Kameny Polynomial 2

`kameny1.1.9`

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Ring: $\mathbb{C}[x]$
Equations:

$(c^2x^2-3)^2+ic^2x^9, \quad c>>1, \quad (c=10^6)$

Polynomial from the REDUCE test suite, supplied by Stan Kameny. Added to the list by D. Bini (Oct. 1996).

1 Characteristics

- Degree 9, real and imaginary parts of the coefficients are integers.
- The polynomial has four simple complex roots with small imaginary parts (10^{-27}) in two close sets (21 common digits):

  $$\pm\,1.73205080756887729352502 8\cdot 10^{-6}+2.41778234660296931896\cdot 10^{-27}\mathtt{i},$$
  $$\pm 1.73205080756887729352986 4\cdot 10^{-6}-2.41778234660296931902\cdot 10^{-27}\mathtt{i}.$$

  The remaining roots are well separated.
- The condition number of the above roots is $cond>10^{20}$. Therefore at least 41 digits of working precision are needed to separate the roots with a floating point computation.
- More difficult situations can be obtained with larger values of the parameter c. For $c=10^{20}$ we have roots with 70 common digits and nonzero imaginary part of modulus less than 10^{-90}. For $c=10^{70}$ we have roots with about 247 common digits and nonzero imaginary part of modulus less than 10^{-315}.

2 Problem

Newton-isolate all the roots.

Newton-isolate all the roots of modulus greater than 1.

2.1 Solution

Algorithm based on simultaneous iterations (Aberth's method) with cluster analysis (see [?]).

Program in Fortran 90 based on the multiprecision package MPFUN, by D. Bailey.

Machine used: IBM Risc 6000.

Time for Newton-isolating all the roots: for $c = 10^6$, 0.9 seconds; for $c = 10^{20}$, 1.3 seconds; for $c = 10^{70}$, 5 seconds.

Time for Newton-isolating all the roots of modulus greater than 1: for $c = 10^6$, 0.3 seconds; for $c = 10^{20}$, 0.34 seconds; for $c = 10^{70}$, 0.44 seconds.

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References

[1] D. Bini. Numerical computation of polynomial zeros by means of aberth's method. *Numerical Algorithms*, 1997. (to appear).

[2] D. Bini and B. Mourrain. Polynomials test suite. 1996.