Synchronous Languages: Embedded Critical Real Time Software

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Critical Software

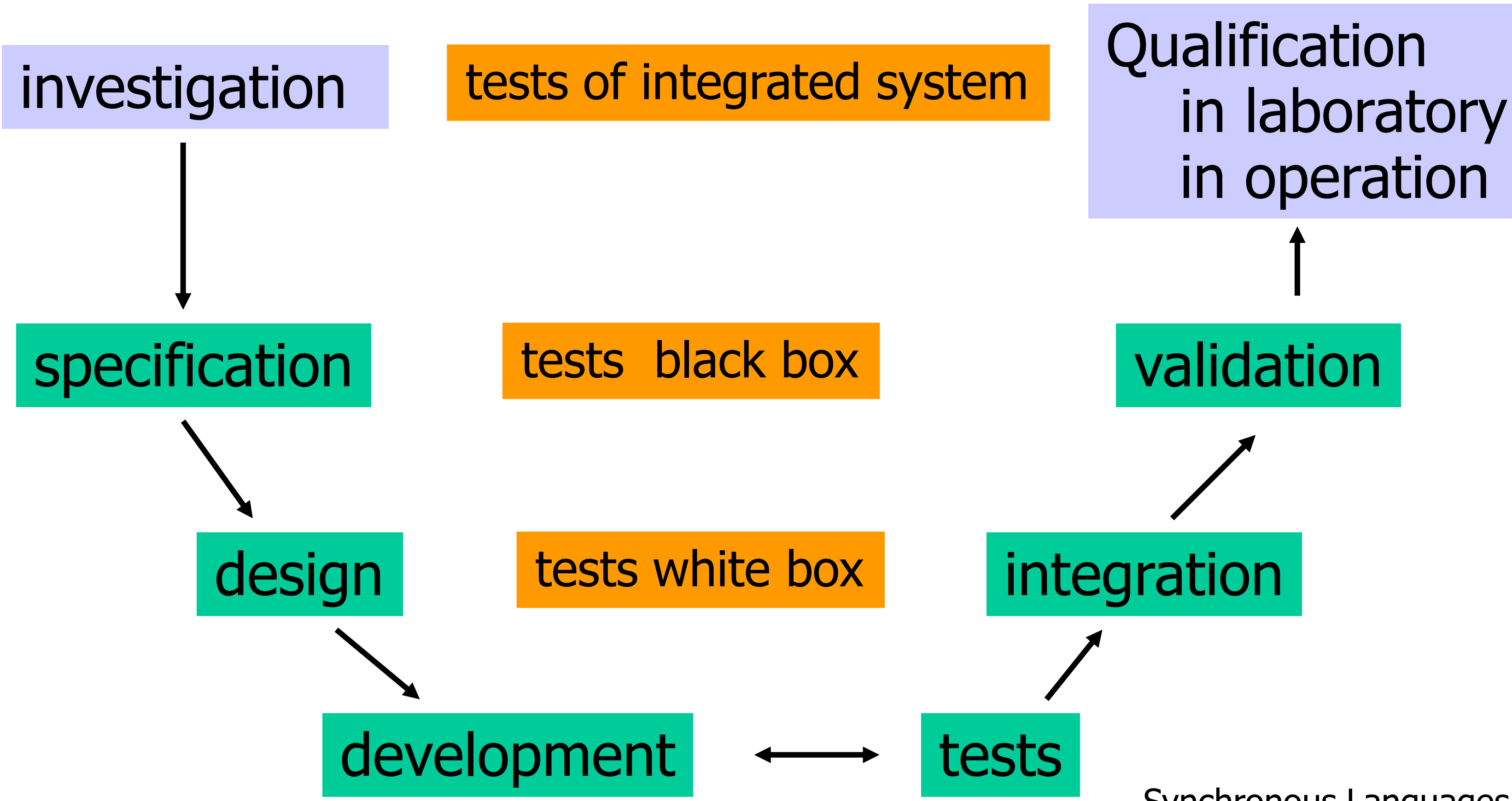
- Roughly speaking a critical system is a system whose failure could have serious consequences
- Nuclear technology
- Transportation
  - Automotive
  - Train
  - Avionics
- …………

Critical Software (2)

- In addition , other consequences are relevant to determine the critical aspect of a software:
  - Financial aspect
    - Loosing of equipment, bug correction
    - Equipment callback (automotive)
  - Bad advertising
    - Intel famous bug

How Develop critical software ?

