A Column Generation Approach for the Exact Solution of Channel Assignment Problems

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Abstract

We propose a 0-1 column generation model for the problem of channel assignment in a cellular network, with the objective of minimizing the unsatisfied channel demand while providing a channel assignment with an acceptable interference level. The formulation takes into account both co-channel and adjacent channel constraints, as well as antenna spacing ones. The 0-1 linear program is solved with a branch-and-cut method where the solution of the continuous relaxation includes a shortest path problem with resource constraints. Preliminary results are presented on an urban network of Bell Mobilité. Performances are compared with those of the other exact approaches of the literature.

Key words: Channel Assignment; Cellular Network; 0-1 Linear Program; Column Generation; Shortest Path; Resource Constraints; Branch-and-Cut.

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1 Introduction

We consider the channel assignment problem where the objective is to optimize the quality of the cellular network while keeping the various interferences at an acceptable level.

Many heuristics have been proposed in the literature to solve the channel assignment problem but usually with the objective of minimizing the order or the span (see, e.g., Gamst and Rave [8], Hurley, Thiel and Smith [10] and Tiourine, Hurkens and Lenstra [20] for a survey of methods and a comparison of computational experiments). More recent heuristics are considering the objective of minimizing the interference level or the unsatisfied channel demand, (see, e.g., Borndörfer *et al.* [3] or Hurkens and Tiourine [4]).

In order to assess the quality of the heuristic solutions, several lower bounds have been proposed. Most of them are obtained by formulating the channel assignment problem as a T-coloring problem and considering relaxations such as the classical coloring or maximum clique problems. More recent works use interior point methods (Warners *et al.* [21]), column generation techniques (Adjakplé [2], Jaumard, Marcotte and Meyer [12]) and tools from polyhedral theory with a relaxation based on a traveling salesman reformulation (Janssen and Kilakos [11]).

Very few exact algorithms have been proposed. Most of them consider a 0-1 linear programming formulation where the basic variable takes the value 1 if a given channel is assigned to a given cell and 0 otherwise (Aardal *et al.* [1], Giortzis and Turner [9], Fischetti *et al.* [7], Mannino and Sassano [17]). Hurkens and Tiourine [4] suggest another formulation with a quadratic 0-1 function.

The paper is organized as follows. In the next section, we provide a detailed statement of the channel assignment problem. In Section 3 we examine a first 0-1 formulation where each variable represents the possible assignment of a channel block (or channel) to a given cell. In Section 4 we propose an original 0-1 column generation model with an auxiliary problem which corresponds to a shortest path problem. In Section 5 we show how to improve the value of the lower bound of the column generation formulation of the previous section through the transfer of several sets of constraints of the master problem in the auxiliary one, leading to a shortest path problem with resource constraints. In Section 6, we discuss the solution of the column generation formulation, i.e., the solution of the auxiliary problem and the branching scheme. In Section 7, we comment some preliminary numerical results on a Bell Mobilité cellular urban network, and how to compare them with those of the other exact algorithms proposed in the literature.

2 Statement of the Channel Assignment Problem

2.1 Basic Definitions

A mobile cellular network can be viewed as a collection of sites where each site is defined as a set of cells (usually between 1 and 5 cells). We denote by N the overall number of cells in the network. Each cell C_i has a given number, usually between 1 and 4, of antennas located at a base station. Denote by H_i the set of antennas of cell C_i. A required number d_i of channels must be assigned to each cell C_i in order to satisfy the demand of users in the area covered by the antennas located at its base station. A tolerance is often associated with this demand ($\underline{d}_i \leq d_i \leq \overline{d}_i, i = 1, 2, \ldots, N$) in order to ensure the existence of a feasible channel assignment.

Channels are taken from a subset of the frequency channel management chart (see, e.g., Lee [16]) that varies from one application to the next with respect to the number of columns.

