Codes in reflectometry
Numerical Schemes and limitations

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The understanding of the turbulent transport is key point for the energy production through the fusion plasma

Transport mechanisms

- Neoclassical (collisions)
- Anomalous => **Turbulence** (δn_e>> δB, δT_e…)

Transport coefficients :
turbulent >> Neoclassical

Diagnostics are needed to access to the turbulence parameters

Too hot to be probed by material tools (except in the edge until the last closed flux surface LCFS)

Only electromagnetic waves can be used

Electromagnetic (EM) wave for probing plasma ?

Quasi optic approximation :

Interferometry
Polarimetry
Contrast Phase imaging
Thompson scattering
Spectroscopy

Microwaves are more appropriated to diagnose turbulence
$\lambda \sim \lambda_f$ (Bragg scattering)

Reflectometry is a versatile diagnostic which is able to provide turbulence parameters:

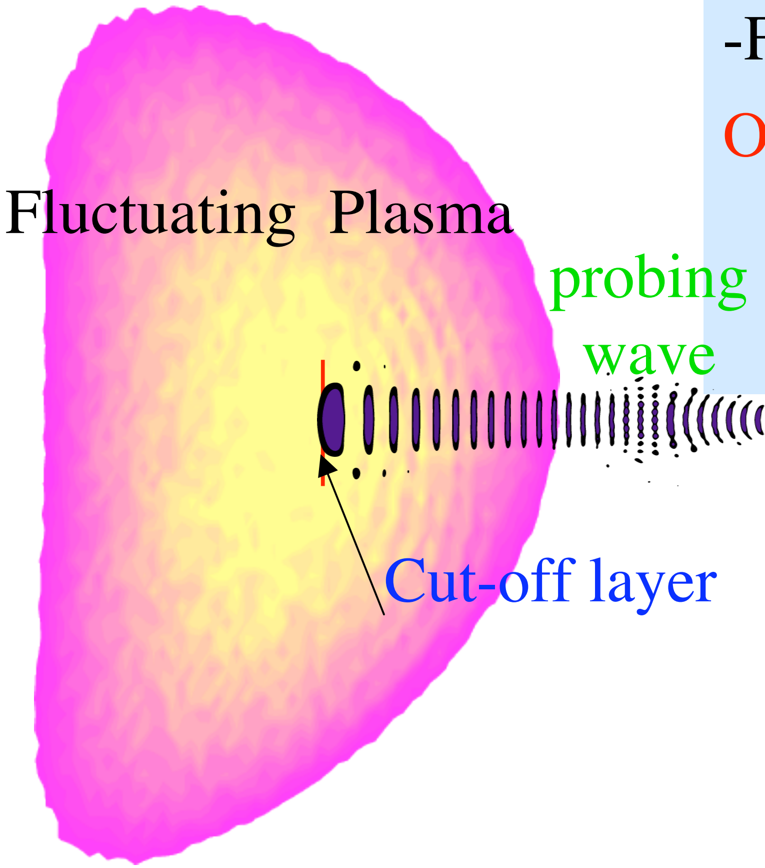
dn_e(r) *absolute density fluctuation profile,*
$S(k_r)$& $S(k_{pol})$ *radial & poloidal wavenumber spectra,*
$S(w_f)$ *frequency spectrum,* l_c *correlation length,*
V_{pol} *turbulence velocity…*

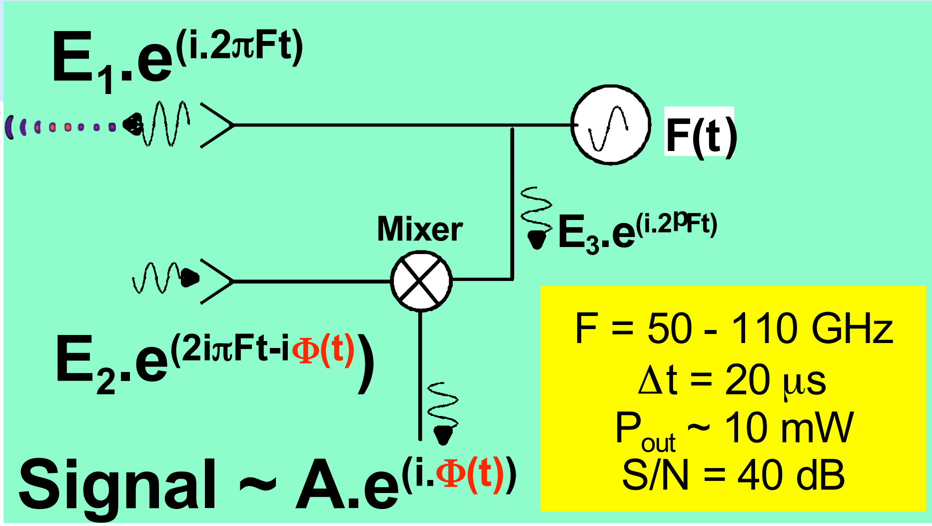
Principle of reflectometry (1)

Diagnostic for density profile

-FMCW Reflectometers mode X or O: $\nu(t) = \alpha\, t + \omega_o$,

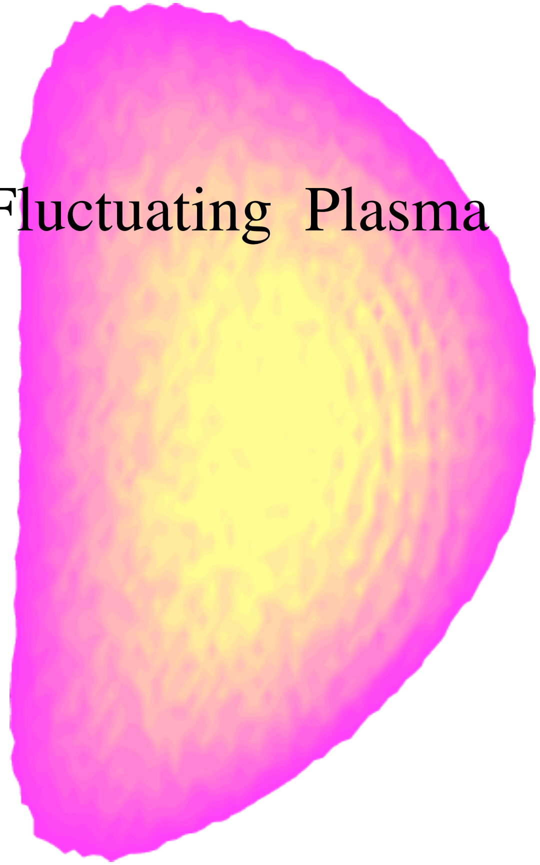
$\delta\varphi(\nu) \rightarrow n_e(r)$,





F. Clairet et al Rev Sci I. (2003) 74, 1481, *F. Clairet et al* PPCF **46**, 1567 (2004).

Principle of reflectometry (1)



