boundary value problems and allow developing highly accurate numerical methods.

In the past, two-phase flow models were derived by considering a mixture of two continuum media governed by conservation laws of mass, momentum and energy coupled with interfacial exchange processes [4]. Here, we propose a new approach which is based on the formalism of thermodynamically compatible systems of hyperbolic conservation laws [5]. It allows us to derive classes of hyperbolic conservation-form equations using generalized potentials and variables. Its core aspect is phenomenological modelling of continuum media, where by using thermodynamic laws we determine a structure of the governing balance laws. In this context, the mixture is assumed to be a continuum in which the multiphase flow character is taken into account. The resulting system of PDEs is hyperbolic and all balance equations can be cast in conservation form [6].

The conservation form of the governing equations provides a straightforward basis for the development of high-order accurate numerical methods [7]. Here, we have developed a finite volume method based on the solution of the Riemann problem. Due to the complexity of the proposed governing equations the solution of the Riemann problem cannot be easily obtained, because the eigenstructure for the general two-phase conservative equations is unknown and even for the linearised equations the eigenstructure problem has no explicit solution and can be solved with the use of numerical linear algebra algorithms only.

Therefore, we present an alternative way to solve the Riemann problem that makes use of the recently proposed GFORCE method [8]. The advantage of the GFORCE is its simplicity and robustness. Test problems are presented and numerical results are discussed.

1. H. Stadtke, B. Worth, G. Francello, On the hyperbolic nature of two-phase flow equations: characteristic analysis and relater numerical methods. In: Toro E.F.(Ed.). Godunov Methods: Theory and Applications, Kluwer Academic/Plenum Publishers, 2001
2. M.R. Baer, J.W. Nunziato, A two-phase mixture theory for the deflagration-to-detonation transition (DDT) in reactive granular materials, Int. J. of Multiphase Flows, V. 12, 861-889, 1986.
3. R. Saurel, R. Abgrall, A multiphase Godunov method for compressible multifluid and multiphase flows, J. Comput. Phys., V. 150, 425-467, 1999.
4. M. Ishii, *Thermodynamics of two-phase flows*. Eyrolles, Paris, 1975.
5. S.K. Godunov, E. Romenski, Elements of continuum mechanics and conservation laws. Kluwer Academic/Plenum Publishers, 2003.
6. E. Romenski, A.D. Resnyansky, E.F. Toro, Conservative hyperbolic formulation for compressible two-phase flow with different phase pressures and temperatures, Quarterly of Appl. Math. (to appear)

7. D. Drikakis, W.J. Rider, High-resolution methods for incompressible and low-speed flows, Springer 2004.
8. E.F. Toro, V.A. Titarev, MUSTA schemes for systems of conservation laws, J. Comput. Phys., V.212, 403-429, 2006.