Assuming that channels are described using integer values (which are in affine correspondence with the channel values in MHz), and that channel f_0 is stored in line 1 and column 1, channel $f = f_0 + \beta - 1 + (\alpha - 1)L$ is stored in line α and column β (where L, most of the time equal to 21, denotes the number of columns of the chart considered). In practice, channels are often assigned one block at a time: each channel block corresponds to a subset of consecutive channels in one of the L columns of the chart. Let K be the set of channel blocks. Two channel blocks are said to be adjacent if they contain a pair of channels, one in each block, that are adjacent, i.e., that correspond to two consecutive numbers. We define the distance between two channel blocks B_k and B_l as the smallest difference in absolute value between two channels, one in each block. We denote by $\Gamma(B_k)$ the set of channel blocks which are adjacent to block B_k. Let $\mathcal{F}$ be the set of channels that can be used in the chart. It is partitioned into sets $\mathcal{F}_{basic}$ and $\mathcal{F}_{exp}$: $\mathcal{F}_{basic}$ is the basic channel spectrum and $\mathcal{F}_{exp}$ the so-called expanded spectrum, i.e., the set of channels that have been added to $\mathcal{F}_{basic}$ in 1986 by the FCC (see, Lee [16]) in order to handle a growing number of users.

2.2 Interferences

We now describe the interference constraints which must be considered when assigning channel blocks (or channels) to cells in order to obtain an acceptable network quality.

Co-Channel interferences. Two cells can use the same channels without any interference if their respective coverages are mutually independent. We assume that this independence is expressed through a symmetric compatibility matrix M^{co} where each element M_{ij}^{co} measures the co-channel compatibility between the cells C_i and C_j. The value M_{ij}^{co} goes from 0 (perfect compatibility) through m^{co} (acceptable compatibility) to 10 (complete incompatibility).

Adjacent channel interferences. Similarly, two cells can use adjacent channels without any interference if their respective coverages are mutually independent. We assume that this independence is expressed through a symmetric compatibility matrix M^{adj} where each element M_{ij}^{adj} measures the adjacent channel compatibility between the cells C_i and C_j. Again, coefficients M_{ij}^{adj} vary between 0 and 10, with m^{adj} representing acceptable compatibility.

We shall assume here that both matrices M^{co} and M^{adj} are symmetrical and that $M_{ij}^{adj} \leq M_{ij}^{co}$ for all $i, j = 1, 2, \ldots, n$. Moreover $m^{co} = m^{adj}$, in order to simplify the mathematical expressions of the models of Sections 3 and 4. There is, however, no difficulty in generalizing them when those simplifying assumptions are not made.

Antenna channel spacing interferences. Typical filtering equipments require a minimum channel spacing, denoted by spa_i, between any two channels assigned to the same antenna of a given cell C_i.

Expanded spectrum. In order to guarantee that users with old phones are able to make their calls, a minimum fraction p_i of channels should be taken in the basic spectrum for each cell C_i of the network.

3 A First Integer Programming Formulation

We now explore a first 0-1 linear formulation that has already been partially explored by various authors in the literature. It is here enriched with the notion of antennas (a given cell may contain more than one antenna). We propose to define a model that guarantees an acceptable solution, i.e., a channel assignment with no co-channel interferences of level higher than m^{co} and no adjacent channel interferences of level higher than m^{adj}, while minimizing the number of non satisfied channel requirements.