Bottollier-Curtet algorithm

or Abel inversion

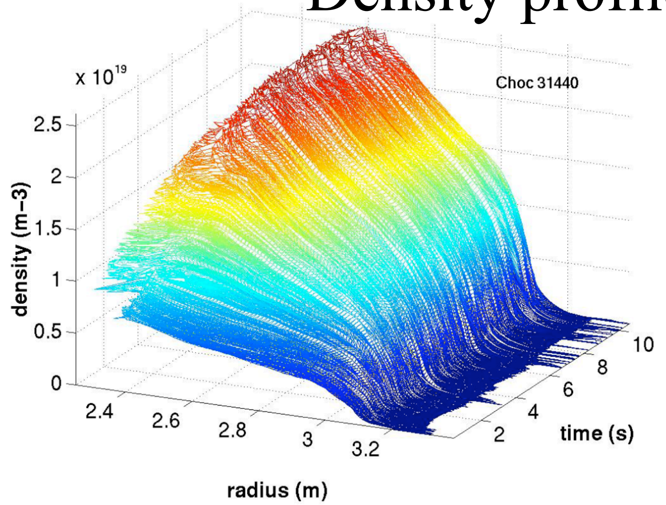
Diagnostic for density profile

-FMCW Reflectometers mode X or O: $\nu(t) = \alpha\, t + \omega_o$,

$\delta\varphi(\nu) \to n_e(r)$,

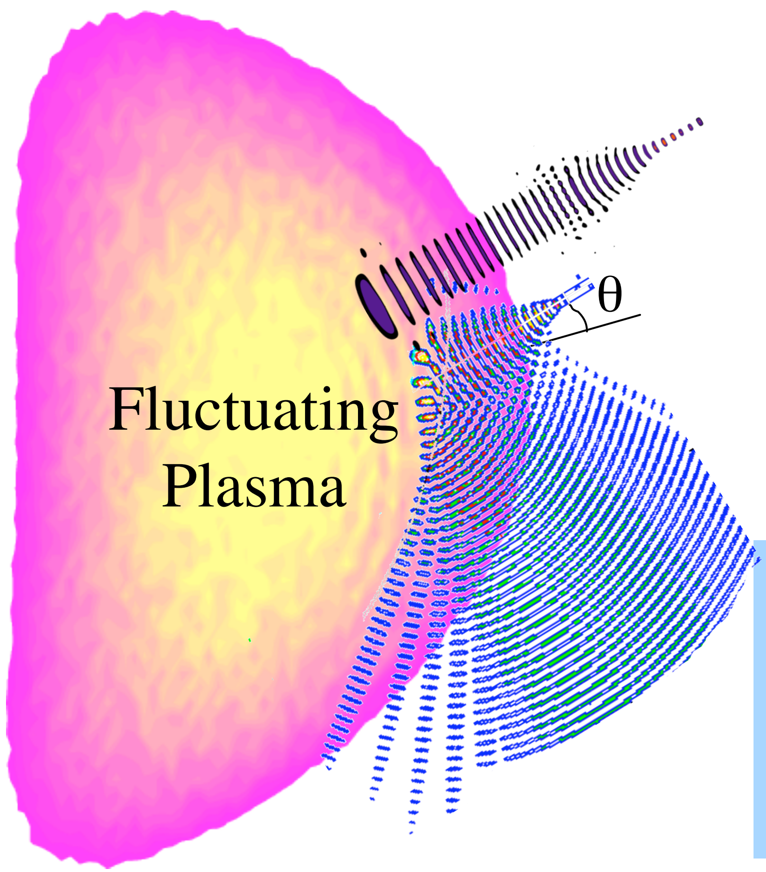
$\delta n_e(r)$, $S(k_r)$ ....

Density profiles



F. Clairet et al Rev Sci I. (2003) 74, 1481, *F. Clairet et al* PPCF **46**, 1567 (2004).

Principle of reflectometry (2)



Diagnostics of the turbulence

-Fluctuation - Reflectometer O-X-mode

(110-155 GHz, fast hopping system)

$\omega = \omega_o$, $\delta\varphi(t)$ -> $\delta n_e(t)$ -> $S(\omega_f)$

sampling rate 1MHz

M. Schubert *Names 2007& EPS 2008*

-Doppler Reflectometer X or O-mode

(50-75, 75-110 GHz, fast hopping system)

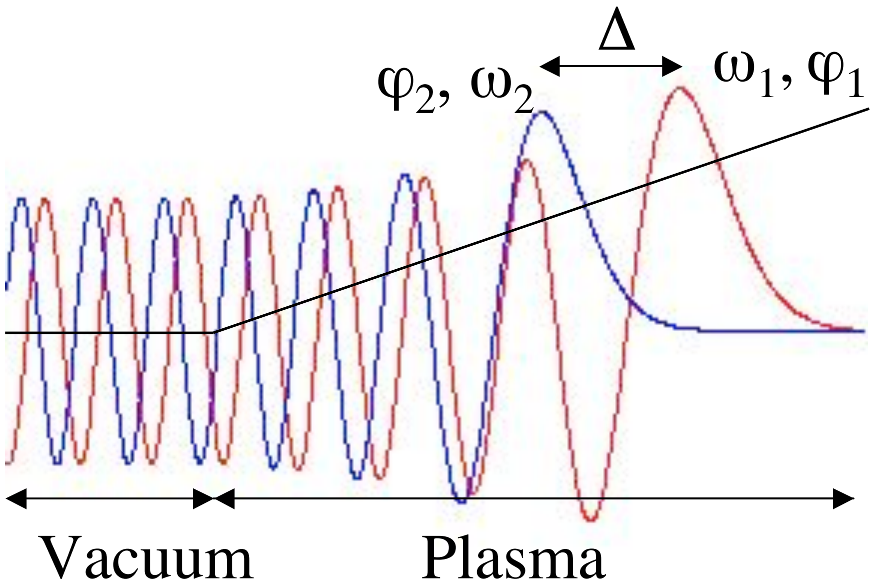
$\omega = \omega_o$, $\delta A_s(t)$ -> V_{pol} , $S(k_{pol})$

R. Sabot *et al*, Int. Journal of Infrared and Millimeter Waves (2004) **25** 229-246.

P. Hennequin *et al*, Rev. Sci. I. (2004) **75** 3881. F. da Silva *et al* Nuclear Fusion **46**, S816 (2006)..

Principle of reflectometry (3)

Radial or poloidal Correlation Reflectometers

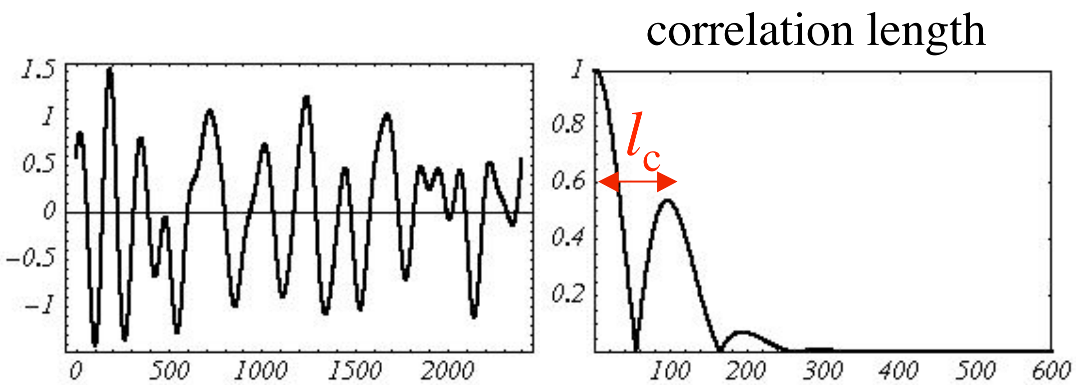


Correlation length measurement

Intercorrelation length

l_{cphase} high

$l_{csignal}$ signal fct of amplitude $\langle E_1 e^{i\varphi 1} E_2 e^{i\varphi 2} \rangle$



2 regimes for small k_f values

linear => l_c α log(Δ)

NL => $l_{cNL} < l_c$ linear

Gusakov et al PPCF (2004) 46, 1393 *Leclert et al PPCF (2006) 48, 1389* *Gusakov et al EPS (2009)*

