Simulations on RF antenna-plasma coupling: RF sheath rectification process taken into transverse currents in $\Omega_{ci}$ range

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Schedule:
- Assumptions associated to the modelling, theory, SEM code
- Applications to ITER
- Reassessment of $L_{//}$
\[ \Delta V_{RF} = \int_{\text{field line}} E_{\parallel RF} \, dl \]

\[ \Phi_{RF} = V_{RF} / T_e \]

IR pictures of Tore Supra ICRH antenna

ICRH Antenna

Magnetic field line

RF sheaths

Flux tube

Sheath \( I(\Phi) \) characteristic
Flux tube sketch

The rectified potential $\phi(x,y,t)$ is computed self-consistently with currents according to this set of equations:

$$j_{\text{pol}} = \frac{L_{\text{para}} \rho_{ci}}{2\Omega_{ci}} \frac{\partial}{\partial t} \Delta \Phi$$

$$j_{\text{para}} = B_0$$

$\phi_{RF}/2$
Assumptions

- $\Phi(x,y,t)$ is computed in a 2D map perpendicular to $B_0$ and is normalized to $T_e$
- $\Phi$ is constant along the flux tube
- Density $n$ is constant all over the map
- Collisional, parallel and transverse RF displacement currents are integrated in the model
- Computation have been completed only with RF transverse currents (other components negligible)
- Electrostatic model

Rectified potential *without* transverse currents

\[
\Phi_0 = \Phi_{fl} + \ln\left(\cosh\left(\frac{\Phi_{RF}}{2}\right)\right)
\]

Current conservation

\[
\Delta I_{perp} = 1 - \exp(\Phi_0 - \Phi)
\]

Momentum equation applied to transverse current

\[
\left(\frac{1}{\Omega_{ci}^2} - \frac{1}{\Omega_{ci}^2}\right) \exp(\phi_0 - \phi) + 1 = \frac{L_{\parallel} \rho_{ci}}{2\Omega_{ci}^2} \frac{d}{dt} \nabla^2 \phi(x,y,t)
\]
Linear modelling in space (for SEM validation)

\[ \Phi(x) - L^2 \Delta \Phi(x) = \Phi_0(x) \]

Assumptions:

\[ \Delta I_{\text{perp}} < 1 \rightarrow \frac{L^2(\omega)\Phi_{RF}}{x_0^2} < 1 \]

\[ L^2(\omega) = \frac{L_{\text{para}} \rho_{ci}}{2} \frac{i \omega / \Omega_{ci}}{1 - \omega^2 / \Omega_{ci}^2} \]

\( \Delta I_{\text{perp}} \) Transverse current normalized to \( 2 j_{\text{sat}} \)

\( L(\omega) \) Characteristic length for RF currents and \( x_0 \) characteristic length for the potential structure

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![Graph showing linear and non-linear limits](image)

linear/non linear limit:

\[ \Phi_{RF} = \Phi \left(1 + \frac{x_0^2}{L^2}\right) \]
spatial resolution of linear modelling

Green's function

\[ \Phi(x) = \int \Phi_0(x') G(x-x')dx' \]

Solution:

\[ \Phi(x) = \Phi_0 \frac{x_0}{x_0^2 - L^2} x_0 e^{-|x|} \left( \begin{array}{cc} -|x| & -|x| \\ x_0 e^{x_0} & -Le^L \\ \end{array} \right) \]

- L is a complex number
- if ||L||>|x_0| then the structure of potential is broadened
- if ||L||<|x_0| the structure remains unchanged
- if Im(L)>Re(L) the potential structure oscillates along x
SEM (Sheath Effect Modelling) code

**Simulation steps**

- RF potential profile perpendicular to $B_0$
- Code SEM
- DC rectified potential profile $(x)$
- Time average
- RF rectified potential $(x,t)$

**Parameters**

- RF Potential evanescent
- Simulation in a poloidal-radial plane
- Map width 10 cm (100 cellules radiales, $\Delta x=1$ mm)
- time step $\Delta t=10^{-12}$ s
- Simulation time: 3 to 4h.