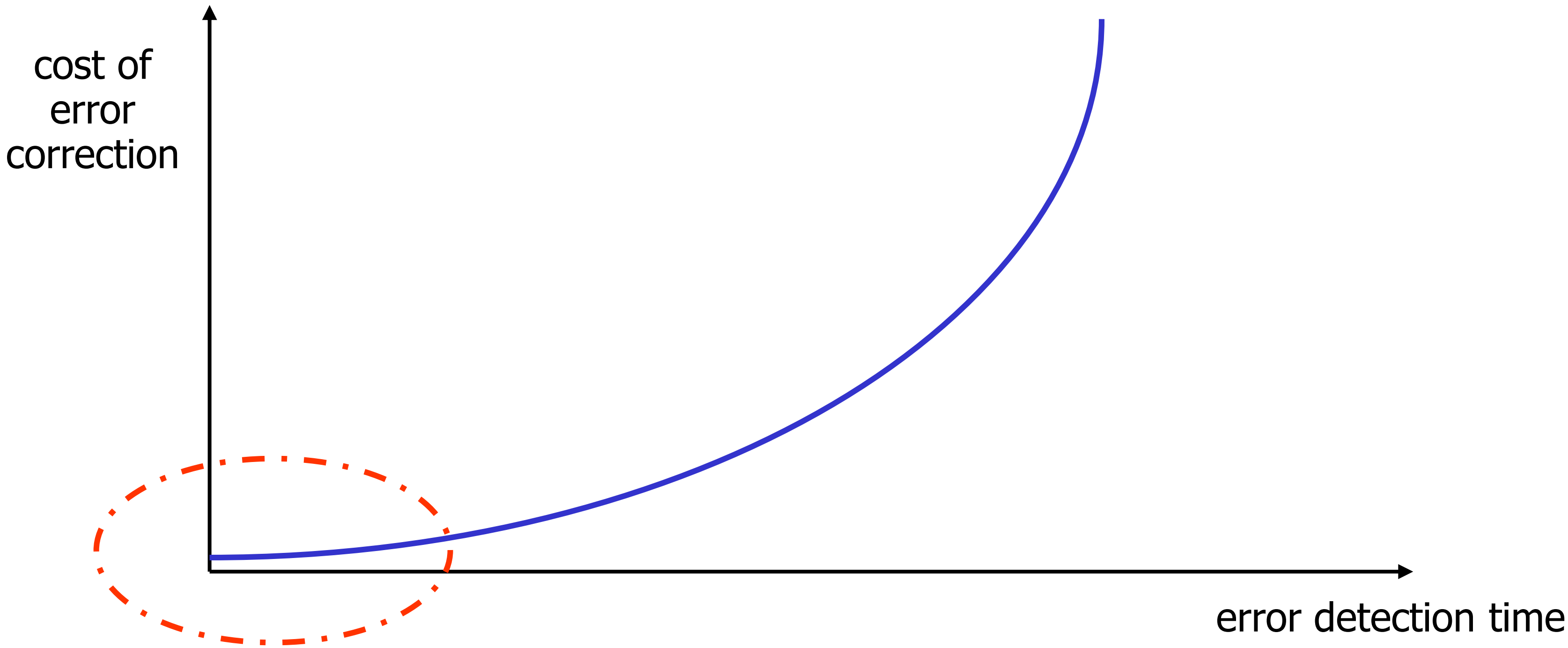
Classical Development V Cycle



How Develop Critical Software ?

- Cost of critical software development:
  - Specification : 10%
  - Design: 10%
  - Development: 25%
  - Integration tests: 5%
  - Validation: 50%
- Fact:
  - Earlier an error is detected, more expensive its correction is.

Cost of Error Correction



Put the effort on the upstream phase

development based on models

How Develop Critical Software ?