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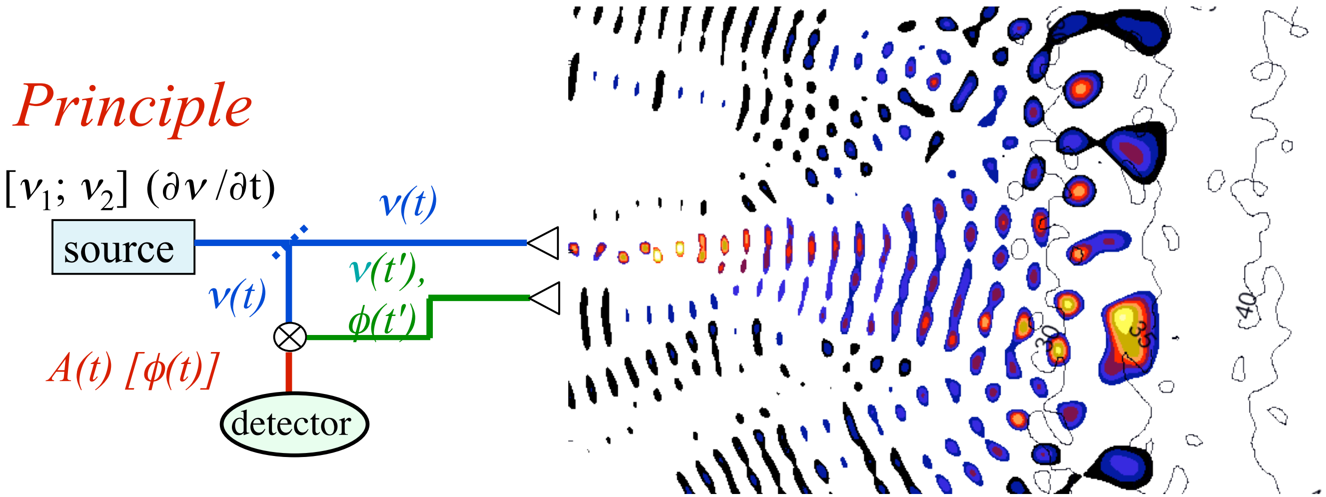
Effects of density fluctuations on the reflectometer signal

IJL

➢ *Principle*



Density fluctuations induce

Density profile

$$\varphi(\tau) = \varphi_{WKB}(\tau) + \delta\varphi(\tau)$$

$$A(\tau) = \langle A(\tau) \rangle + \delta A(\tau)$$

Fixed frequency –> $\delta\varphi(t)$ => $S(\omega_f)$

Frequency sweep –> $\delta\varphi(\nu)$ => $S(k_f)$

Heuraux *et al.*, Rev. Sci. I (2003) **74**, 1501, L. Vermare *et al,* Plasma Phys Cont Fusion **47**, 1895 (2005)

T. Gerbaud et al **77,** 10E928 (2006).

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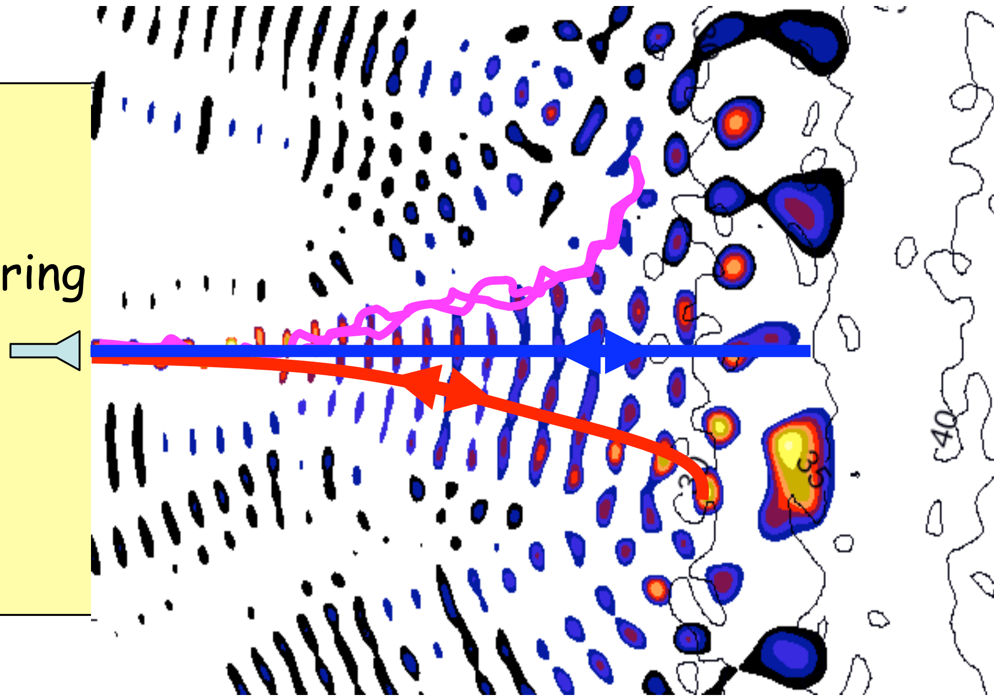
Effects of density fluctuations on the reflectometer signal

In magneta

Forward scattering

Beam widening

Doppler shift V



In blue

Reflection at cut-off layer

$N_{O,X}(x_c) = 0$

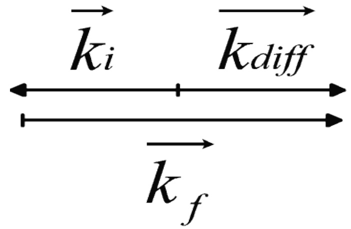
Amplitude modulation

Destructive interference

Doppler shift n(t)

In red

Bragg backscattering => $k_f = 2\,k_{loc}(x)$
phase fluctuations



Localized process in $n_e(r)$

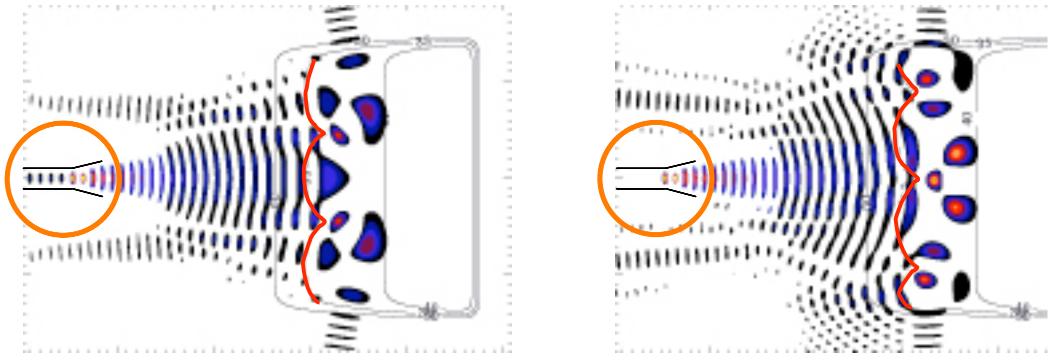
F. Da Silva et al Nuclear Fusion **46**, S816 (2006);
F. Da Silva et al & A. Popov et al IRW8 reflectometry meeting 2-4 May 2007