3.1 Variables

The basic 0-1 variables $x_{i_h k}$ of the model are defined as follows:

$$x_{i_h k} = \begin{cases} 1 & \text{if channel block } B_k \text{ is assigned on antenna } A_h \text{ of cell } C_i \\ 0 & \text{otherwise.} \end{cases}$$

In order to simplify the algebraic expressions which do not require to distinguish the antennas from the cells, we introduce the 0-1 variables y_{ik} defined as follows:

$$y_{ik} = \sum_{h \in H_i} x_{i_h k} = \begin{cases} 1 & \text{if channel block } B_k \text{ is assigned on cell } C_i \\ 0 & \text{otherwise.} \end{cases}$$

3.2 Objective Function

We consider here the minimization of the unsatisfied channel demand which can be expressed as follows:

$$\min \sum_{i=1}^{N} \left(\overline{d}_i - \sum_{k \in K} |B_k| y_{ik} \right).$$

Assuming that we do not allow oversatisfiyng the channel demands, this is equivalent to maximizing the number of channels used throughout the network, i.e., to

$$\max \sum_{i=1}^{N} \sum_{k \in K} |B_k| y_{ik}.$$

3.3 Constraints

Channel demand constraints:

$$\underline{d}_i \leq \sum_{k \in K} |B_k| y_{ik} \leq \overline{d}_i, \qquad i = 1, \ldots, N.$$

Co-channel constraints:

$$y_{ik} + y_{jk} \leq 1 \qquad i, j = 1, 2, \ldots, N,\ i < j \text{ and } M_{ij}^{co} > m^{co}, \qquad k \in K.$$

Adjacent channel constraints:

$$y_{ik} + y_{j\ell} \leq 1 \qquad i, j = 1, 2, \ldots, N, \quad i \leq j \text{ and } M_{ij}^{adj} > m^{adj}$$
$$k, \ell \in K : B_k \text{ and } B_\ell \text{ are adjacent}$$

Antenna channel spacing constraints:

$$x_{i_h k} + x_{i_h \ell} \leq 1 \qquad h \in H_i; i = 1, 2, \dots, N \qquad k, \ell \in K : d(B_k, B_\ell) < spa_i$$

Basic spectrum limitations:

$$\sum_{k \in K} y_{ik} \, |B_k \cap \mathcal{F}_{basic}| \geq \; p_i d_i \qquad i = 1, 2, \dots, N.$$

4 First Column Generation Formulation

We propose below another 0-1 formulation that assumes a column generation scheme. In spite of their exponential number of variables (beware that they are implicitly taken into account in column generation methods), such formulations have proved to be quite successful for problems arising, e.g., in classification (du Merle *et al.* [6]) or airline scheduling (Desrosiers *et al.* [5]), and for the graph coloring problem (Mehrotra and Trick [18]).

4.1 Variables

For each antenna A_h of a given cell C_i, we define a set of 0-1 variables $z_t (t \in T_i^h)$ such that z_t is associated with a potential channel block (or channel) set to be assigned to the corresponding antenna, i.e., with a column vector $a_{.t} = (a_{1t}, a_{2t}, \dots, a_{|K|t})$ defined as follows.

$$a_{kt} = \begin{cases} 1 & \text{if channel block } B_k \text{ is assigned toantenna } A_h \text{ of cell } C_i \\ 0 & \text{otherwise} \end{cases} \qquad t \in T_i^h, h \in H_i, i = 1, 2, \dots, N.$$

Hence, T_i^h defines the index set of the variables z_t associated with the antenna A_h of cell C_i. Variable z_t $(t \in T_i^h, h \in H_i)$ will be set to 1 if the channel block set described by $a_{.t}$ is selected for antenna A_h of C_i, and to 0 otherwise. We denote by T_i the union of the sets $T_i^h, h \in H_i$, i.e., $T_i = \bigcup_{h \in H_i} T_i^h$, and T as the union of the T_i sets: $T = \bigcup_{i=1}^{N} T_i$.

The variables $x_{i_h k}$ and y_{ik} of the first formulation are related to the variables z_t of the second formulation by the equalities:

$$x_{i_h k} = \sum_{t \in T_i^h} a_{kt} z_t \qquad h \in H_i, \quad i = 1, 2, \dots, N, \quad k \in K \tag{1}$$

$$y_{ik} = \sum_{t \in T_i} a_{kt} z_t, \qquad i = 1, \dots, N, \quad k \in K. \tag{2}$$

4.2 Constraints

The following constraints ensure that at most one potential channel block (or channel) set is assigned to each antenna (some antenna may be unused):

$$\sum_{t \in T_i^h} z_t \leq 1 \qquad h \in H_i; \; i = 1, 2, \dots, N.$$

The remaining constraints are obtained from the first formulation by replacing $x_{i_h k}$ or y_{ik} by their expression in (1) and (2). Note that the linearity is preserved.

