A fully adaptive and accurate semi-Lagrangian scheme for the Vlasov-Poisson equation.

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In this talk, we propose a new adaptive scheme for solving the one-dimensional Vlasov-Poisson equation and give an a priori error estimate. In order to save computational costs while approximating the complex and thin structures that may appear in the solutions, several adaptive schemes have been proposed in the past few years, inspired from the semi-Lagrangian method of Cheng and Knorr, later revisited by Sonnendrücker, Roche, Bertrand and Ghizzo. Yet no one has been proven to converge.

A key feature of our new scheme relies on the simple structure of the graded dyadic quadrangulations which are used to build hierarchical finite element approximations of the phase space distribution functions f(t_n,.,.). Based on the regularity analysis of the numerical solution and how it gets transported by the numerical flow, the method performs an accurate evolution of the adaptive mesh from one time step to the next one, in the sense that the accuracy of the scheme is monitored by a prescribed tolerance parameter epsilon which represents the local interpolation error at each time step.

As a consequence, the numerical solutions are proved to converge in L infinity towards the exact ones as epsilon and Dt tend to zero, provided the initial data is in W{1,infinity} and W{2,1}. The rate of convergence is in O(Dt2 + espilon/Dt), and several numerical tests illustrate the effectiveness of our approach for generating the optimal adaptive meshes.