Effects of density fluctuations: destructive interference

electric field for ≠ positions of the islands, fixed ω

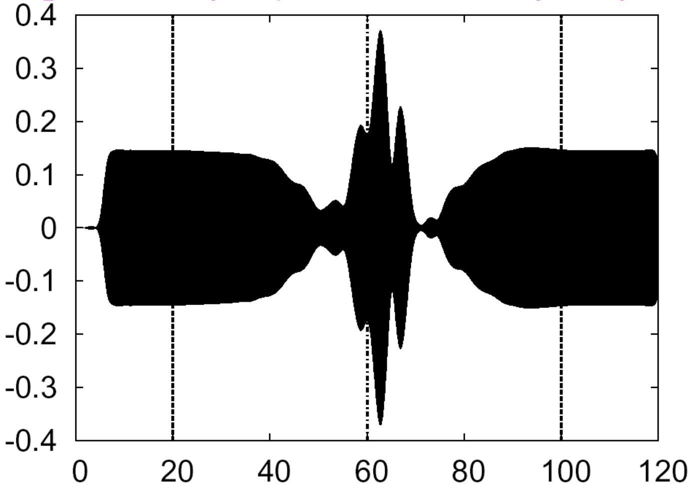
O Mode
Frequency sweep
25-40 GHz

Island length $10\lambda_o$
width $4\lambda_o$

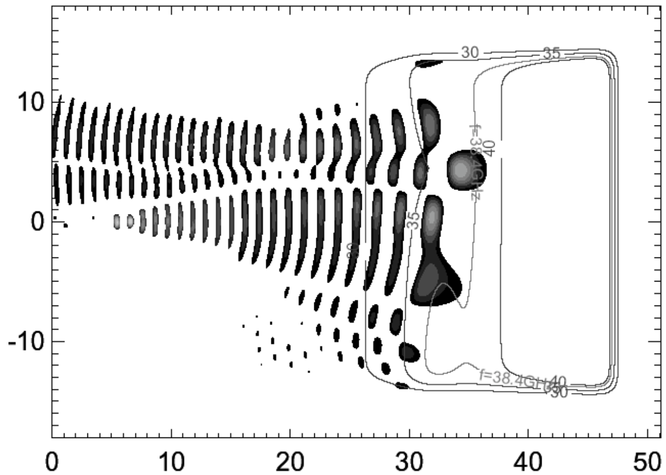


O point on the waveguide axis X point on the waveguide axis

Amplitude of reflected wave fct of ω



E_z island length 40 λ_o



Shifted Position / pt X

Same behaviour for single structure

island + fluctuation −> < amplitude > ↘

F. Da Silva *et al* Rev. Sci. Instrum. **74**, 1497-1501 (2003)

Why Reflectometry Simulations ?

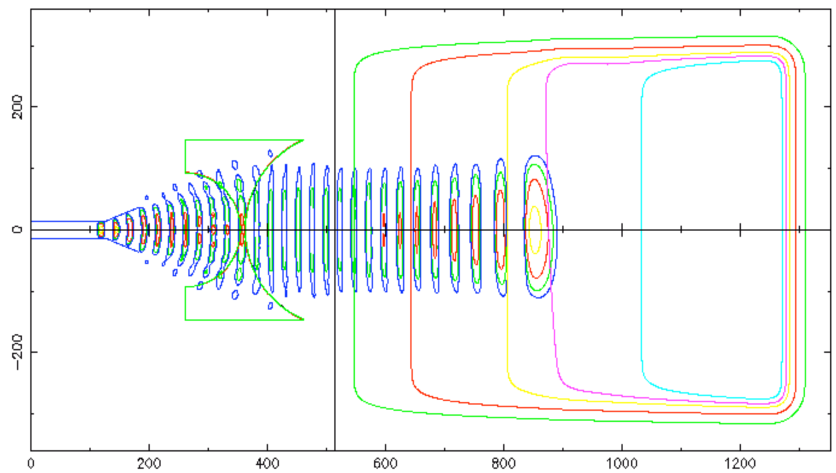
Reflectometry Diagnostics in Tokamaks

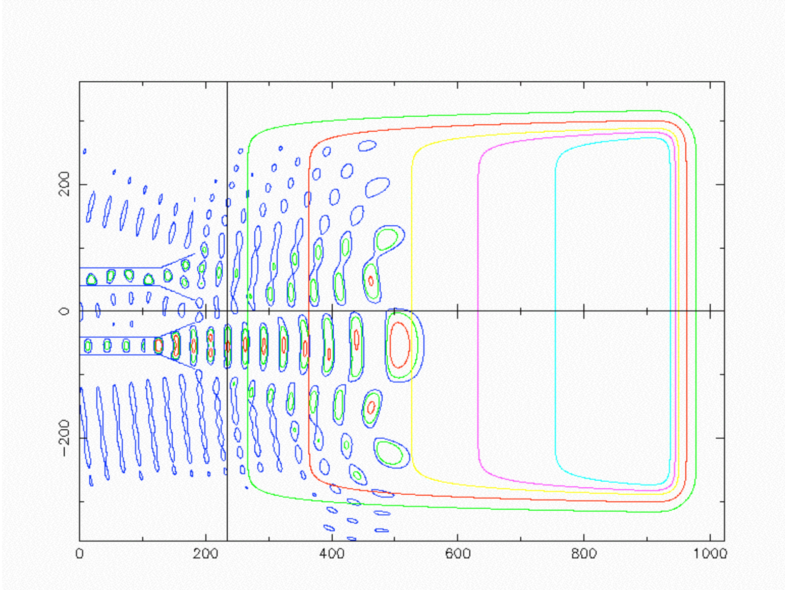
ng

- -Fast sweep frequency reflectometers O or X-mode
- -Plasma Position Reflectoemter
- -Fluctuation reflectometer
- -Doppler Reflectometer
- -Correlation Reflectometer

Solutions to some simulation problem

Monomode Wave Injection in oversized wave guide
Realistic description of EM probing beam
Unidirectional Transparent Source (UTS) for frequency sweep





UTS needed

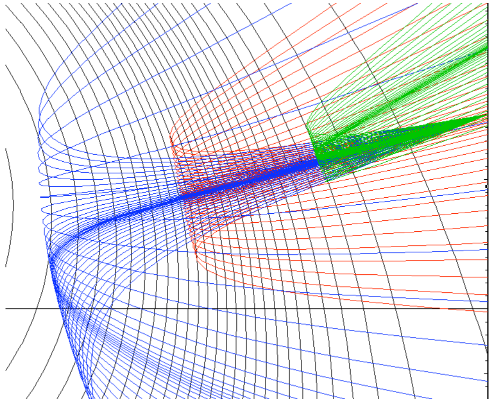
J. of Computational Physics **174**, 1 (2001), J. of Computational Physics **203**, 467 (2005),
J. Plasma Physics **72**, 1205 (2006), RSI 79, 10F104 (2008)

From ray tracing to wave equation (1)

Quasi-optic description without scattering

Ray tracing

Hyp. WKB : $\left|\frac{dk}{dx}\right| \ll k^2, \quad \left|\frac{d^2k}{dx^2}\right| \ll \left|\frac{dk}{dx}k\right|$



Single mode description $D(\omega,\mathbf{k},\mathbf{r},t)=0$

Set of coupled Odes to solve   RK45

$$\begin{cases} \dfrac{\partial \vec{r}}{\partial \tau} = -\dfrac{\partial D(\omega,\vec{k},\vec{r},t)}{\partial \vec{k}} \\ \dfrac{\partial \vec{k}}{\partial \tau} = \dfrac{\partial D(\omega,\vec{k},\vec{r},t)}{\partial \vec{r}} \end{cases} \qquad \begin{cases} \dfrac{\partial t}{\partial \tau} = \dfrac{\partial D(\omega,\vec{k},\vec{r},t)}{\partial \omega} \\ \dfrac{\partial \omega}{\partial \tau} = -\dfrac{\partial D(\omega,\vec{k},\vec{r},t)}{\partial t} \end{cases}$$

Can be extended to Gaussian beam propagation by one ODE associated to amplitude

From ray tracing to wave equation (2)

Monochromatic and single polarisation probing system

Helmholtz's equation (full-wave)

Hyp: monochromatic wave, steady state plasma (Δt or $\tau_{corr} \gg 4r_c/c$)

