# Kameny Polynomial 4

`kameny.1.14`

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**Ring:** $\mathbb{R}[x]$
**Equations:**

$$x^{14} + 2 \cdot 10^{24} \cdot x^{11} + 10^{48} \cdot x^8 + 4 \cdot x^7 - 4 \cdot 10^{24} \cdot x^4 + 4$$

Polynomial from the REDUCE test suite, supplied by Stan Kameny. Added to the list by D.A. Bini (Oct. 1996).

## 1 Characteristics

- Degree 14, integer coefficients
- The polynomial has a cluster of 8 roots of modulus $1.89 \cdot 10^{-6}$, a set of 6 roots having modulus $10^8$. The cluster made up by 8 roots has: two real roots having 21 common digits

  $$-0.0000011892071150027210667183, \; -0.0000011892071150027210667167,$$

  two pairs of complex roots having small real parts, and imaginary parts with 21 common digits

  $$-5.4525386633262882960501 \cdot 10^{-28} \pm 0.000001189207115002721066718045\mathtt{i},$$
  $$5.4525386633262882960201 \cdot 10^{-28} \pm 0.000001189207115002721066716954\mathtt{i},$$

  The set made up by 6 roots of modulus $10^8$ has two very close real roots

  $$-1.00000000000000000000000000009 \cdot 10^8, \; -9.99999999999999999999999999906 \cdot 10^7,$$

  matching in their first 28 digits, and 2 very close complex pairs

  $$4.99999999999999999999999999953 \cdot 10^7 \pm 8.66025403784438646763723170835 \cdot 10^7\mathtt{i},$$
  $$5.00000000000000000000000000047 \cdot 10^7 \pm 8.66025403784438646763723170671 \cdot 10^7\mathtt{i},$$

  whose real and imaginary parts match in their first 27 digits.
- For the condition number of the above roots we have $cond(-1.00000000000000000000000000009 \cdot 10^8) > 10^{35}$, $cond(-9.99999999999999999999999999906 \cdot 10^7) > 10^{35}$, $cond(-0.00000118920711500272106671$ $10^{14}$, $cond(-0.0000011892071150027210667167) > 10^{14}$.

This means that in order to separate the former two real roots by using a floating point computation, at least 63 digits of working precision are needed, while for the latter two roots at least 35 digits are needed. A similar result holds for the complex roots.

# 2 Problem

Newton-isolate all the roots.

Newton-isolate the real roots.

Count the number of real roots.

Newton-isolate the roots of modulus greater than 1.

Count the number of roots of modulus greater than 1.

## 2.1 Solution

Algorithm based on simultaneous iterations (Aberth's method) with cluster analysis (see [?]).

Program in Fortran 90 based on the multiprecision package MPFUN, by D. Bailey.

Machine used: IBM Risc 6000.

Time for isolating all the roots: 2.9 seconds;

Time for isolating only the real roots: 1.5 seconds;

Time for isolating the roots of modulus greater than 1: 1.5 seconds;

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# References

[1] D. Bini. Numerical computation of polynomial zeros by means of aberth's method. *Numerical Algorithms*, 1997. (to appear).

[2] D. Bini and B. Mourrain. Polynomials test suite. 1996.