- Goals of critical software specification:
  - Define application needs
    - $\Rightarrow$ specific domain engineers
  - Allowing application development
    - Coherency
    - Completeness
  - Allowing application functional validation
    - Express properties to be validated

$\Rightarrow$ Formal models usage

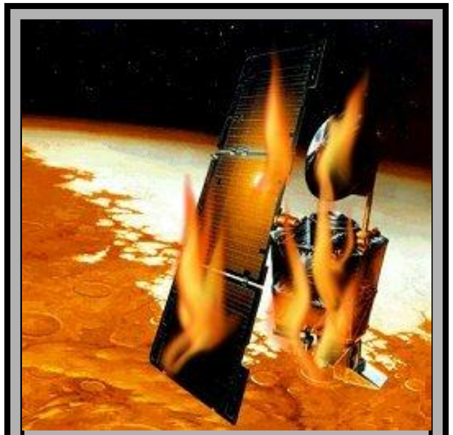
Critical software specification

- First Goal: must yield a formal description of the application needs:
  - Standard to allowing communication between computer science engineers and non computer science ones
  - General enough to allow different kinds of application:
    - Synchronous (and/or)
    - Asynchronous (and/or)
    - Algorithmic

Example of bad understanding

Nasa lost a $125 million Mars Orbiter because one engineering team used metric units while another used English metrics for a key spacecraft operation

For that reason, information failed to transfer between the Mars Climate Orbiter spacecraft team in Colorado and the mission navigation team in California



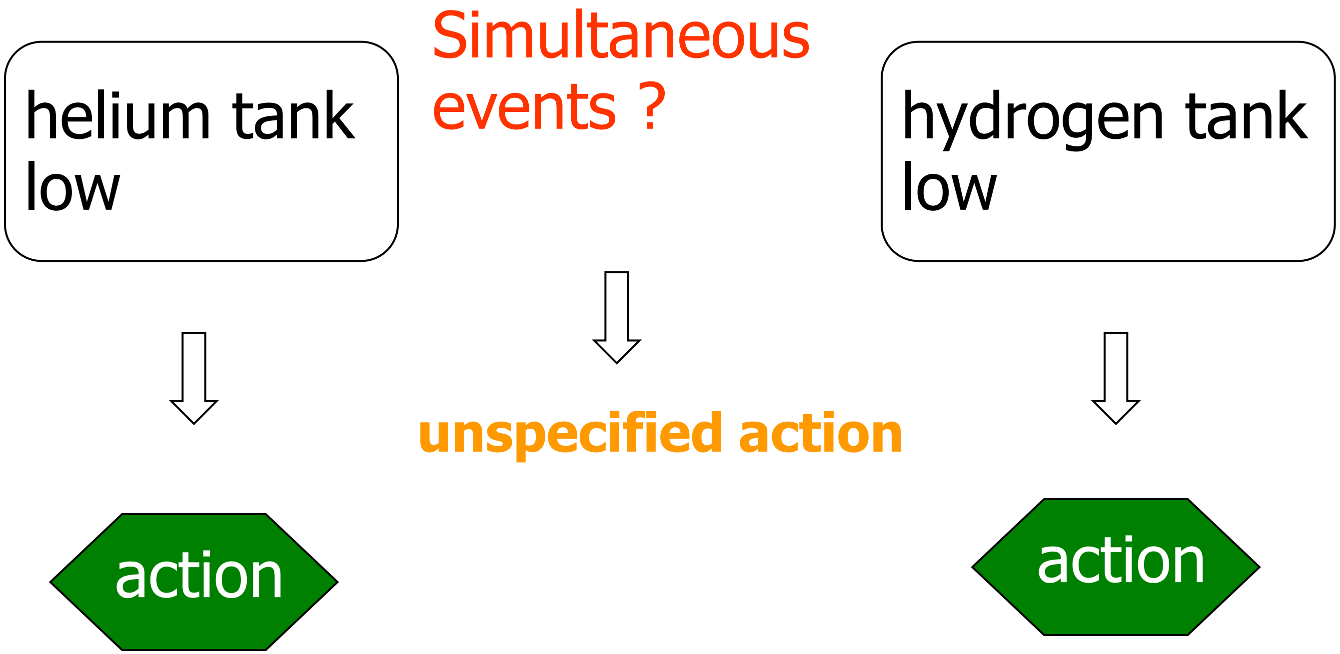
Nasa climate orbiter was lost September 23, 1999

Critical software specification

- Second Goal: allowing errors detection carried out upstream:
  - Validation of the specification:
    - Coherency
    - Completeness
    - Proofs
  - Test
    - Quick prototype development
    - Specification simulation