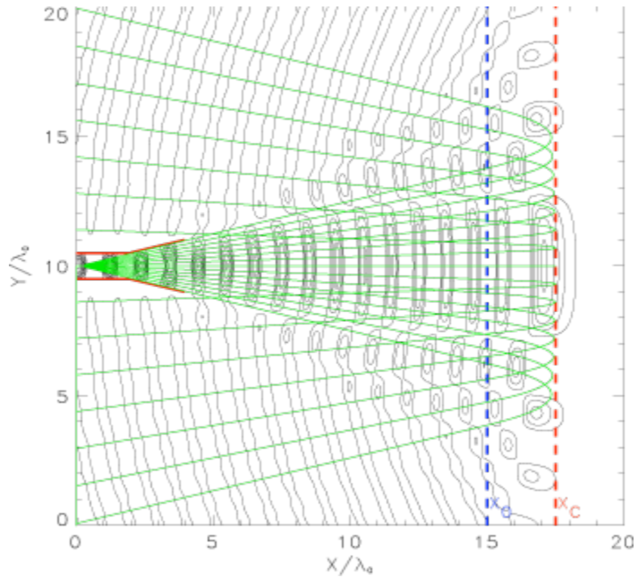
Single mode description: Computation of the index *N(r)*

$$\Delta\vec{E} + N^2(\vec{r})\vec{E} = 0$$

Finite Difference
4th order (Numerov)

Be careful in multi dimensional case, possible cross derivatives more complicated to solve

No Doppler



Finite Element Method

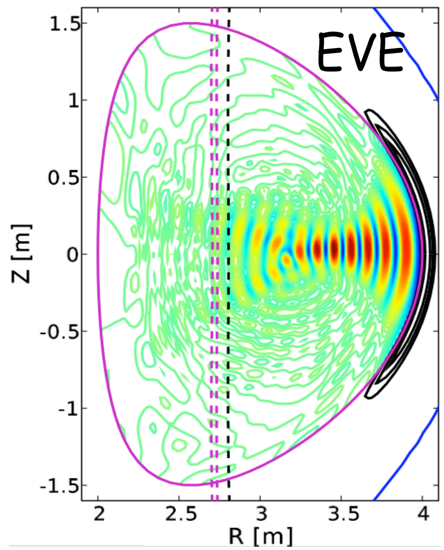
Monochromatic multi-polarisation probing system

Actually only few developments on FEM with dispersive media

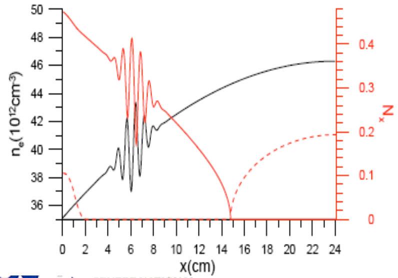
In plasma only using equivalent dielectric (Ph Lamalle for ICRH or F. Braun & L. Colas) for ICRH

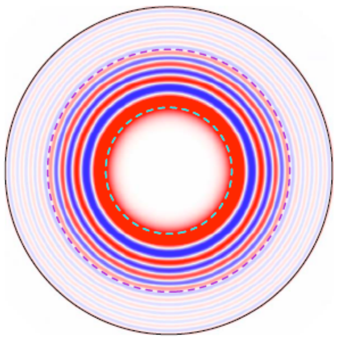
Accurate method in vacuum and in complex geometry (commercial software)

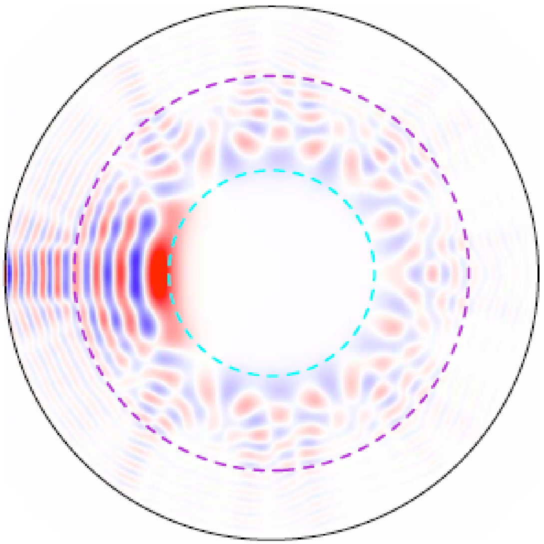
ALCYON was ICRH code based on functionals, if will be replaced by EVE code developed by R. Dumont (CEA_cadarache) and needs a lot of memory (~10-20 Gbytes)



In the case of reflectometry possible ? Yes







From ray tracing to wave equation (3)

Quasi steady state plasma

Shrödinger like's equation (full-wave)

Hyp: quasi-monochromatic wave $\omega >> \partial_t$

quasi steady state plasma (Δt or τ_{corr} >> $4r_c/c$)

Single mode description: Computation of the index *N(r)*

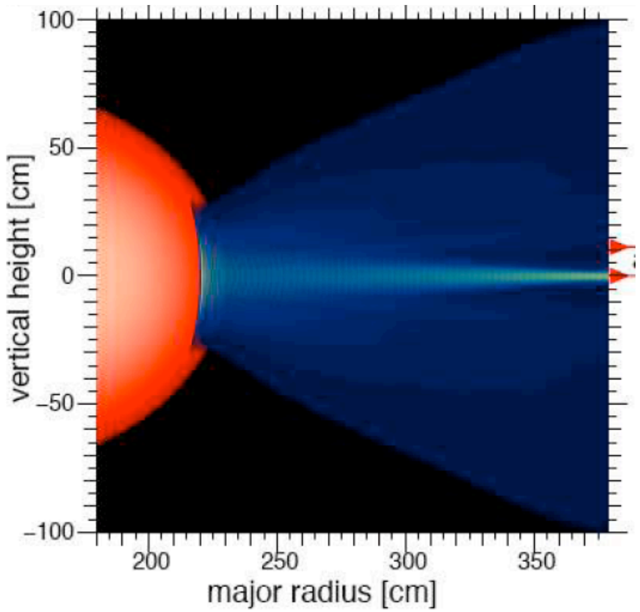
$$i\partial_t \vec{E} + \Delta \vec{E} + N^2(\vec{r})\vec{E} = 0$$

Parabolic

Restriction on dispersion effects,

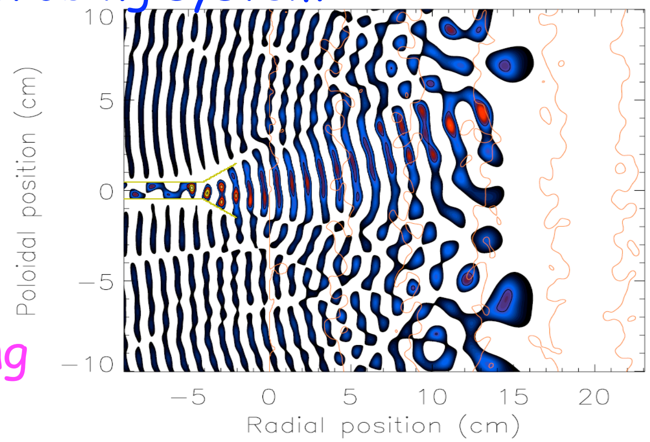
Quasi-paraxial approximation

Lin et al, Plasma Phys. Cont. Fusion 40 L1 (2001)



From ray tracing to wave equation (4)

Time dependent physical processes or probing system



O-mode or isotrpic plasma

wave equation (quasi-steady state medium)

Hyp: $(t_f, \Delta t \text{ or } \tau_{corr} \gg 4r_c/c)$

$$\partial_t^2 \vec{E} - c^2 \Delta \vec{E} + \omega_{pe}^2(\vec{r})\vec{E} = 0$$

Finite Difference + $\omega_p E$ rewritting

Hacquin et al, J. of Computational Physics **174**, 1 (2001),

$$\begin{cases} \frac{\partial^2 E_x}{\partial t^2} + c^2 \frac{\partial^2 E_x}{\partial x \partial y} - c^2 \frac{\partial^2 E_x}{\partial y^2} + \omega_p^2 E_x = \omega_p^2 v_y \\ \frac{\partial^2 E_y}{\partial t^2} + c^2 \frac{\partial^2 E_y}{\partial x \partial y} - c^2 \frac{\partial^2 E_y}{\partial x^2} + \omega_p^2 E_y = \omega_p^2 v_x \\ \frac{\partial}{\partial t} v_x = -\omega_c v_y - \omega_c E_x \\ \frac{\partial}{\partial t} v_y = \omega_c v_x - \omega_c E_y \end{cases}$$