4.3 Master Problem

Because of the exponential number of variables z_t, the problem we have just described cannot be solved directly. The vectors $a_{.t}$, however, can be defined implicitly (i.e., by sets of constraints), and the problem can thus be solved by column generation techniques.

The so-called master problem is described below.

$$\max\left(f(z) = \sum_{t\in T}\sum_{k\in K}|B_k|a_{kt}z_t\right)$$

subject to:

$$\sum_{t\in T_i^h} z_t \leq 1 \qquad h\in H_i; i=1,2,\ldots,N \tag{3}$$

$$\underline{d}_i \leq \sum_{t\in T_i}\sum_{k\in K}|B_k|a_{kt}z_t \leq \overline{d}_i \qquad i=1,\ldots,N \tag{4}$$

$$\sum_{t\in T_i} a_{kt}z_t + \sum_{t\in T_j} a_{kt}z_t \leq 1 \qquad \begin{array}{c} k\in K: B_k\in S_i\cap S_j \\ i,j=1,2,\ldots,N,\ i<j \text{ and } M_{ij}^{co} > m^{co}\end{array} \tag{5}$$

$$\sum_{t\in T_i} a_{kt}z_t \leq 1 \qquad \begin{array}{c} k\in K: B_k\in S_i \\ i=1,2,\ldots,N\end{array} \tag{6}$$

$$\sum_{t\in T_i} a_{kt}z_t + \sum_{t\in T_j} a_{\ell t}z_t \leq 1 \qquad \begin{array}{c} k,\ell\in K: B_k\in S_i \text{ and } B_\ell\in S_j \text{ adjacent} \\ i,j=1,2,\ldots,N,\ i\leq j \text{ and } M_{ij}^{adj} > m^{adj}\end{array} \tag{7}$$

$$(a_{kt}+a_{\ell t})z_t \leq 1 \qquad \begin{array}{c} t\in T_i^h; h\in H_i; i=1,2,\ldots,N \\ d(B_k,B_\ell)\leq spa_i;\ k,\ell\in K\end{array} \tag{8}$$

$$\sum_{t\in T_i}\sum_{k\in K}|B_k\cap\mathcal{F}_{basic}|a_{kt}z_t \geq p_i d_i \qquad i=1,2,\ldots,N \tag{9}$$

$$z_t\in\{0,1\} \qquad t\in T. \tag{10}$$

Constraints (3) express that we must select at most one potential channel block (or channel) set for each antenna (some antenna may be unused). Constraints (4) describe the channel requirements. Constraints (5) and (6) are the co-channel ones (in most cases, constraints (6) will be implied by (5)), while the constraints (7) describe the adjacent channel ones. Constraints (8) express the antenna channel spacing constraints. Those last constraints are numerous and maybe be written in a more compact form:

$$\sum_{t\in T_i^h} a_{kt}z_t + \sum_{t\in T_i^h} a_{\ell t}z_t \leq 1 \qquad \begin{array}{c} h\in H_i; i=1,2,\ldots,N \\ d(B_k,B_\ell)\leq spa_i;\ k,\ell\in K.\end{array}$$

We will see in the next section how to handle them. Finally, constraints (9) describe a threshold on the number of channels in the basic spectrum.

