Mignotte-like Polynomial 1

`mignotte1.1.n`

February 18, 1997

Ring: $\mathbb{C}[x], \mathbb{R}[x]$
Equations:

$$x^n + (ax+1)^m, \quad n >> m > 1, \ |a| > 1, \quad (n = 20, m = 3, a = 100i)$$

The polynomial is a modification of Mignotte's polynomial $x^n + (ax+1)^2$ having two close roots whose distance is close to the Mahler bound. Supplied by D. A. Bini (Oct. 1996).

1 Characteristics

- The polynomial has m very close roots around $-1/a$.
- For $n = 20, m = 3, a = 100i$, there are 3 roots close to $-1/100i$, matching in the first 13 digits. Their condition numbers are greater than 10^{25} so that at least 38 digits of working precision are needed for isolating the roots. The remaining roots are well conditioned and 15 digits of working precision are sufficient to isolate them.
- Difficult situations can be obtained by tuning n, m and $|a|$.
- Interesting situations are: the case of large degree and small cluster, i.e., $n = 100, 500$, $m = 3$, $a = 100i$; the case of large degree and large cluster, i.e., $n = 100, 500$, $m = 31$, $a = 1000$.

2 Problem

Newton-isolate all the roots.

Newton-isolate all the roots of modulus greater than 1.

Count the number of roots of modulus greater than 1.

2.1 Solution

Algorithm based on simultaneous iterations (Aberth's method) with cluster analysis (see [?]).

Program in Fortran 90 based on the multiprecision package MPFUN, by D. Bailey.

Machine used: IBM Risc 6000.

For $n = 20$, $m = 3$, $a = 100i$, time needed for isolating all the roots: 1.0 seconds; time for isolating the roots of modulus greater than 1: 0.4 seconds.

For $n = 100$, $m = 3$, $a = 100i$, time needed for isolating all the roots: 5.5 seconds; time for isolating the roots of modulus greater than 1: 0.6 seconds.

For $n = 500$, $m = 3$, $a = 100i$, time needed for isolating all the roots: 106 seconds; time for isolating the roots of modulus greater than 1: 3.4 seconds.

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References

[1] D. Bini. Numerical computation of polynomial zeros by means of aberth's method. *Numerical Algorithms*, 1997. (to appear).

[2] D. Bini and B. Mourrain. Polynomials test suite. 1996.