**Conditions de bord d’un Tokamak:**

- $T_e = T_i = 20$ eV
- $N_0 = 10^{18}$ m$^{-3}$
- $\Omega_{ci} = 32$ MHz (Deuterium plasma)
- $F = 53$ MHz
- $L_{para}^{eff} = [1 - 30]$ m
- $\Phi_{RF} = [10 - 150]$
2D fluid code: SEM

- Potential = Cste along magnetic field line
- Algorithm: finite differences, implicit in space and explicit in time

\[ A = \frac{L_i \rho_{ci}}{2\Omega_{ci}} \]

\[ \frac{A}{\Delta t} \left[ \frac{\Phi_{i+1,j+1} - 2\Phi_{i,j+1} + \Phi_{i-1,j+1}}{\Delta x} + \frac{\Phi_{i,j+1} - 2\Phi_{i,j} + \Phi_{i,j-1}}{\Delta y} \right] = \]

\[ 1 - \exp \left[ \Phi_j + \ln \left( \frac{\Phi_{E}}{\Phi_{i,j}} \right) \right] + \frac{A}{\Delta t} \left[ \frac{\Phi_{i+1,j+1} - 2\Phi_{i,j} + \Phi_{i-1,j}}{\Delta x} + \frac{\Phi_{i,j+1} - 2\Phi_{i,j} + \Phi_{i,j-1}}{\Delta y} \right] \]

Matrix penta-diagonal solve at each time step

- \( \Phi_{E} \)
- \( \Phi_{i,j} \)
- \( \Phi_{E+1} \)
- \( \Phi_{i+1,j} \)
- \( \Phi_{i,j+1} \)
- \( \Phi_{i-1,j} \)
- \( \Phi_{i,j-1} \)

\[ \begin{bmatrix}
- \Phi_{E} & -\Phi_{E} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} \\
\Phi_{E} & \Phi_{E} & -\Phi_{E} & \Phi_{E} & \Phi_{E} & \Phi_{E} & \Phi_{E} \\
-\Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & -\Phi_{E} & \Phi_{E} & \Phi_{E} \\
\Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & -\Phi_{E} & \Phi_{E} \\
-\Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & -\Phi_{E} \\
\Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} & \Phi_{E+1} \\
\end{bmatrix} \]

\[ = \int_{\Phi_{E}(i,j,t), \Phi_{i,j}} \]

\[ \begin{bmatrix}
\Phi_{E} \\
\Phi_{E} \\
-\Phi_{E} \\
\Phi_{E} \\
-\Phi_{E+1} \\
\Phi_{E+1} \\
\end{bmatrix} \]
Numerical scheme for explicit time computation

\[
\frac{1}{\Omega_{ci}^2 \Delta t^2} \left( \phi_{0}^{t+1} - 2\phi_{0}^{t} + \phi_{0}^{t-1} - \phi_{t+1} + 2\phi_{t} - \phi_{t-1} \right) \exp \left( \phi_{0}^{t} - \phi^{t} \right) - \\
\frac{1}{4\Omega_{ci}^2 \Delta t^2} \left( (\phi^{t+1})^2 - 2\phi^{t+1}\phi^{t-1} + (\phi^{t-1})^2 \right) + 2(\phi^{t+1} - \phi^{t-1})(\phi_{0}^{t+1} - \phi_{0}^{t-1}) + (\phi_{0}^{t+1} - \phi_{0}^{t-1})^2 - 1
\]

\[
\exp \left( \phi_{0}^{t} - \phi^{t} \right) + 1 = \frac{L_{\parallel} \rho_{ci}}{2 \Omega_{ci}} \frac{\nabla^2 \phi^{t+1} - \nabla^2 \phi^{t}}{\Delta t}
\]

Then $\Phi_{t+1}$ can be deduced from

\[
\frac{1}{\Omega_{ci}^2 \Delta t^2} \left( \phi_{0}^{t+1} - 2\phi_{0}^{t} + \phi_{0}^{t-1} + 2\phi_{t} - \phi_{t-1} \right) \exp \left( \phi_{0}^{t} - \phi^{t} \right) - \\
\frac{1}{4\Omega_{ci}^2 \Delta t^2} \left( (\phi^{t})^2 - 2\phi^{t}\phi^{t-1} + (\phi^{t-1})^2 \right) + 2(-\phi^{t-1})(\phi_{0}^{t+1} - \phi_{0}^{t-1}) + (\phi_{0}^{t+1} - \phi_{0}^{t-1})^2 - 1
\]

\[
\exp \left( \phi_{0}^{t} - \phi^{t} \right) + 1 + \frac{L_{\parallel} \rho_{ci}}{2 \Omega_{ci}} \frac{\nabla^2 \phi^{t}}{\Delta t} = \left[ \frac{L_{\parallel} \rho_{ci}}{2 \Omega_{ci}} \frac{\nabla^2}{\Delta t} + \frac{1}{4\Omega_{ci}^2 \Delta t^2} \left( 4 + 2(\phi_{0}^{t+1} - \phi_{0}^{t-1}) \right) \right] \phi^{t+1}
\]

