Mignotte-like Polynomial 2

`mignotte2.1.n`

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Ring: $\mathbb{C}[x], \mathbb{R}[x]$
Equations:

$(x^{n-k} + (ax+1)^m)(ax+1)^k, \quad n >> m > 1,\ n >> k > 1,\ |a| > 1,\ (n = 20, m = 3, k = 3, a = 100i)$

The polynomial is a modification of Mignotte's polynomial $x^n + (ax+1)^2$ having two close roots whose distance is close to the Mahler bound. Supplied by D. A. Bini (Oct. 1996).

1 Characteristics

- The polynomial has a cluster of m very close roots around $-1/a$ where $-1/a$ is also a multiple root of multiplicity k.
- Difficult situations can be obtained by tuning n, m, k and $|a|$, say $n = 500$, $m = 10$, $k = 2$, $a = 10000$.

2 Problem

Newton-isolate all the roots up to within a relative error 10^{-100} for nonisolated roots.

Newton-isolate all the root with automatic detection of multiplicity.

Newton-isolate all the roots of modulus greater than 1.

Count the number of roots of modulus greater than 1.

2.1 Solution

Algorithm based on simultaneous iterations (Aberth's method) with cluster analysis (see [**?**]).

Program in Fortran 90 based on the multiprecision package MPFUN, by D. Bailey.

Machine used: IBM Risc 6000.

For $n = 20$, $m = k = 3$, $a = 100i$, time needed for Newton-isolating all the roots up to within a relative error 10^{-100} for nonisolated roots: 3.8 seconds;

with automatic detection of multiplicity: 7 seconds; time for isolating the roots of modulus greater than 1: 0.88 seconds.

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References

[1] D. Bini. Numerical computation of polynomial zeros by means of aberth's method. *Numerical Algorithms*, 1997. (to appear).

[2] D. Bini and B. Mourrain. Polynomials test suite. 1996.