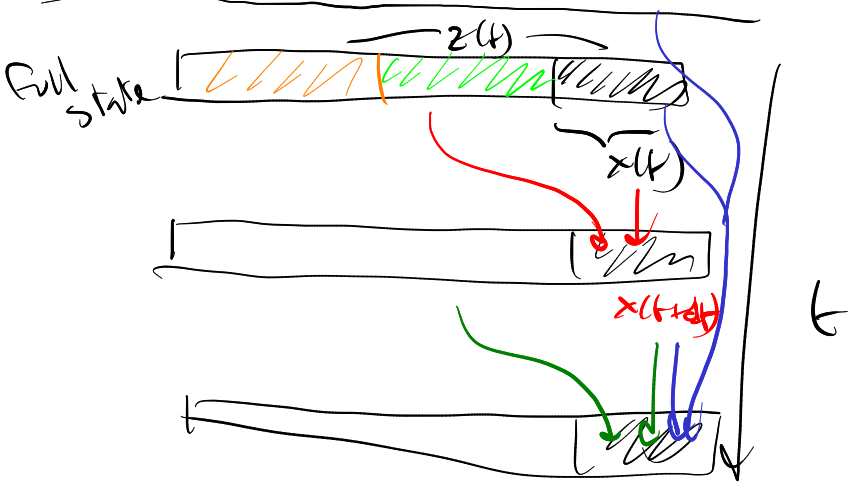
Dynamical systems (continued)

→ partly-observable systems

full state $z(t)$ — observable state $x(t)$

$$\frac{dz}{dt} = f(z,t) \qquad \frac{dx}{dt} = g(x,t)$$ (not all information needed)

Mori-Zwanzig / Takens's theorem



$$\frac{dx}{dt} = f(x(t), x(t-\tau), x(t-2\tau), \dots)$$ (delays)

history of x

⇒ provide the history of x to better predict x

⇒ delay differential equations DDE

Other approach:

$$\frac{dx}{dt} = f(x,t)$$ (observable: not sufficient)

$x^\ddagger = (x(t), x_+(t))$ ← supplementary variables "augmented" ⟺ memory term in a recurrent network

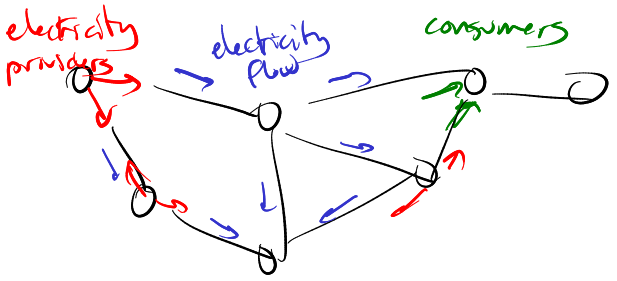
$$\frac{dx^\ddagger}{dt} = g(x^\ddagger, t)$$

Solvers

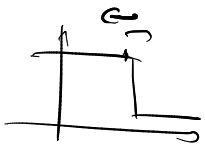
Learn the operator: initial state, boundary conditions, mesh / domain → solution of the equations

ex: Deep statistical solvers

↳ electricity grid in France

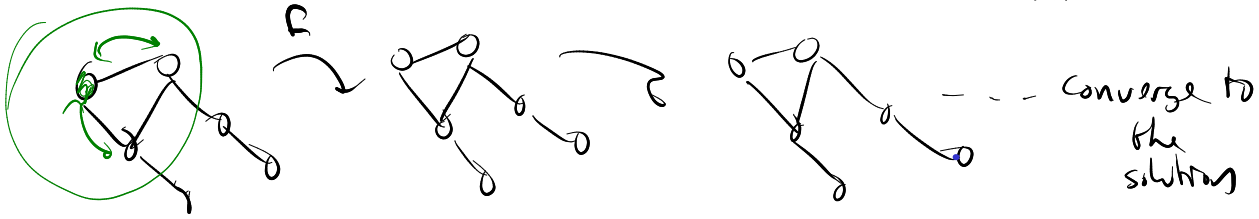


RTE



production
consumption
network
⟶ electricity flow

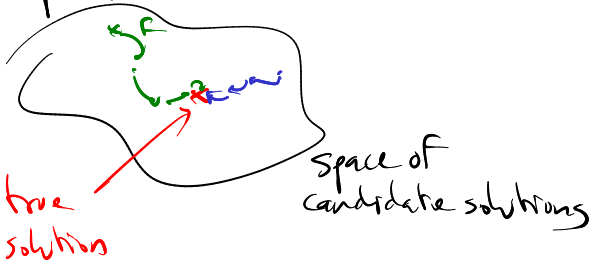
graph-NN
recurrent



$\cdots f \circ f \circ f \circ f(x_0) \longrightarrow x_T$

need to apply N times the graph-NN
with $N \geq$ diameter(graph)

⇒ search for a contractive operator



Classical setup:

- design the right contractive operator by hand
- apply it until convergence

ex: $AX = B$ solve $\rightarrow X$?

Pt NN: $\min_X \|AX - B\|$

approximate/iterative solvers

$X_{t+1} = f(X_t) \longrightarrow$ solution of $AX = B$

property (desired): X^* is a fixed point of f

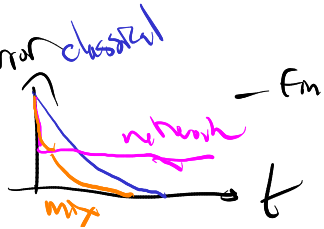
$$X_{t+1} = X_t + (AX_t - B) \times \alpha$$

variations: - Jacobi
- conjugate gradient

$\alpha < 0$ and close to 0

Mixing the 2:

- do a certain number of steps with the trained operator (without guarantees)
- finish with the classical solver (guaranteed to converge)



pros	cons
very fast at test time guarantees & high precision	no guarantee & bad precision slow

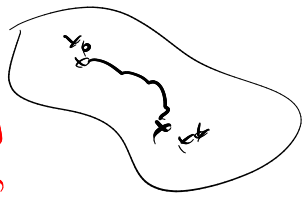
Implicit layers:

So far: learn an operator f

- apply it iteratively until cv $\rightarrow$ fixed point) very naive way to find the fixed point off

use a classical solver meant for finding fixed points

$\Rightarrow$ faster, more accurate, with guarantees



$x_0 \rightarrow [f] \rightarrow x_1$ ~~$\rightarrow x_2$~~

solver $\left(\rightarrow [f] \rightarrow , x_0, \text{desired precision} \right)$

$\hookrightarrow$ output: fixed point x_F

Loss $= \|x_F - x^*\|^2$

inf f s.t. $\|x_F - x^*\|$

$\neq$ from:

$x_0 \mapsto [f] \rightarrow x_1 \xrightarrow{f} x_2 \xrightarrow{f} x_3 \cdots \xrightarrow{f} x^*$

$\hookrightarrow$ x_F $\quad f(x_F) \simeq x_F \rightarrow x^*$