Example of non completeness

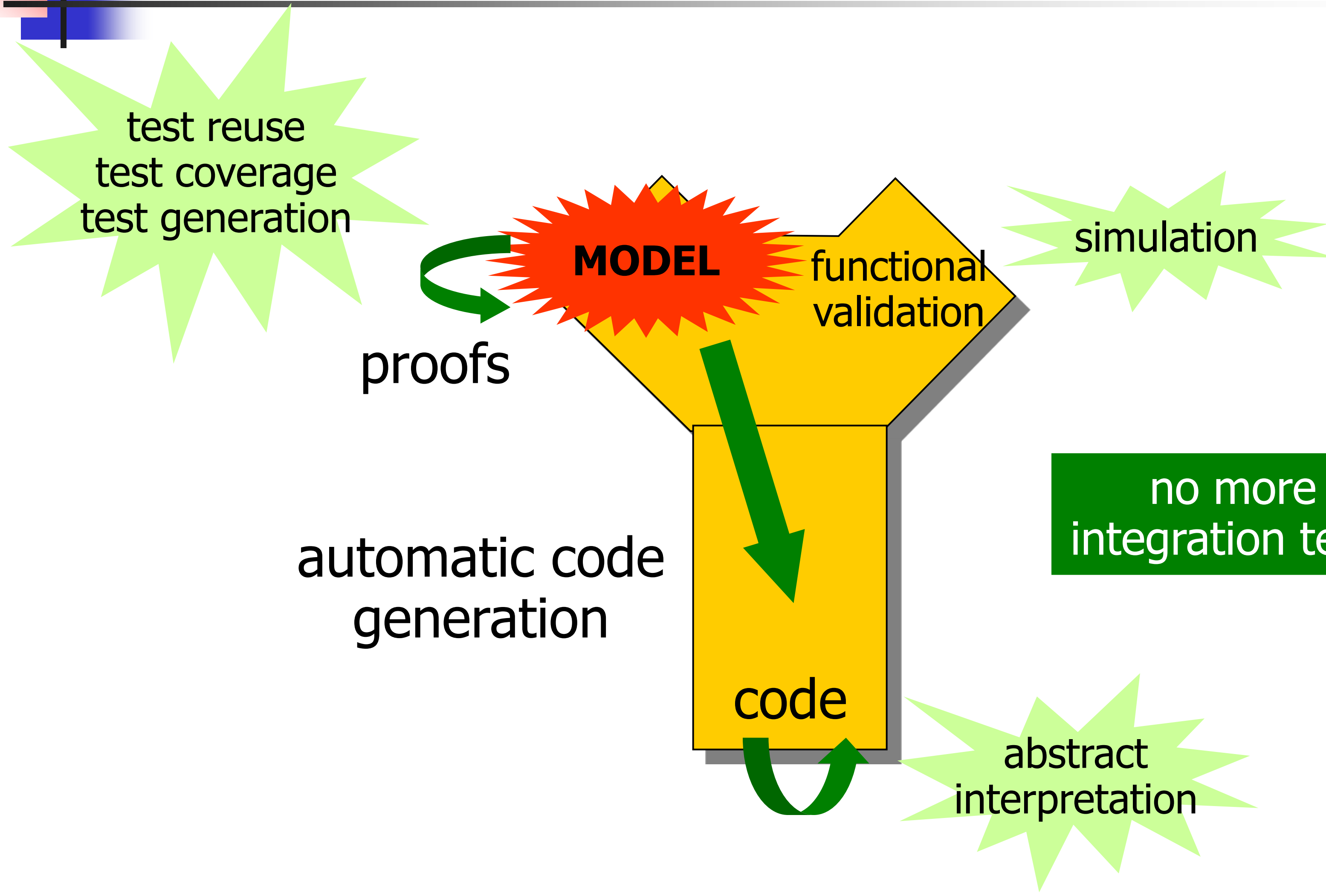
From Ariane 5:



Critical Software Specification (3)

- Third goal: make easier the transition from specification to design (refinement)
  - Reuse of specification simulation tests
  - Formalization of design
  - Code generation
    - Sequential/distributed
    - Toward a target language
    - Embedded/qualified code