Set of coupled partial differential equations associated to X-mode

$V = V/V_D$ where $V_D = E_o/B_o$

and $E = E/E_o$

Cohen et al, Plas. Phys. Cont Fusion **40**, 75 (1998),

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From ray tracing to wave equation (5)

Turbulence dynamics, fast events

wave equation (time-depend medium)

Hyp: single mode polarisation

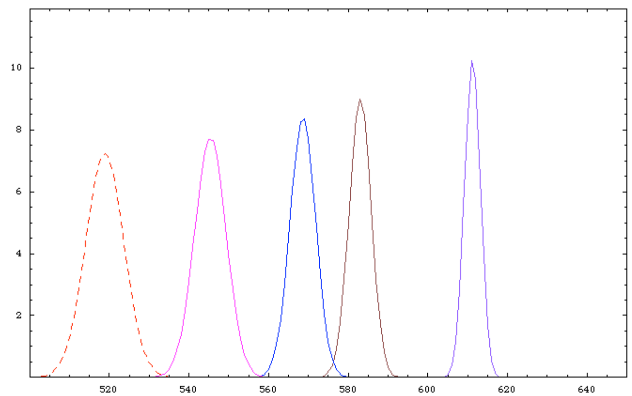
Fast gradient motion,
up or down frequency shift
amplitude variation

$$\begin{cases} \partial_t^2 \vec{E} - c^2 \Delta \vec{E} + \omega_{pe}^2(\vec{r},t)\vec{E} = \dfrac{e}{\varepsilon_o}\vec{v}\,\partial_t n \\ \partial_t \vec{v} = -\dfrac{e}{m_e}\vec{E} \end{cases}$$

O-mode or isotrpic plasma

Finite
Difference +
ω_pE rewritting
+ RK45

Just to add ∂_tn in the
Set of coupled partial
differential equations
associated to X-mode



Frequency upshift with ∂_tn ↗

Cross polarisation simulations

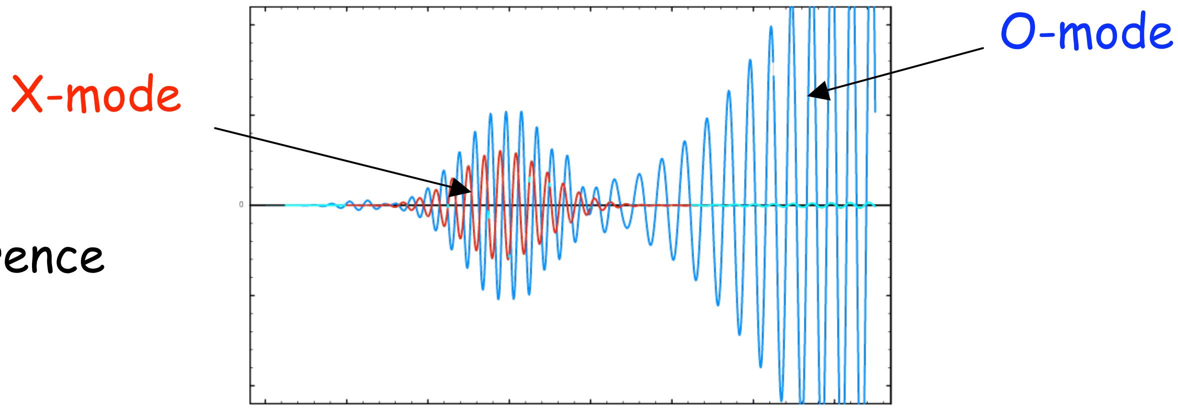
δB measurements

1D Case: O-mode and X-mode

Hojo et al, J. of Phys. Soc Jpn. **67**, 2574 (1998),

$$\begin{cases} \partial_t^2 E_z - c^2 \partial_x^2 E_z + \omega_{pe}^2(x,t) E_z = C_{OX}\left(E_x, E_y\right) \\ \partial_t^2 E_x + \omega_{pe}^2(x,t) E_x = -\omega_{pe}^2(x,t) v_y + C_{XOx}\left(E_z\right) \\ \partial_t^2 E_y - c^2 \partial_x^2 E_y + \omega_{pe}^2(x,t) Ey = \omega_{pe}^2(x,t) v_x + C_{XOy}\left(E_z\right) \\ \partial_t \vec{v} = -\dfrac{e}{m_e}\vec{E} - \dfrac{e}{m_e}\vec{v} \times \vec{B} \end{cases}$$

Finite difference



Full description: Maxwell's equations

Velocity field mapping, Shear layer detection

Hyp: linear response of the plasma

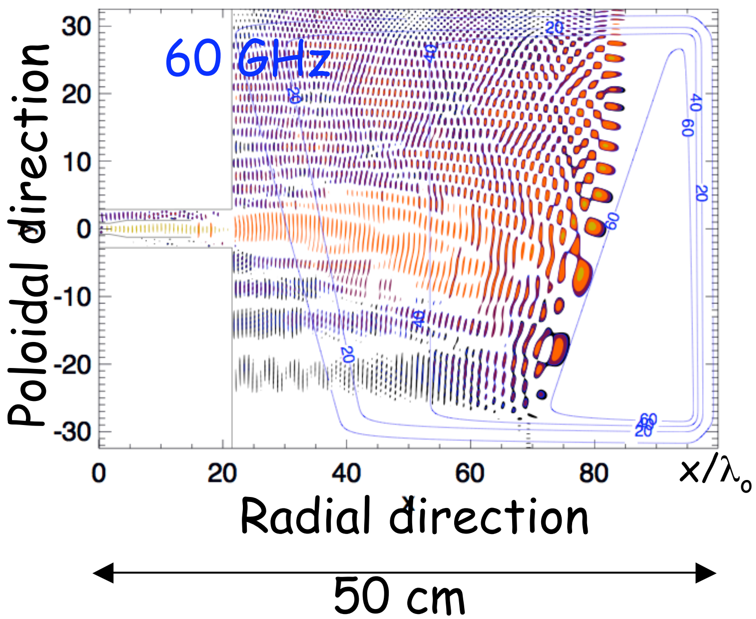
ρ total density of charges

j current density

Associated model fluid or kinetic

$$\begin{cases} \nabla.\vec{B} = 0 \\ \nabla.\vec{E} = \dfrac{\rho}{\varepsilon_o} \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_o \vec{j} + \dfrac{1}{c^2}\dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

