A load study for intersatellite links in Satellite constellations*

Afonso Ferreira† Jérome Galtier‡ Jean-Noël Petit† Patrick Bergé §

INRIA Sophia Antipolis

1 Introduction

Telecommunications systems are using satellites since the fifties. Most of them are using the geostationary orbit where satellites appear immobile to a terrestrial user. But at this time, there are few places left on this orbit. Therefore, the high altitude (36000 km) leads to large transmission delays and losses. Today, several telecommunications systems use nearer orbits like LEO (Low Earth Orbit with an altitude between 400 and 1500 km) or MEO (Medium Earth orbit with an altitude between 5000 and 13000 km). These orbits are free and the delay is quite short (between 20 and 100 ms round trip). But satellites on this kind of orbits do not appear immobile and one needs to use many orbits and satellites on each orbit to provide a global Earth coverage. These systems are called satellite constellations [1]. Of course, the architecture of a constellation (type of orbit, number of satellites, use of inter-satellites links [3–5]or not) must be chosen depending on the type of services it addresses. In constellations which do not use inter-satellite links, satellites are only relays. Call from users are directly sent to a gateway. Then, routing calls towards end users use terrestrial networks. We consider here constellations with inter-satellites links, forming a network in space. There, calls between users are routed as much as possible on the constellation itself. We want to study the impact of different architectural choices on the load of the inter-satellite links when a number of communications are in the system. In addition to probabilistic studies to evaluate the mean load of the links, we have implemented a satellite constellation simulator.

2 Problem

Two types of satellites constellations (LEO or MEO) have been proposed until now, depending on the kind of orbits they use.

The first uses near polar orbits, and is therefore called polar constellation. Ascending nodes of the orbits on this type of constellations are spread on 180 degrees (hence the alternate denomination, $\pi - constellation$). This configuration includes a seam, a place where satellites on two adjacent orbits are traveling in opposite directions. Today's technology does not allow the maintenance of inter-satellite links through the seam, and we will not consider them in the present paper. Due to this lack of inter-satellite links in the area of the seam, a call between two users each one on a different side of the seam must be routed through a pole. We will see what is the impact on the inter-satellite links load.

The other family of constellations is in general designed with inclined orbits. Ascending nodes of the orbits are there spread on 360 degrees ($2\pi - constellation$). These constellations do not include a seam. But the coverage of an area can be done by satellites of completly different orbits, running in reverse way each other. This configuration can lead to have a long route between two close users.

For sake of simplicity, we consider only polar

---

*This research was partly supported by the INRIA action COMMOBIL and the RNRT project Constellation.

† aferrei@sophia.inria.fr jnpetit@sophia.inria.fr SLOOP – CNRS-I3S-INRIA – BP 93 – F-06902 Sophia-Antipolis – France.

‡ jgaltier@sophia.inria.fr CNET – SLOOP – CNRS-I3S-INRIA – BP 93 – F-06902 Sophia-Antipolis – France.

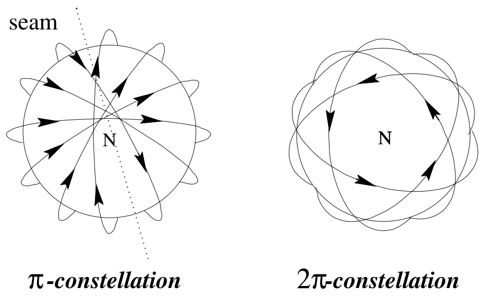
§ ENS Lyon – 46 allée d'Italie – 69364 Lyon cedex 07 – France.

orbits in this paper. Moreover, the inter-satellite links connect only neighboring satellite having the same latitude or the same longitude.

Then, we need to choose a routing policy. Indeed, whatever architectural choices are made in a constellation, the load on the links depends on the routing algorithm. For example, inter-orbital links between two satellites near the pole are shorter than near the equator. We have chosen a shortest path algorithm which minimizes the number of hops while keeping an XY shape.

Next, we want to study the impact of the traffic repartition on the inter-satellite link load. At first we use an uniform repartition of users on earth and all the satellites handle the same amount of calls. Then we consider a repartition of the users by latitude.

3 Probabilistic studies



In the following, we call Y the links which connect two successive satellites on the same orbit (i.e. having the same longitude). And we call X the links which connect two satellites on adjacent orbits (i.e. having the same latitude). We study a constellation where each satellite is linked with four of its neighbours. The previous and the next on the same orbit, and one on each of the adjacents orbits.