Relying on Formal Methods



no more integration tests

Synchronous Languages Verification

Critical Software Validation

- What is a correct software?
  - No execution errors, time constraints respected, compliance of results.
- Solutions:
  - At model level :
    - Simulation
    - Formal proofs
  - At implementation level:
    - Test
    - Abstract interpretation

Validation Methods

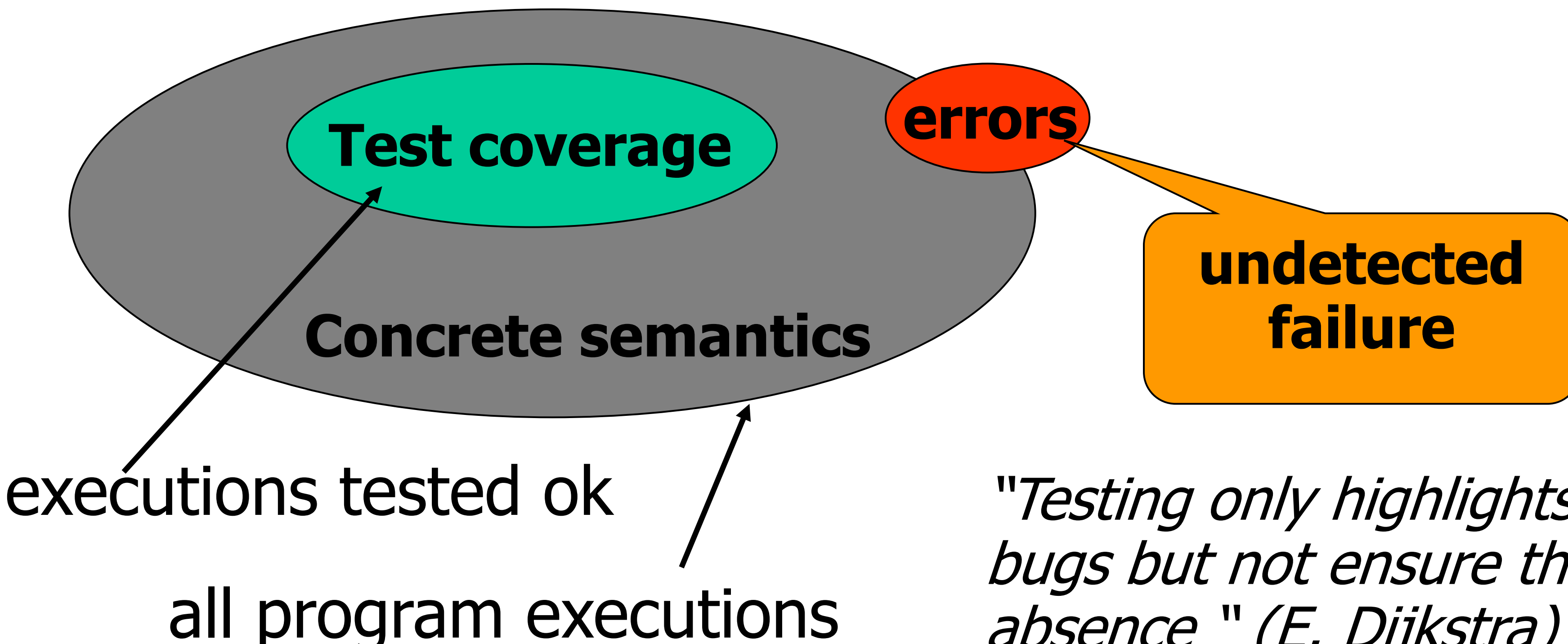
- Testing
  - Run the program on set of inputs and check the results
- Static Analysis
  - Examine the source code to increase confidence that it works as intended
- Formal Verification
  - Argue formally that the application always works as intended

Testing

- Dynamic verification process applied at implementation level.
- Feed the system (or one if its components) with a set of input data values:
  - Input data set not too large to avoid huge time testing procedure.
  - Maximal coverage of different cases required.

Testing (2)

Program Testing



"Testing only highlights bugs but not ensure their absence " (E. Dijkstra)

Static Analysis

- The aim of static analysis is to search for errors without running the program.
- *Abstract interpretation* = replace data of the program by an abstraction in order to be able to compute program properties.
- Abstraction must ensure :
  - $\mathcal{A}$(P) "correct" $\Rightarrow$ P correct
  - But $\mathcal{A}$(P) "incorrect" $\Rightarrow$?