Stable numerical scheme + results validated by 2D Particle in cell code
RF potential induced by the slow wave

\[ \Phi_0(x, t) = \Phi_0(t) e^{-\frac{x}{x_0}} \]

RF potential profiles perpendicular to B0

\( x_0 = \text{Skin depth} \rightarrow 5 \text{ mm} \)
The non linear broadening is smaller than the one expected by linear modelling.
Non linear L as a function of linear L and the RF potential

The combination of 80 simulations (8 values for $\phi_{RF}$ and 10 for L) gives a surface for $L_{nl}$ as a function of linear L and $\phi_{RF}$.

$L_{nl} = \frac{x_0}{\ln(\Phi(0)) - \ln(\Phi(x_0))}$

logarithmic slope

- $L_{nl}$ increases quasi-linearly with L except for high values of $\phi_{RF}$
- $L_{nl}$ exponentially decreases with $\phi_{RF}$

The increase of $\phi_{RF}$ tend to make $L_{nl}$ decrease down to a limit value of 2 cm for these parameters.

--> How justify this value?

--> it means: high RF potentials = low broadening
Projection on the \((L_{nl}, L)\) plane

Each iso-curve represents a value of \(\phi_{RF}\)

\[
L_0 = x_0 + \sqrt{\frac{L_{\text{para}} \rho_{ci}}{2}}
\]

\(L_{nl}\) evaluation

\[
L_{nl} \sim x_0 + L
\]

\(L_{nl} < L_0 \text{ for } \Phi_{RF} < 50\)

\(L_0\) is a good approximation for a upper value of the structure broadening.
Application to ITER
4 scenarios have been simulated with 3 phasing for the 4 straps of the ITER antenna: $00\pi\pi$, $0\pi0\pi$, $0\pi\pi0$

The tabular below summerizes simulation parameters: density, temperature, magnetic connexion length in the SOL for each scenario made by Alberto Loarte (Turin).

<table>
<thead>
<tr>
<th></th>
<th>Sc2 short</th>
<th>Sc2 long</th>
<th>Sc4 short</th>
<th>Sc4 long</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$ (T)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>tilt angle (°)</td>
<td>15</td>
<td>15</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$f$ (MHz)</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>$l_x$ (m)</td>
<td>0,1</td>
<td>0,1</td>
<td>0,1</td>
<td>0,1</td>
</tr>
<tr>
<td>$l_y$ (m)</td>
<td>3,5</td>
<td>3,5</td>
<td>3,5</td>
<td>3,5</td>
</tr>
<tr>
<td>$n_0$ (m$^{-3}$)</td>
<td>$10^8$</td>
<td>$10^8$</td>
<td>$10^8$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>$L_{para}$ (m)</td>
<td>2,8 &amp; 20</td>
<td>2,8 &amp; 20</td>
<td>2,8 &amp; 35</td>
<td>2,8 &amp; 35</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$T_i$ (eV)</td>
<td>20</td>
<td>20</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>$dt$ (s)</td>
<td>$10^{12}$</td>
<td>$10^{12}$</td>
<td>$10^{12}$</td>
<td>$10^{12}$</td>
</tr>
</tbody>
</table>

RF field maps have been computed with TOPICA (D Milanesio) coupled with FELICE for the absorption of the slow wave. Electric field are calculated for a 1V feeder and are normalized to 20 MW of coupled power.
Density profiles for each scenario
RF potential map in the radial-poloidal plane

\[ V_{RF} = \int_{L_{sol}} E_{\text{para}} \, dl \]
DC rectified potential map after SEM code run ($L_{\text{para}}=2.8\text{m}$)
DC map Potential profiles: average over $y$