TE and TM are usually treated separately



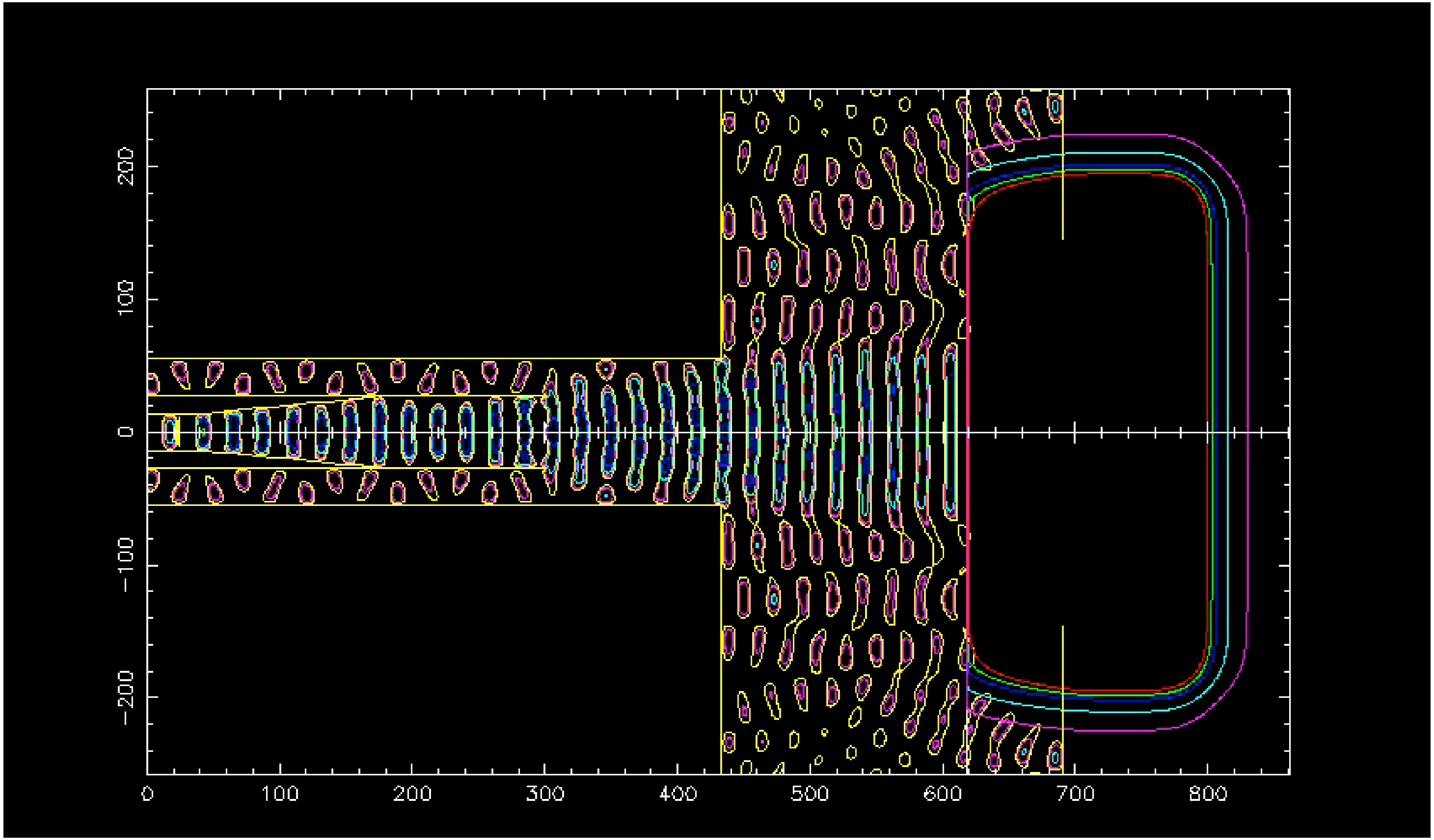
Yee's algorithm

+

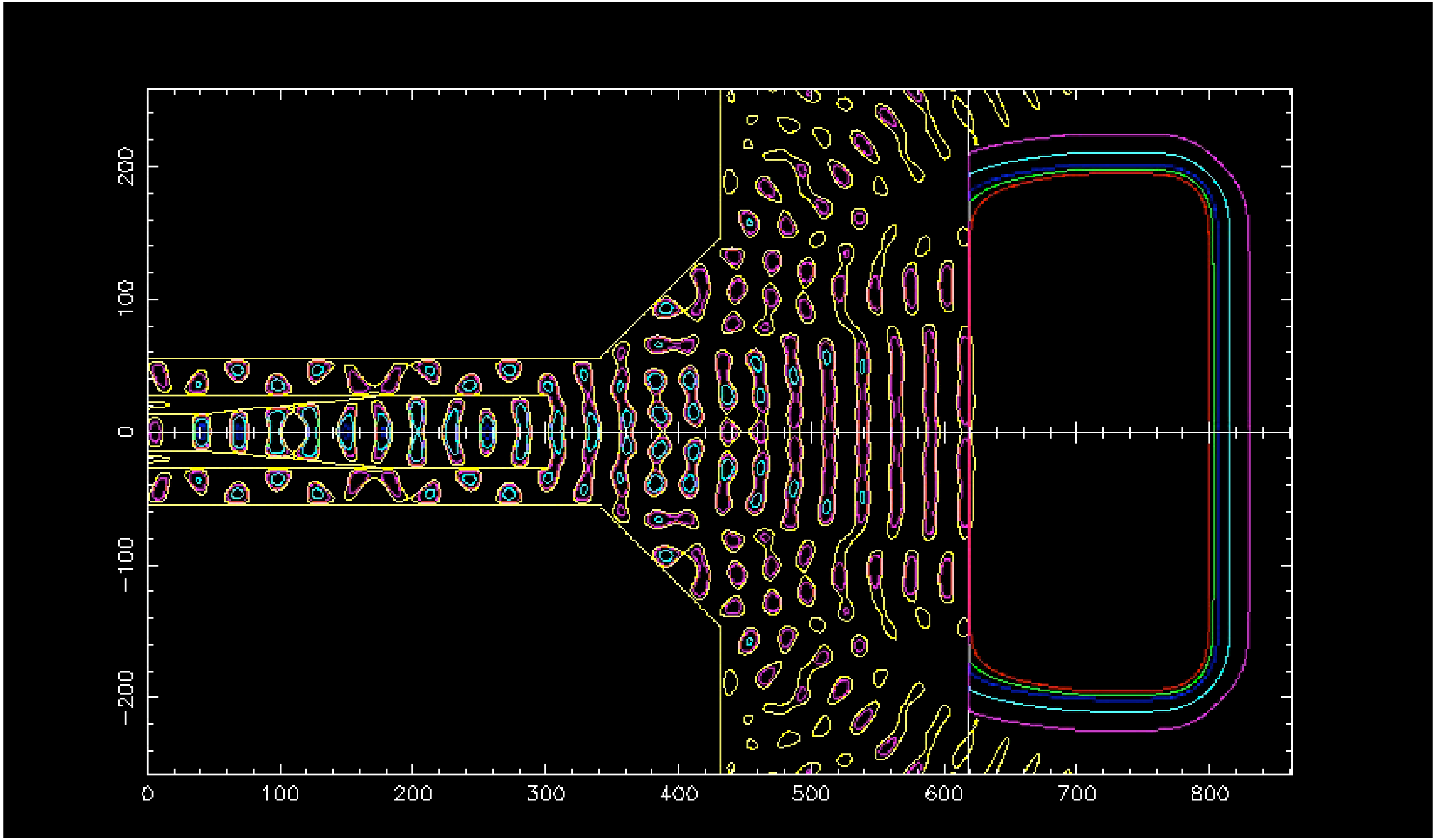
J solver

F. da Silva et al , J Plasma Phys. **72** 1205 (2006) and Rev. Sci Instr. 79, 10F104 (2008)

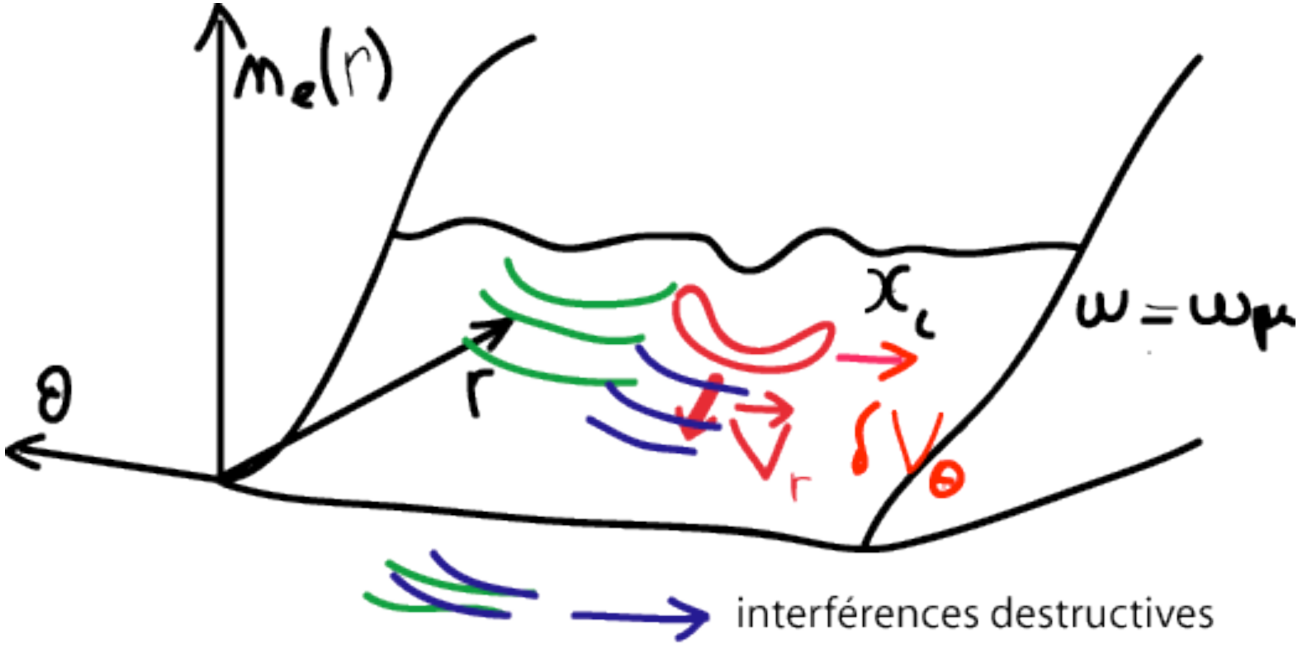
One example: ITER Plasma Position Reflectometer



One example: ITER Plasma Position Reflectometer



Long Terms Projects

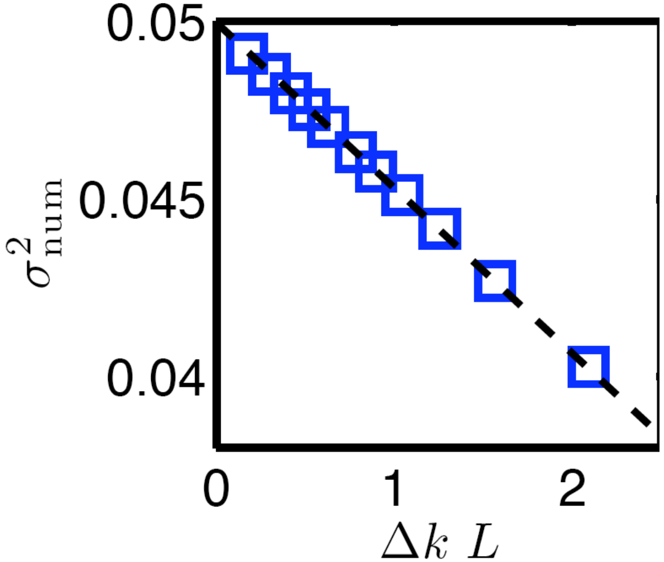


Blob signature, single event detection (condition requirements)

Role of the velocity shear layers (spectrum wings ?)

Reflectometry Computation Requirements (1)

To describe the forward scattering effects (long wavelength contribution)



To recover the theoretical results of the forward scattered power much larger mesh size is required

Usefulness of the testing of the code by using analytical results

Be careful on the choice of the turbulence generator: modes summation, burst superposition, …. or coming from *turbulence code* BUT has intrinsic limitations

Reflectometry Computation Requirements (2)

To describe ITER case full size:

1000 vacuum wavelengths -> Helmholtz code (4th order)-> 14 pts/wavelength
-> FDTD code -> 20pts/wavelengh and 40 pts/period

Helmholtz scheme characteristics : with UMF pack library

Memory α N^2
computation time α N^3

Time series long enough to have a good statistics results for the forward scattered power better to use Yee's algorithm

Absorbing boundary conditions to avoid can satisfy to resonant conditions,
Needs of real transparent boundary conditions

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European Reflectometry Computation consorsium

Full-wave european codes in Reflectometry :

Helmholtz's code (1D&2D, O and X) CEA, IJL, LPTP

Wave equation code (1D &/or 2D, O &/or X) TEXTOR, IJL

Maxwell's equations code (2D, O or X) IST, IJL, CIEMAT, ASDEX, Stuttgart

