Proof that PCPT is Unbiased

We use standard Monte Carlo methodology. The expected value of our estimator (Eq. (9) in the main paper) is given as follows:

$$E\left[\frac{1}{k}\sum_{k=1}^{K}\frac{I(i,\xi_k)}{p_i^{conn}(\xi_k)}\right] = \frac{1}{k}\sum_{k=1}^{K} E\left[\frac{I(i,\xi_k)}{p_i^{conn}(\xi_k)}\right] \tag{1}$$

$$= E\left[\frac{I(i,\xi_1)}{p_i^{conn}(\xi_1)}\right] \tag{2}$$

$$= \sum_{j=1}^{M}\frac{I(i,j)}{p_i^{conn}(j)}p_i^{conn}(j) = \sum_{j=1}^{M} I(i,j) = I_i \tag{3}$$

We use linearity of expected value on the first line and the fact that ξ_k are IID from Eq. (1) to Eq. (2) and the definition of expected value to go from Eq. (2) to Eq. (3).

Thus p_i^{conn} can be chosen to be an arbitrary distribution, as long as it is a PMF, i.e., it sums to 1 and $p_i^{conn}(j) \neq 0$ whenever $I(i,j) \neq 0$. The latter property is guaranteed by blending with the uniform PMF.