4.4 Column Generation Method and Auxiliary Problem

The auxiliary problem, denoted (P_AUX), corresponds to the problem of finding the variable or column with the largest reduced cost. The basic idea of column generation methods is as follows (see Lasdon [15] for more details). The master problem is first solved with a small set of columns possibly generated with a heuristic. Then new columns are added as long as the auxiliary problem is able to produce columns with a positive reduced cost. When it fails to do so, then an optimal solution of the continuous linear relaxation has been obtained. We now describe the auxiliary problem.

The objective function can be rewritten:

$$f(z) = \sum_{t \in T} c_t z_t,$$

where $c_t = \sum_{k \in K} |B_k| a_{kt}$. As $T = \bigcup_{i=1}^{N} \bigcup_{h \in H_i} T_i^h$, solving (P_AUX) amounts to find the column with the largest reduced cost for each antenna A_h, and then to consider the largest of them.

When considering the auxiliary problem, it is often wise to transfer to its constraint set, the constraints of the master problem which impose conditions on the coefficients of a single column of the constraint matrix. This is the case of constraints (8) as well as the co-channel and adjacent channel constraints with respect to the channel blocks assigned to the same antenna of a given cell.

For a given antenna A_h of a cell C_i, since $\max \overline{c}_t = -\min -\overline{c}_t$, we have,

$$(\text{P\_AUX})_{h,i} \qquad \min\{-\overline{c}_t : a_{kt} + a_{\ell t} \leq 1 \text{ for } k, \ell \in K \text{ such that } d(B_k, B_\ell) \leq spa_i\},$$

where the analytical expression of the reduced cost is as follows:

$$\begin{aligned}
\overline{c}_t = & \sum_{k \in K} |B_k| a_{kt} - u_i^3 + \sum_{k \in K} |B_k| \underline{u}_i^4 a_{kt} - \sum_{k \in K} |B_k| \overline{u}_i^4 a_{kt} + \sum_{k \in K} |B_k \cap \mathcal{F}_{basic}| u_i^9 a_{kt} \\
& - \sum_{j: i<j, M_{ij}^{co} > m^{co}} \; \sum_{k: B_k \in S_i \cap S_j} u_{ijk}^5 a_{kt} - \sum_{j: i>j, M_{ji}^{co} > m^{co}} \; \sum_{k: B_k \in S_i \cap S_j} u_{jik}^5 a_{kt} - \sum_{k: B_k \in S_i} u_{ik}^6 a_{kt} \\
& - \sum_{j: i<j, M_{ij}^{adj} > m^{adj}} \; \sum_{k, \ell: d(B_k, B_\ell) \leq 1} u_{ijk\ell}^7 a_{kt} - \sum_{j: i>j, M_{ij}^{adj} > m^{adj}} \; \sum_{k, \ell: d(B_k, B_\ell) \leq 1} u_{jik\ell}^7 a_{kt}
\end{aligned}$$

with all dual variables being non negative.

In order to solve it efficiently, the auxiliary problem $(\text{P\_AUX})_{h,i}$ can be reformulated as a shortest path problem $(\text{PCC\_AUX})_{h,i}$ defined as follows. Consider a graph $G_i = (V_i, U_i)$ where each vertex, except for the source s and the sink p, is associated with a channel block:

$$V_i = \{v_k : v_k \equiv B_k, k \in K\} \cup \{v_0 \equiv s, v_{|K|+1} \equiv p\},$$

and each arc with a possible combination of two channel blocks:

$$U_i = \Big\{(v_k, v_\ell) : k < \ell; d(B_k, B_\ell) \geq spa_i\Big\} \cup \Big\{(v_0, v_k), (v_k, v_{|K|+1}) : k \in K\Big\}.$$

As $spa_i > \frac{L}{2}$ for all $i = 1, 2, \ldots, N$ (see Section 2.1 for a definition of L), we can assume, without loss of generality, that channel blocks are indexed in such a way that there are no pairs of blocks

(B_k, B_ℓ) with $\ell > k + spa_i$ such that $d(B_k, B_\ell) < spa_i$. This entails that: *(i)* no path involving pairs of channel blocks B_k, B_ℓ such that $d(B_k, B_\ell) < spa_i$ exists; and *(ii)* the graph G_i is acyclic. We also introduce two dummy blocks B_0 and $B_{|K|+1}$ respectively associated with the source and the sink.

