N-th Roots

`nthroot.1.n`

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Ring: $\mathbb{C}[x], \mathbb{R}[x]$
Equations:

$x^n - a, \quad (n = 20, a = 1)$

Supplied by D. A. Bini (Oct. 1996).

1 Characteristics

- Sparse polynomial.
- The polynomial has roots equidistributed along a circle, for $a = 1$ they are the n-th roots of the unity.
- Even though the roots come close together as n grows, they are numerically well conditioned. Their condition number is at most $2/n$.
- Interesting cases are given by large values of n, say, $n = 500, 1000, 10000$.
- Due to its specificity, the polynomial can be assigned as a straight-line program.

2 Problem

Newton-isolate all the roots.

2.1 Solution

Algorithm based on simultaneous iterations (Aberth's method) with cluster analysis (see [**?**]).

Program in Fortran 90 based on the multiprecision package MPFUN, by D. Bailey.

Machine used: IBM Risc 6000.

Time for Newton-isolating all the roots: for $n = 20$, $a = 1$: 0.1 seconds; for $n = 200$, $a = 1$: 0.57 seconds; for $n = 500$, $a = 1$: 3.4 seconds.

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References

[1] D. Bini. Numerical computation of polynomial zeros by means of aberth's method. *Numerical Algorithms*, 1997. (to appear).

[2] D. Bini and B. Mourrain. Polynomials test suite. 1996.