We also consider non-uniform distributions of users. For this, we introduce a latitude density function f of users ranking from $-\pi$ to π (or equivalently from $90S$ to $90N$), that governs the presence of users. This function allows to take into account many characteristics of the distribution of the demand around the globe. We show in figure 1 examples of such a function. The first one gives a probability equal to the *area* of a zone. The second is based on market considerations de-

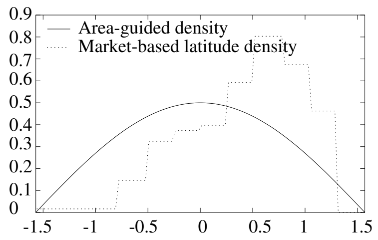


FIG. 1 – *Examples of latitude density functions.*

rived from [6]. For sake of simplicity, we suppose $f\left(\frac{\pi}{2}+x\right)=f(\frac{\pi}{2}-x)$ and $f(x+2\pi)=f(x)$.

3.1 $\pi-constellation$

Average load of a Y link due to a communication

Proposition 1 *We denote by n_0 the number of orbits of the constellation and n_s the number of satellites per orbit, then the probability that a communication caused by a uniform distribution uses a given Y link is given by :*

$$P_y=\frac{\lfloor\frac{n_s}{2}\rfloor\lceil\frac{n_s}{2}\rceil}{n_o n_s^2}$$

Proof: With uniform density of call on the satellites, the average length of the Y links is given by the average length of a path between two nodes selected with equal probability inside a cycle of length n_s. When the number of satellites is odd, the sum of the length of the paths from one satellite to all the others is given by

$$2\left(\frac{\lceil\frac{n_s}{2}\rceil(\lceil\frac{n_s}{2}\rceil-1)}{2}\right)$$

When the number is even, (i.e $1-\lceil\frac{n_s}{2}\rceil+\lfloor\frac{n_s}{2}\rfloor=1$) we must add the distance to the opposite satellite in the cycle: $\lceil\frac{n_s}{2}\rceil$.

According to this, the average length in terms of Y links of a communication is given by:

$$\frac{(\lceil\frac{n_s}{2}\rceil-1)\lceil\frac{n_s}{2}\rceil+(1-\lceil\frac{n_s}{2}\rceil+\lfloor\frac{n_s}{2}\rfloor)\lceil\frac{n_s}{2}\rceil}{n_s}=\frac{\lceil\frac{n_s}{2}\rceil\lfloor\frac{n_s}{2}\rfloor}{n_s}.$$

The result is then derived from the fact that the communication can start with uniform probability from any of the $n_0 n_s$ satellites of the constellation.

□

Proposition 2 *The probability of use of a Y link by a communication with non-uniform user distribution governed by latitude density f is given by:*

$$P_y(\lambda) = \frac{1}{n_o \cdot s(\lambda - \frac{2\pi}{n_s},\lambda + \frac{2\pi}{n_s})} \times$$
$$\left[2 \sum_{\substack{1+i+j < \frac{n_s}{2} \\ i \geq 0 \\ j \geq 0}} s(\lambda+\tfrac{2i\pi}{n_s},\lambda+\tfrac{2(i+1)\pi}{n_s})\cdot s(\lambda-\tfrac{2(j+1)\pi}{n_s},\lambda-\tfrac{2j\pi}{n_s}) \right.$$
$$\left. + \sum_{\substack{1+i+j = \frac{n_s}{2} \\ i \geq 0 \\ j \geq 0}} s(\lambda+\tfrac{2i\pi}{n_s},\lambda+\tfrac{2(i+1)\pi}{n_s})\cdot s(\lambda-\tfrac{2(j+1)\pi}{n_s},\lambda-\tfrac{2j\pi}{n_s}) \right]$$

with $s(a,b) = \int_a^b f(t)dt$

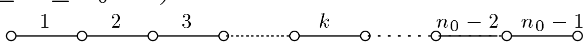
Proof: Omitted. □

Average load of a X link due to a communication

Proposition 3 *We denote by n_0 the number of orbits of the constellation and n_s the number of satellites per orbit, then the the probability that a communication caused by a uniform distribution uses a given X link is given by:*

$$P_x(k) = \frac{2k(n_o - k)}{{n_o}^2 n_s}.$$

where k is the index of the link, defined as follows ($1 \leq k \leq n_0 - 1$)*.*



In particular the probability that a communication uses a given central link (at equal distance from seams) is :

$$P_x\left(\frac{n_o}{2}\right) = \frac{2\left(\frac{n_o}{2}\right)^2}{{n_o}^2 n_s} = \frac{1}{2n_s}.$$

Proof: In a configuration with uniform load, a link k is used by any path that connects one of the k satellites before it to one of the $n_0 - k$ satellites after it, and vice versa. Therefore :

$$\frac{\frac{k}{n_0}\frac{n_0-k}{n_0}}{n_s} + \frac{\frac{n_0-k}{n_0}\frac{k}{n_0}}{n_s}.$$

Hence the result.

