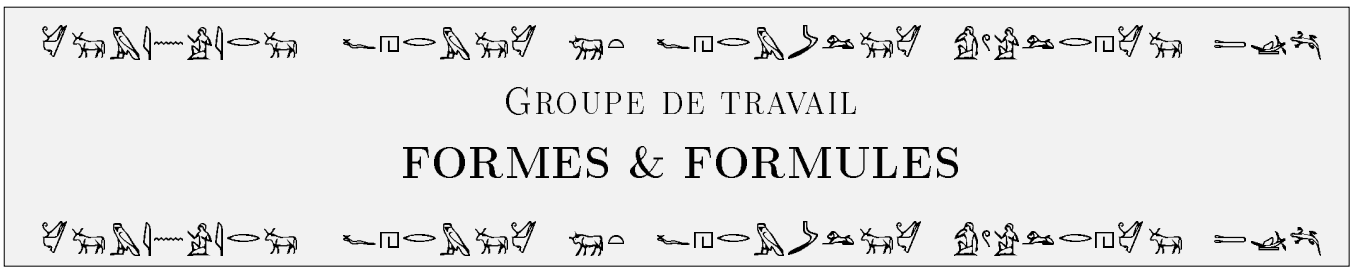

Le Jeudi 9 Mai à 14 Heures, Bat. Math :

Victor Y. Pan

« Optimal Algorithms for Approximating Polynomial Zeros »

We substantially improve the known algorithms for approximating all the complex zeros of an n-th degree polynomial $p(x)$. Our new algorithms save both Boolean and arithmetic sequential time, versus the previous best algorithms of Schönhage 1982, Pan 1987 and Neff and Reif 1994. In parallel (NC) implementation, we dramatically decrease the number of processors, versus the parallel algorithm of Neff 1994, which was the only NC algorithm known for this problem so far. Specifically, under the simple normalization assumption that the variable x has been scaled so as to confine the zeros of $p(x)$ to the unit disc $\{x : |x| \leq 1\}$, our algorithms (which promise to be practically effective) approximate all the zeros of $p(x)$ within the absolute error bound 2^{-b}, by using order of n arithmetic operations and order of $(b+n)n^2$ Boolean (bitwise) operations (in both cases up to within polylogarithmic factors). Orders of n and $(b+n)n^2$ of arithmetic and Boolean processors, respectively, suffice in parallel polylogarithmic time implementation of our algorithms. All the cited bounds on the computational complexity are *within polylogarithmic factors from the optimum* (in terms of n and b) under both arithmetic and Boolean models of computation [in the Boolean case, under the additional (realistic) assumption that $n = O(b)$].

Key Words: complex polynomial zeros, approximation, polynomial factorization, computational complexity, parallel algorithms, sequential algorithms.