After rewriting $a_{kt} = \sum_{\ell:(v_k,v_\ell)\in U_i} e_{k\ell}$ where $e_{k\ell} = 1$ if both channel blocks B_k and B_ℓ are assigned to antenna A_h of cell C_i, and 0 otherwise, the reduced cost can be rewritten as follows:

$$\overline{c}_t = c_0 + \sum_{(v_k,v_\ell)\in U_i} w_{k\ell}, \tag{11}$$

where $c_0 = -u_i^3$ is a constant and $w_{k\ell}$ are linear functions of variables $e_{k\ell}$. As $\sum_{\ell:(v_k,v_\ell)\in U_i} e_{k\ell} \in \{0,1\}$ $(k \in K)$, the auxiliary problem is then equivalent to a shortest path problem on an acyclic graph, and hence easy to solve.

5 Second Column Generation Formulation

In order to be able to transfer more constraints in the auxiliary problem, and therefore to improve the upper bound of the continuous relaxation, we redefine in this section, the variables of the master problem as follows. For each cell C_i, we define variables $z_t (t \in T_i)$, each of which associated with a potential channel block (or channel) set to be assigned on this cell, i.e., with a column vector $a_{.t} = (a_{1t}, a_{2t}, \ldots, a_{|K|t})$ defined as follows:

$$a_{kt} = \begin{cases} 1 & \text{if channel block } B_k \text{ is assigned on cell } C_i, \\ 0 & \text{otherwise} \end{cases} \qquad t \in T_i; i = 1, 2, \ldots, N.$$

and satisfying
$\mathcal{K} = \{k : a_{kt} = 1\}$ can be partitioned in at most $|H_i|$ subsets such that pairs B_k, B_ℓ of channel blocks belonging to the same subset satisfy the antenna spacing constraints, i.e., $d(B_k, B_\ell) \geq spa_i$.

A first master problem can be written:

$$\max\ f(z)$$

subject to:

$$\sum_{t\in T_i} z_t = 1 \qquad i = 1, 2, \ldots, N \tag{12}$$
$$\text{constraints } (4) \text{ to } (7), (9), (10)$$

(Beware that the sets T_i are however defined in a different way). As we assume that $\underline{d}_i > 0$, we must select one potential channel (block) set for each cell C_i $(i = 1, 2, \ldots, N)$ as expressed in constraints (12).

With the new definition of the variables z_t, we observe that constraints (4), (6) and (9) can be transferred to the auxiliary problem as each of them impose conditions on the coefficients of a single column. This implies that the constraints (5) which remain in the master problem will impose the inter-cell co-channel ones (while the intra-cell ones are imposed in the auxiliary problem). Similarly, the intra-cell adjacent channel constraints (7) can also be transferred to the auxiliary problem.

In such a case, the master problem becomes:

$$\max\{f(z) : \text{ constraints } (12), (5), (7) \text{ and } (10)\}.$$

The auxiliary problem can be then formulated as a shortest path problem with resource constraints on an acyclic graph with three type of resources to express: (R1) the channel demand constraints; (R2) the threshold on the minimum number of channels to be taken in the basic spectrum; (R3) the antenna channel spacing constraints. The last ones are the most difficult to model. They require to introduce a consumption $|H_i|$-uple for each resource expressed as an index set of channel blocks. They can be handled using a generalization of the two-phase algorithm of Jaumard, Vovor and Semet [14].

