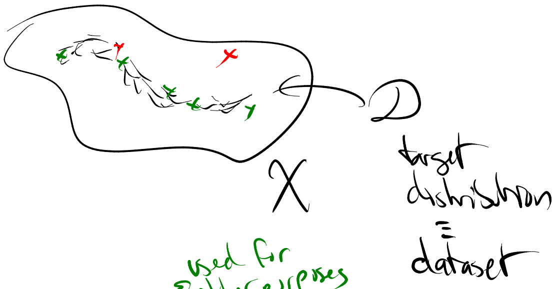
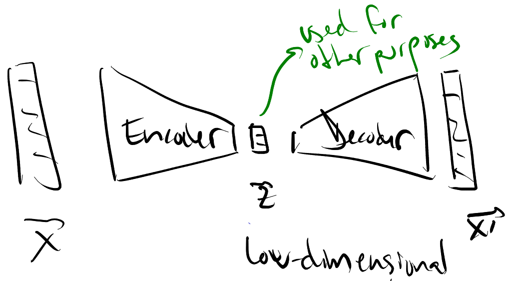
Generative models

Auto-encoders

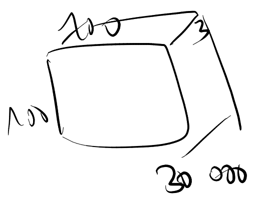


task: generate new points that "look like" the ones in the training set



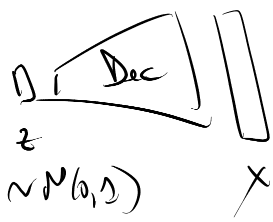
Low-dimensional representation of x

Latent representation



$x' \simeq x$

Loss: $\mathbb{E}_{x \sim D}\left[\|x' - x\|^2\right]$ reconstruction error



Generative model: decodes noise

⚠ decode parts of Z not trained

Solution: make
- $\underbrace{Enc(D)}_{\text{train}}$ to be close to $\underbrace{\mathcal{N}(0,1)}_{\text{sample}}$
- $\underbrace{Dec(\mathcal{N}(0,1))}_{\text{generated samples}}$ to be close to $\underbrace{D}_{\text{dataset}}$

. choice of the "metric" to compare distributions: $KL(D \| D_G)$ Kullback-Leibler

↳ Shannon div

↳ $\underbrace{\int_X p_D(x) \log \frac{p_D(x)}{p_{D_G}(x)} dx}$

$\underbrace{\mathbb{E}_{x \sim p_D}\left[\quad " \quad \right]}$ Monte Carlo estimate

$\simeq \frac{1}{N} \sum$ large minibatch of $x \sim D$

.

⚠ move to variational inference:

Enc & Dec are stochastic

$P_{Enc}(z|x)$ modeled as a Gaussian $\mathcal{N}(\mu(x), \Sigma(x))$

$P_{Dec}(x|z)$ - - - - - - - $\mathcal{N}(\mu'(z), \Sigma'(z))$

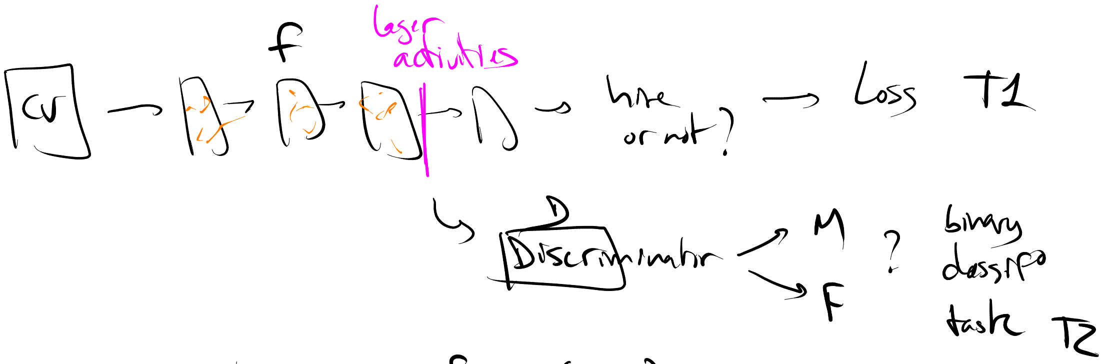
$x \xrightarrow{Enc} \left.\begin{matrix}\rightarrow \mu \\ \rightarrow \Sigma\end{matrix}\right\}$ w/ sample $z \xrightarrow{Dec} \left.\begin{matrix}\rightarrow \mu' \\ \rightarrow \Sigma'\end{matrix}\right\}$ w/ sample x'