□

Proposition 4 *The average probability of use of a X link of index k by a communication with non-uniform user distribution governed by latitude density f is given by:*

$$P_x(k,\lambda) = \frac{2k(n_o - k)}{n_o^2 \cdot \int_{\lambda-\frac{2\pi}{n_s}}^{\lambda+\frac{2\pi}{n_s}} f(t)dt}$$

Proof: Omitted. □

3.2 $2\pi - constellation$

A user is connected to one of the satellites that cover him, choosen at random. The distribution is uniform.

Proposition 5 $P_y = \frac{\lfloor\frac{n_s}{2}\rfloor\lceil\frac{n_s}{2}\rceil}{n_o n_s^2}$ *and* $P_x = \frac{\lfloor\frac{n_0}{2}\rfloor\lceil\frac{n_0}{2}\rceil}{n_s n_0^2}$

Proof: This is an easy adaptation of the previous one. □

3.3 Overload probability

We want to know how many chanels are required between two satellites to maintain the communications. This number must be specified to make negligible communications which are cut or which can not be established.

We define the reject rate as the ratio between the number of communications cut or not established and the total number of generated communications.

3.3.1 Exact overload

As we saw in the previous section, for each communication, horizontal links who are the farthest from the seam have the highest probability to be used. They are the critical ones.

The probability that N independent communications arising with the probability p use more than C chanels on a link is given by:

$$\sum_{i=C}^{N} \binom{N}{i} p^i (1-p)^{N-i}.$$

For example, with $N = 30$, $n_s = 11$ and $p = \frac{1}{2n_s}$, links must have a minimum capacity of 7 to keep the probability of dropping a call close to zero.

3.3.2 Asymptotical overload

Using a stochastic study, we establish an upper bound of this rate according to the number N of available chanels on an intersatellite link and the average number of communications $< n_{ij} >$ using this link between satellites i and j. We suppose n_{ij} be a random variable whose expectation comes from statistical data on communications calls from one point to another on Earth. These data depend only on the geographical coordinates and time of call on the source and the destination terminals . Then, we get $< n_{ij} >$ from the routing algorithm and position of the satellites at this time. In fact, n_{ij} is a time-dependent variable

Random variable n_{ij} is a sum of random variables (in fact the number of calls which use the link ij). These variables are supposed to be independent and come from a temporal process.

Proposition 6 *We suppose the link ij allows an infinite number of chanels. We suppose the probability law of this variable is a Poisson law whose parameter is $< n_{ij} >$:*

$$Pr[n_{ij}] = e^{-<n_{ij}>} \frac{< n_{ij} >^{n_{ij}}}{n_{ij}!}.$$

Then we have:

$$Pr[n_{ij} \geq N] \leq \left(\frac{e < n_{ij} >}{N} \right)^N.$$

Proof: To bound the reject rate due to the link ij, we consider $Pr[n_{ij} > N]$.

For any positive random variable X we have the following Markov inequality:

$Pr[X \geq u] \leq E[X]/u$.

We suppose f a positive increasing function. We get

$$Pr[X \geq u] = Pr[f(X) \geq f(u)] \leq E[f(X)]/f(u).$$

This result can be applied to the function $f(x) = e^{\alpha x}$, choosing the parameter α to reduce the right member of the inequality above.

So we write:

$$\begin{aligned} E[f(X)] &= \sum_{k \geq 0} \frac{e^{\alpha k} \cdot e^{-<n_{ij}>} <n_{ij}>^k}{k!} \\ &= e^{-<n_{ij}>} e^{[e^{\alpha} \cdot <n_{ij}>]} \end{aligned}$$

Then, we consider the value:

$$T_\alpha = \frac{E[f(X)]}{f(N)} = \frac{e^{-<n_{ij}>} \cdot e^{[e^{\alpha} \cdot <n_{ij}>]}}{e^{\alpha N}}$$

which is minimized when:

$$\alpha = \alpha^* = \log \left[\frac{N}{< n_{ij} >} \right]$$

and we get:

$$P_r[n_{ij} \geq N] \leq T_{\alpha^*} = e^{-<n_{ij}>} \left(\frac{e < n_{ij} >}{N} \right)^N$$

$\square$

The expression $Pr[n_{ij} > N]$ bounds the probability the link ij with N chanels refuses a call. If we sum on all the links, we get a overvaluation of the reject rate:

$$\rho \leq \sum_{ij} \left(\frac{e < n_{ij} >}{N} \right)^N.$$

Then, if $< n_{ij} >$ is not homogeneous, with N more than 1000, and the right members being exponential, we can approximate this sum keeping only the terms whose ij couple has the highest $< n_{ij} >$, denoted n_{max}:

$$\rho \leq \left(\frac{e \cdot n_{max}}{N} \right)^N.$$

Whatever the reject rate we want, we are able to establish implicitly the capacity of a link, according to the statistical data and the routing algorithm chosen.