Project: 3D code Maxwell's equation O and X-mode (ITM group)

To do What ?

fundamental studies (forward scattering effects,….)
new diagnostic development (S(kr) fast sweep and radial correlation, …)
interpretation of experiments (Doppler, correlation fluctuation,…)
ITER design (plasma position reflectometer,…)

Pb turbulence modelling which one, mode superposition, burst emission,….

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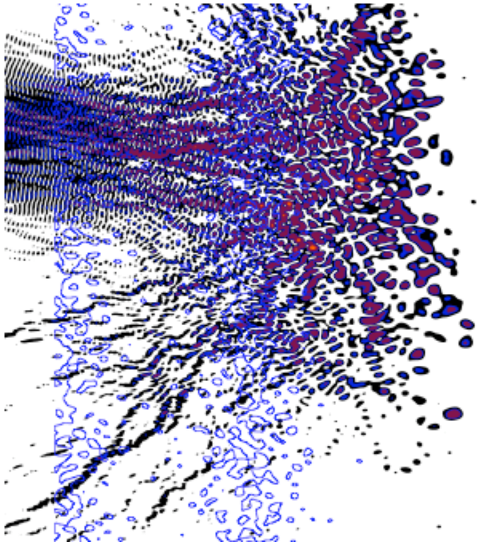
Conclusions and proposals of further studies (1/2)

-2D full-wave simulations seem to show that it is possible to determine the position of the LCFS with ITER spatial resolution specification when the probing beam corresponds to the perpendicular of the LCFS.

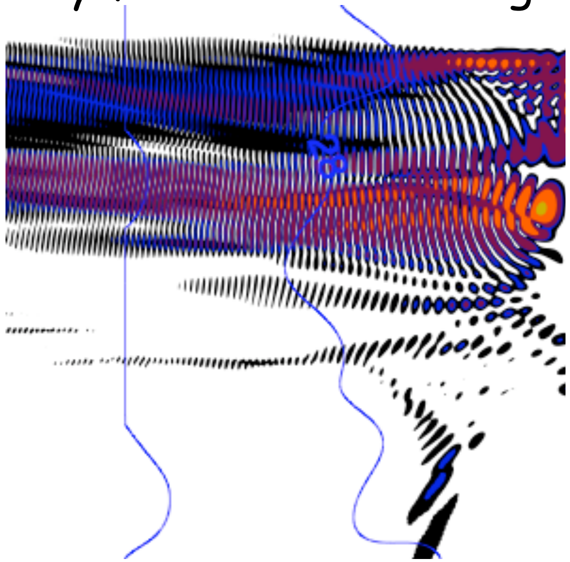
-Reconstruction density profile should be improved to treat the parasitic resonances.

-Full-wave simulations including high amplitude of edge density fluctuations has to be done according to the electric field structure see below (role of the k - spectrum and of the peeling modes)

dominated by Bragg backscattering



and by forward scattering



Conclusions and proposals of further studies (2/2)

-High density fluctuation amplitude at the edge induces modifications of the reconstructed density profile as show in F. da Silva *et al* paper. This effect should be also taken into account in further studies.

-The effect (toroidal deviation) of the shear magnetic field on the probing beam propagation has been neglected until now, this point should be verified .

-The parasitic resonances should be also studied in details (3D full-wave simulations are required, should be done in vacuum)

F. da Silva et al EPMESC IX, 22-27 November Macau "Computational methods in Engineering and science"ed A.A Balkema ISBN 9058095673, p233 (2003).