Roots in geometric progression along a spiral

`spiral.1.n`

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Ring: $\mathbb{C}[x]$
Equations:

$$\prod_{j=1}^{n}(x+\frac{a^j-1}{a-1}), |a| << 1, (n=20, a=i/1000, a=(3+4i)/10)$$

Suppplied by D. A. Bini (Oct. 1996).

1 Characteristics

- The polynomial has roots along a spiral centered in $1/(a-1)$, where the distances of the j-th zero from the center of the spiral approach exponentially 0.
- For $n=20$, $a=i/1000$ the condition number of the roots ranges from 10^{30} to 10^{138}. The closest roots have about 60 common digits. In this way a working precision of 198 digits is needed for separating the closest roots. The numerators and the denominators of the coefficients have at most 132 digits.
- Large values of n, say $n=500, 1000, 2000$, constitute an interesting test for polynomial root-finders.

2 Problem

Newton-isolate all the roots.

2.1 Solution

Algorithm based on simultaneous iterations (Aberth's method) with cluster analysis (see [?]).

Program in Fortran 90 based on the multiprecision package MPFUN, by D. Bailey.

Machine used: IBM Risc 6000.

Time for Newton-isolating all the roots: for $a = (3 + 4i)/10$, $n = 20$: 46 seconds.

Dario Andrea Bini, Dipartimento di Matematica, via Buonarroti 2, 56127 Pisa, ITALY.
`bini@dm.unipi.it`. (October 15, 1996)

References

[1] D. Bini. Numerical computation of polynomial zeros by means of aberth's method. *Numerical Algorithms*, 1997. (to appear).

[2] D. Bini and B. Mourrain. Polynomials test suite. 1996.