A natural question that this development raises is which link has the maximum mean load? Let us compare the load of Y link to the maximum load of a X link. When do we have

$$\frac{\lfloor \frac{n_s}{2} \rfloor \lceil \frac{n_s}{2} \rceil}{n_o n_s^2} < \frac{1}{2n_s}?$$

Taking the approximation $\lfloor \frac{n_s}{2} \rfloor \lceil \frac{n_s}{2} \rceil \approx \frac{n_s^2}{4}$ this inequality becomes true when $n_s < 2n_0$. The furthest X links from the seams are the most loaded for $\pi-$constellations whose number of satellites on each orbit is less than twice the number

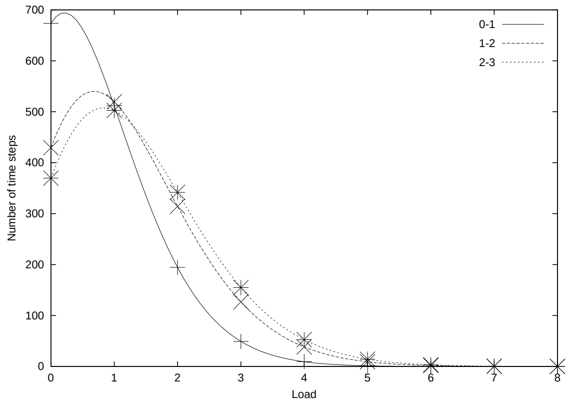


FIG. 2 – *Density of load of the inter orbital links - Model*

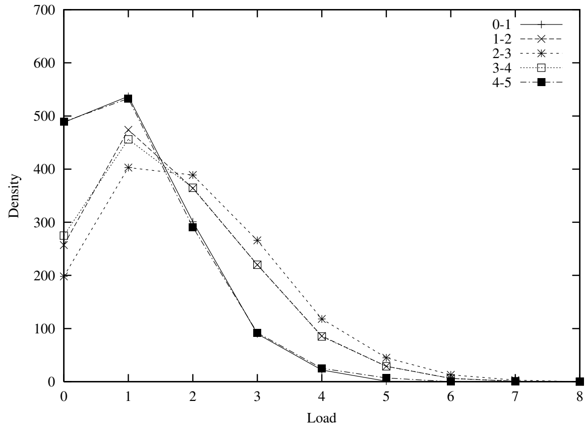


FIG. 3 – *Density of load of the inter orbital links - Simulation*

of orbits. As a result of proposition 3, in a uniformly loaded constellation, one link of index k will be required for an incoming communication with probability :

$$P^{'}(k) = \frac{2k(n_0 - k)}{n_0^2}$$

If we suppose that the communications already in the system follow a Poisson distribution (see Figure 2), the probability that this link is already at maximum capacity C is, according to proposition 6 :

$$P[n_{ij} \geq C] \leq \left(\frac{e\mu P(k)}{C}\right)^C.$$

We then bound the failure probability by : $p = P^{'}(k)\left(\frac{e\mu P(k)}{C}\right)^C$ and the total number of failures can be bounded by Mp, where M is the number of incoming communications. This is at the basis of figure 4, with $M = 1440$, $\mu = 30$, $n_0 = 6$, $n_s = 11$, $k = 1, 2, 3, 4, 5$ and $C = 5, 6, 7$.

4 Constellation simulator

We implemented a constellation simulator where all the significant parts of a satellite constellation are taken into account : satellites, gateways, users, calls. We oriented it to simulate a constellation with inter-satellite links. A simulation takes into account both the call routing and rerouting due to handovers. We can use different kinds of traffic repartition and routing policy. At first, we took the Iridium constellation as a model, but we are able to modify the simulator to handle constellations with different architectural choices, like inclined orbits or number of inter-satellite links. The simulator core use Prosit [2] which provides features like time step simulation, object programming (C++), thread programming on different hardware architectures. The time steps of the simulation must be equal or smaller to the delay between two handovers for a call.