6 Algorithms

6.1 Solving the Continuous Relaxation

The solution of the continuous relaxation of the two master problems presented in the previous sections, has been outlined in Section 4.4. It can be improved both with respect to its optimal value (i.e., a lower bound on the optimal integer solution) and with respect to the speed of the column generation using the recent results of polyhedral theory for the vertex packing problem. Indeed, observe that constraints (5) and (7) (plus (6) if we consider the first column generation formulations) can be viewed as particular vertex packing ones, and hence reinforced with the addition of valid inequalities (see, e.g., Nemhauser and Sigismondi [19]). Moreover, in practice it is wise not to insert all the constraints at the beginning of the column generation method, but as needed as the iterations go along, i.e., only those which are violated by the current solution.

6.2 Branching Scheme

We describe the branching scheme for the first column generation formulation.

It is well known that it is usually more efficient to define a branching scheme on the variables of the auxiliary problem than on the variables of the master problem. We now describe a method for branching on the variables of the auxiliary problem.

Let $\tilde{z}$ be the optimal vector of the linear relaxation of the master problem of Section 4.3 with the constraints (8) transferred in the auxiliary problem. Denote by $s_{i,h}$ the slack variable of constraint $(3)_{i,h}$. Let $t_1 \in T_i^h$ be such that $\tilde{z}_{t_1}$ is fractional. We have one of the two following cases. Either there exists another $t_2 \in T_i^h$ such that $\tilde{z}_{t_2}$ is fractional or $\tilde{s}_{i,h}$ is fractional.
In the first case, let $k \in K$ be such that $a_{kt_1} \neq a_{kt_2}$ (such an index always exists since no two columns are identical). If there also exists an index ℓ such that $a_{\ell t_1} = a_{\ell t_2}$, we need to consider the following two possibilities: either $a_{kt} = a_{\ell t}$ or $a_{kt} + a_{\ell t} = 1$. In order to simplify the solution of the auxiliary problems, we propose to consider four branches: (B1) $a_{kt} = a_{\ell t} = 0$; (B2) $a_{kt} = a_{\ell t} = 1$; (B3) $a_{kt} = 1 - a_{\ell t} = 0$; (B4) $a_{kt} = 1 - a_{\ell t} = 1$. If there is no such index ℓ, we propose to perform

a binary branching on variable a_{kt}, $t \in T_i^h$.
In the second case, we perform a binary branching on variable $s_{i,h}$ (decide whether to use or not to use antenna A_h of cell C_i). In such a case, constraint $(3)_{i,h}$ either becomes an equality (variable $s_{i,h}$ fixed at 0, i.e., antenna A_h must be used) or becomes redundant and entails elimination of variables z_t, $t \in T_i^h$ (they are all fixed at 0).
We always give first priority to branchings involving two fractional variables $\tilde{z}_{t_1}$ and $\tilde{z}_{t_2}$, next to binary branching on the slack variables $s_{i,h}$ and then to the binary branching on variables a_{kt}.

In the branching scheme for the second formulation, it becomes simpler as we do not need to consider binary branching on some slack variables as constraints (12) are equalities.

The branching scheme entails additional constraints in the auxiliary problem. Looking carefully at each of the various cases, we observe that they all can be taken account either by merging or deleting one or two vertices, and by an adequate definition of the cost on the arcs leaving or going out of a vertex corresponding to an imposed or a forbidden channel block.

7 Preliminary Computational Results and Conclusion

Using a variant of the second column generation model formulation, we have solved the continuous relaxation of the master problem for an urban instance of the Bell Mobility network with 50 cells, $|K| = 63$, and $7 \leq d_i \leq 22$, $\mathcal{F} = 393$. It should be observed that finding a solution in reasonnable time for such an instance is unlikely with the first integer formulation as it requires 3,150 0-1 variables.

It can be often shown than column generation models provide better continuous relaxation bounds than the classical 0-1 linear models. This is indeed the case here for the channel assignment problem (see, Jaumard *et al.* [13]). Moreover, it is expected that they may allow to solve larger instances as the key point of the solution corresponds to the one of the auxiliary problems, here shortest path problems on acyclic graphs with resource constraints.

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