Kameny Polynomial 3

`kameny2.1.9`

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Ring: $\mathbb{R}[x]$
Equations:

$(c^2x^2 - 3)^2 + c^2x^9, \quad c = 10^6$

Polynomial from the REDUCE test suite, supplied by Stan Kameny. Added to the list by D.A. Bini (Oct. 1996).

1 Characteristics

- Degree 9, integer coefficients.
- The polynomial has two extremely close real roots (20 common digits)

  $-0.000001732050807568877293530865, -0.000001732050807568877293524027,$

  plus a complex pair with extremely small imaginary part.

  $$1.732050807568877293527446 \cdot 10^{-6} \pm 3.419260585432166 \cdot 10^{-27}\mathtt{i}.$$

  The remaining roots are well separated, one of them is real: -251.189.
- The condition number of the above clustered roots is greater than 10^{20}
- More difficult situations can be obtained with larger values of the parameter c. For $c = 10^{20}$ we have roots with 70 common digits and nonzero imaginary part of modulus less than 10^{-90}. For $c = 10^{70}$ we have roots with about 247 common digits and nonzero imaginary part of modulus less than 10^{-315}.

2 Problem

Newton-solate all the roots.

Newton-isolate the real roots.

Newton-isolate the roots of modulus greater than 1.

Count the number of real roots.

2.1 Solution

Algorithm based on simultaneous iterations (Aberth's method) with cluster analysis (D. Bini, Numerical computation of polynomial zeros by means of Aberth's method, to appear in Numerical Algorithms).

Program in Fortran 90 based on the multiprecision package MPFUN, by D. Bailey.

Machine used: IBM Risc 6000.

Time for isolating all the roots: for $c = 10^6$, 0.88 seconds; for $c = 10^{20}$, 1.22 seconds; for $c = 10^{70}$, 4.6 seconds.

Time for Newton-isolating all the roots of modulus greater than 1: for $c = 10^6$, 0.3 seconds; for $c = 10^{20}$, 0.3 seconds; for $c = 10^{70}$, 0.45 seconds.

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References

[1] D. Bini and B. Mourrain. Polynomials test suite. 1996.