5 Simulation results

We show here only the results we get using a $\pi - constellation$. We stress that the previous study showed that this type of constellation contains at least all the irregularity phenomenons observed in a $2\pi - constellation$. Calls are uniformly spread and their duration is between 10 and 50 minutes (So the mean number of communications present in the system is also $\mu = 30$). Results of the simulation are shown in Figure 3 and Figure 5. We denote i-$i+1$ the category of

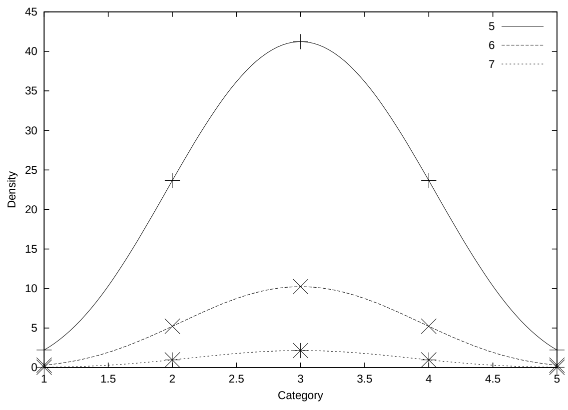


FIG. 4 – *Call failures - Model*

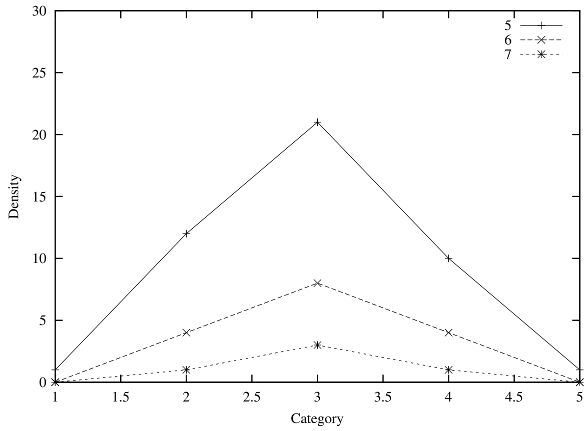


FIG. 5 – *Call failures - Simulation*

inter-orbital links between the i^{th} and the $(i+1)^{th}$ orbit, where the orbits are indexed from 0 to 5. Figure 3 presents for each possible load state of a link, the average number of time steps spent in this state by a link of the category. We saw that links $2-3$ are in general more loaded than anyother. Therefore, only this category reaches the load of 7 ; the behavior of the curves is quite similar to that of the Poisson model (see Figure 2) although the links seem to be a little bit more loaded for medium capacities. The figure also confirms that the furthest link from the seams is the most loaded. The Figure 5 presents for 3 different capacities and for each category of link how many calls were refused or cut. Clearly, the real amount of call failures is under the upper bound given by the statistical model. For small capacities ($C = 5$) the bound is quite far from the reality (say twice the real value) but it gets better when the capacity increases.

6 Conclusion

Given a routing algorithm and a call distribution, we have established the probability a communication uses the X and Y links in a π or 2π constellation. A surprising result is that in our Iridium-like constellation, the central X link appears to be the most loaded. These results, of course, appeal further work. Among topics of future research, we include the consideration of inclined orbits, other user distribution models, and various routing algorithms.

Références

[1] M. D. Kennedy and P. L. Malet. Application for authority to construct, launch and operate the Celestri multimedia LEO system. Technical report, Motorola Global Communications, Inc., june 1997. Filed before the Federal Communications Commission.

[2] G. Siegel. *Prosit: Un environnement pour la programmation de simulations à événements discrets.* PhD thesis, Université de Nice - Sophia Antipolis, September 1997.

[3] M. Werner. A dynamic routing concept for ATM-based satellite personal communication networks. *IEEE Journal on Selected Areas in Communications*, 15(8):1636–1648, October 1997.

[4] M. Werner, C. Delucchi, and K. Burchard. ATM networking for future ISL-based LEO satellite constellations. In *International Mobile Satellite Conference (IMSC'97)*, pages 295–300. JPL Publications, 1997.

[5] M. Werner, C. Delucchi, H.-J. Vogel, G. Maral, and J.-J. De Ridder. ATM-based routing in LEO/MEO satellite networks with intersatellite links. *IEEE Journal on Selected Areas in Communications*, 15(1):69–82, January 1997.

[6] M. Werner and G. Maral. Traffic flows and dynamic routing in LEO intersatellite link networks. In *International Mobile Satellite Conference (IMSC'97)*, pages 283–288. JPL Publications, 1997.