Profil RF

$\Phi_{\max} < \Phi_{RF}/2$

Profil DC $L_{\text{para}} = 2.8 \text{m}$

Profil RF $L_{\text{para}} = 20 \text{ & } 35 \text{ m}$
For long SOL scenarios the skin depth is shorter than L and then the profile is broadened by transverses RF currents. On the contrary for short SOL the effect of transverses currents is not visible ($x_0=L$).
Utility to take rectified potential in front of the antenna

- $V_{DC}$ amplitude $\Leftrightarrow$ magnitude of $v_{\text{ExB}}$
- $V_{DC}$ decay length $\Leftrightarrow$ cells extention
- $\Rightarrow$ Local density $\uparrow$ @ mouth

Then energy flux deposition can be computed $\Rightarrow Q = e n(V_{DC}) C_s V_{DC}$

Conclusion

Double probe model applied to a flux tube exchanging RF transverse currents

--> linear modelling for solving DC rectified potential structures

--> potential amplitude condition [Faudot2006] :
\[ \Phi(x) < \frac{\Phi_{RF}(x)}{2} \]

--> broadening condition (Green function) : - if \( ||L|| < x_0 \) --> \( x_0 \)

- if \( ||L|| > x_0 \) --> \( ||L|| \)

with

\[ L(\omega) = \sqrt{\frac{L_{\text{eff}}^{\rho_{ci}}}{2} \frac{i \omega / \Omega_{ci}}{1 - \omega^2 / \Omega_{ci}^2}} \]

--> non linear modelling

--> SEM code

--> Validation of the amplitude for DC potentials

--> Rectification criterion: linear/non linear

--> Validation of the broadening with the parameter \( L \), \( L_0 \) is a upper value for the non linear broadening :
\[ L_0 = x_0 + \sqrt{\frac{L_{\text{eff}}^{\rho_{ci}}}{2}} \]

--> broadening criterion: linear/non linear

--> non linear effect make increase the DC amplitude and decrease the broadening

--> Evaluation of an effective connexion length for transverse RF currents with experimental probe measurements :

\[ L_{\text{eff}}^{\rho_{ci}} < 2m \quad \Rightarrow \quad \text{Needs 3D} \]
2-D map of $V_{\text{float}}$ versus $(\delta r, Z_{Q5})$ connected side for $P_{Q5} = 1.5 \text{ MW}$. Dashed lines: side limiter radial position, and antenna box vertical extension. Arrows: sketch of $v_{E \times B}$ induced by $V_{\text{float}}$


reciprocating Langmuir probes very useful tool
Evaluation of the effective $L_{\text{para}}$ from probe measurements

Upper value for $L_{\text{para}}$

$$L_{\text{perp}} = x_0 + \sqrt{\frac{L_{\text{para}}^\text{up} \rho_{ei}}{2}}$$

Mean value for $L_{\text{para}}$

$$L_{\text{perp}} = x_0 + \sqrt{\frac{L_{\text{para}}^\text{mean} \rho_{ei}}{2 \left( 1 - \omega^2 / \Omega_{ci}^2 \right)}}$$

With
- $B_0 = 3 \text{ T}$
- $x_0 = 5 \text{ mm}$
- $T_e = T_i = 20 \text{ eV}$
- $f = 53 \text{ MHz}$
- connection length = 7 m

<table>
<thead>
<tr>
<th>$L_{\text{perp}}$</th>
<th>$L_{\text{para}}^\text{mean}$</th>
<th>$L_{\text{para}}^\text{up}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td>13 cm</td>
<td>46 cm</td>
</tr>
<tr>
<td>1.5 cm</td>
<td>55 cm</td>
<td>1.85 m</td>
</tr>
</tbody>
</table>

1 cm $< L_{\perp} < 1.5$ cm

Shot 41869

Probe connected to antenna (J. Gunn)
Computation of the RF potential map

\[ V_{RF} = \int_{L_{sol}} E_{para} \, dl \]

Sample of a 2D DC rectified potential map in front of an ICRF antenna
1D RF rectified potential

\[ L^2(\omega) = 5 \times 10^{-4} \text{ m}^2 \]

Code SEM

RF rectified potential profile \( \Phi(x,t) \)

 transient time during which the AC amplitude decreases

\[ L^2(\omega) = 5 \times 10^{-3} \text{ m}^2 \]