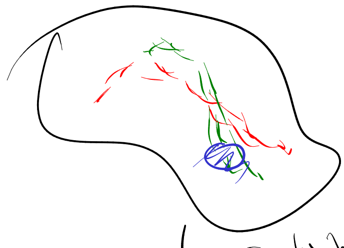
Adversarial setting

→ GAN : Generative Adversarial Network

→ DANN : Domain-Adversarial Neural Network

ex: biases:





$$\sup_{F} \inf_{D} T2(D, F)\ \text{error}$$

(Kolmogorov test)

$$\inf_{F} T1\ \text{error}$$

training loop

unstable

for f: loss: $T1 - \alpha T2$

for D: loss: $T2$

Going further:

optimal transport

Kantorovich–Rubinstein duality

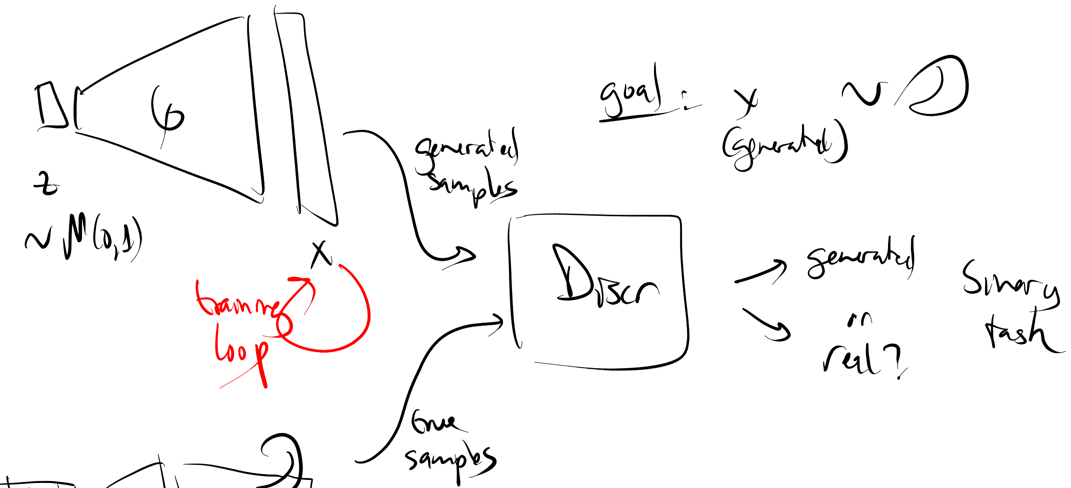
$$OT(P_G, P_{Target}) = \sup_{\substack{f \\ \text{1-Lipschitz}}} \left(\mathbb{E}_{x \sim P_G}[f(x)] - \mathbb{E}_{x \sim P_T}[f(x)] \right)$$

generated

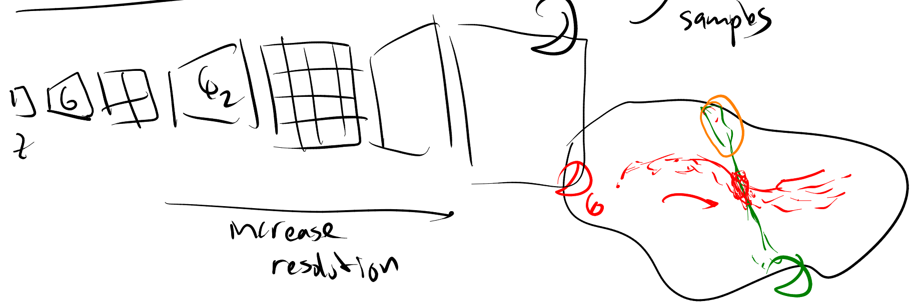
$$\left\| \frac{df}{dx} \right\| \leq 1 \qquad + \mathrm{ReLU}\left(\left\| \frac{df}{dx} \right\| - 1 \right)$$

GAN:

goal: x (generated) ~ D



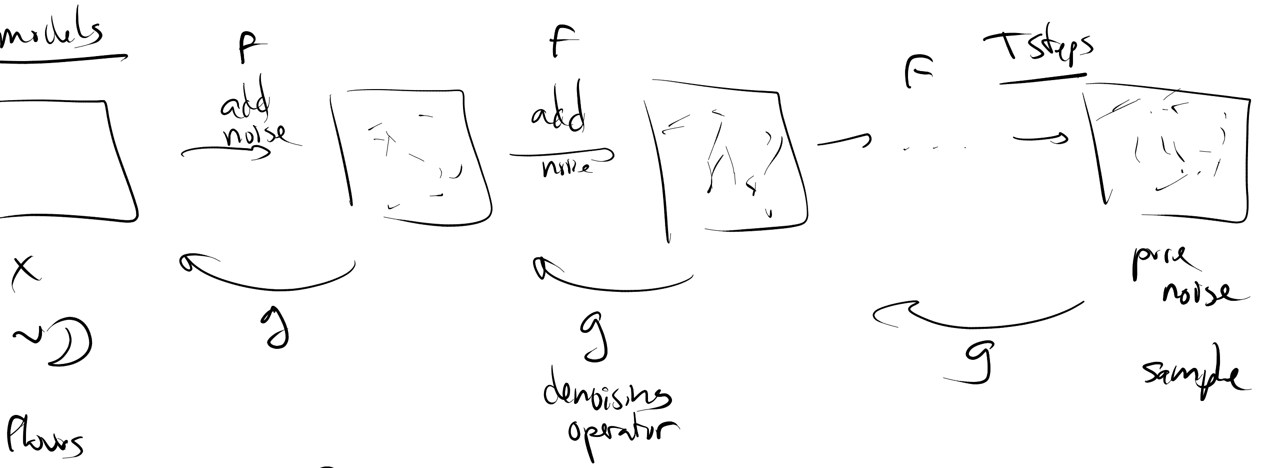
Scale/Resolution:



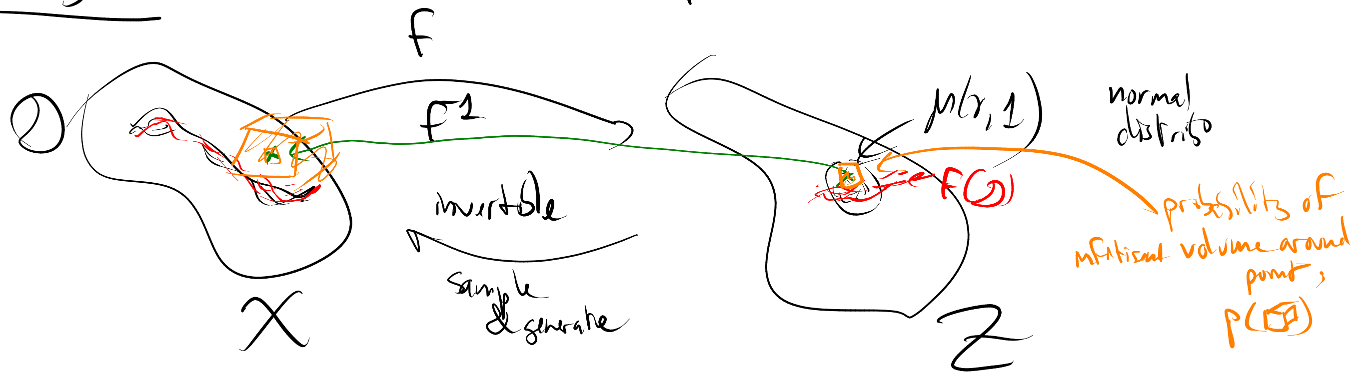
⚠ mode-drop

(no feedback about missing modes (points))

Diffusion models



Normalizing Flows

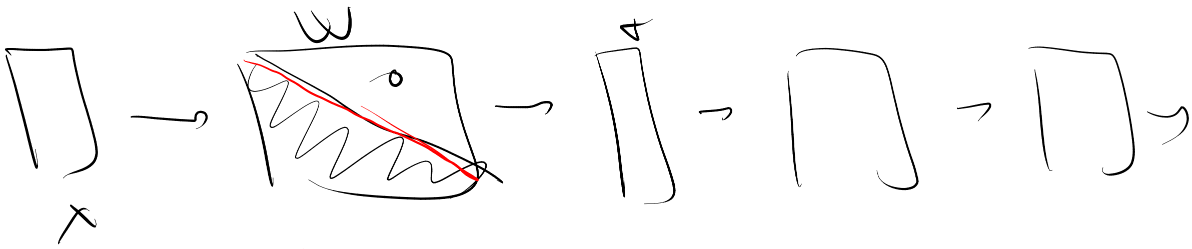


Loss: $KL(F(D) \,\|\, \mathcal{N}(0,1))$ or $KL(\mathcal{N}(0,1) \,\|\, F(D))$ in Z space

$KL(D \,\|\, f^{-1}(\mathcal{N}(0,1)))$ or $KL(f^{-1}(\mathcal{N}(0,1)) \,\|\, D)$ in X space

$$KL(p\|q) = \underbrace{\int p}_{\mathbb{E}[\quad]} \log \frac{p}{q} \leftarrow ?$$

$$p_\theta(x) = \underset{\sim\mathcal{N}}{p_z}(f(x)) \times |\det J_F|$$



$$\det W = \prod_i w_{ii}$$

$$J(f) = \prod J_{W_l}$$

$$\det J_F = \prod_l \underline{\det W_l} \times \sigma'$$