Static Analysis: example

abstraction: integer by intervals

```
1: x:= 1;
2: while (x < 1000) {
3:   x := x+1;
4: }
```

$$x1 = [1,1]$$

$$x2 = x1 \cup x3 \cap [-\infty, 999]$$

$$x3 = x2 \oplus [1,1]$$

$$x4 = x1 \cup x3 \cap [1000, \infty]$$

Abstract interpretation theory $\Rightarrow$ values are fix point equation solutions.

Formal verification

- What about functional validation ?
  - Does the program compute the expected outputs?
  - Respect of time constraints (temporal properties)
  - Intuitive partition of temporal properties:
    - Safety properties: something bad never happens
    - Liveness properties: something good eventually happens

Safety and Liveness Properties

- Example: the beacon counter in a train:
  - Count the difference between beacons and seconds
  - Decide when the train is ontime, late, early

```
node train (sec, bea : bool) returns (ontime, early, late: bool)
  let
      diff = (0 ->pre diff) + (if bea then 1 else 0) + (if sec then -1 else 0);
      early = (true -> pre ontime) and (diff > 3) or
                  (false -> pre early) and (diff > 1);
      late  = (true -> pre ontime) and (diff < -3)  or
                 (false -> pre late) and (diff < -1);
      ontime = not (early or late);
  tel
```

Safety and Liveness Properties

- Some properties:
  1. It is impossible to be late and early;
  2. It is impossible to directly pass from late to early;
  3. It is impossible to remain late only one instant;
  4. If the train stops, it will eventually get late
- Properties 1, 2, 3 : safety
- Property 4 : liveness

It refers to unbound future

Safety and Liveness Properties Checking

- Use of model checking techniques
- Model checking goal: prove safety and liveness properties of a system in analyzing a model of the system.
- Model checking techniques require:
  - model of the system
  - express properties
  - algorithm to check properties on the model ($\Rightarrow$ decidability)

Model Checking Techniques

- Model = automata which is the set of program behaviors
- Properties expression = temporal logic:
  - LTL : liveness properties
  - CTL: safety properties
- Algorithm =
  - LTL : algorithm exponential wrt the formula size and linear wrt automata size.
  - CTL: algorithm linear wrt formula size and wrt automata size

Properties Checking

- Liveness Property Φ :
  - $\Phi \Rightarrow$ automata $B(\Phi)$
  - $\mathbb{L}(B(\Phi)) = \varnothing$ décidable
  - $\Phi \models \mathcal{M} : \mathbb{L}(\mathcal{M} \otimes B(\sim\Phi)) = \varnothing$
- Scade allows only to verify safety properties, thus we will study such properties verification techniques.

Safety Properties

- CTL formula characterization:
  - Atomic formulas
  - Usual logic operators: not, and, or ($\Rightarrow$)
  - Specific temporal operators:
    - EX $\varnothing$, EF $\varnothing$, EG $\varnothing$
    - AX $\varnothing$, AF $\varnothing$, AG $\varnothing$
    - EU($\varnothing_1$,$\varnothing_2$), AU($\varnothing_1$,$\varnothing_2$)

Safety Properties Verification (1)

- Mathematical framework:
  - S : finite state, ($\mathcal{P}(S)$, $\subseteq$) is a complete lattice with S as greater element and $\emptyset$ as least one.
  - f : $\mathcal{P}(S) \longrightarrow \mathcal{P}(S)$:
    - f is monotonic iff $\forall$ x,y $\in \mathcal{P}(S)$, $x \subseteq y \Rightarrow f(x) \subseteq f(y)$
    - f is $\cap$-continue iff for each decreasing sequence $f(\cap\, x_i) = \cap\, f(x_i)$
    - f is $\cup$-continue iff for each increasing sequence $f(\cup\, x_i) = \cup\, f(x_i)$

Safety Properties Verification (2)

- Mathematical framework:
  - if S is finite then monotonic $\Rightarrow$ $\cap$-continue et $\cup$-continue.
  - x is a fix point iff of f iff f(x) = x
  - x is a least fix point (lfp) iff $\forall$y such that f(y) = y, x $\subseteq$ y
  - x is a greatest fix point (gfp) iff $\forall$y such that f(y) = y, y $\subseteq$ x

Safety Properties Verification (3)

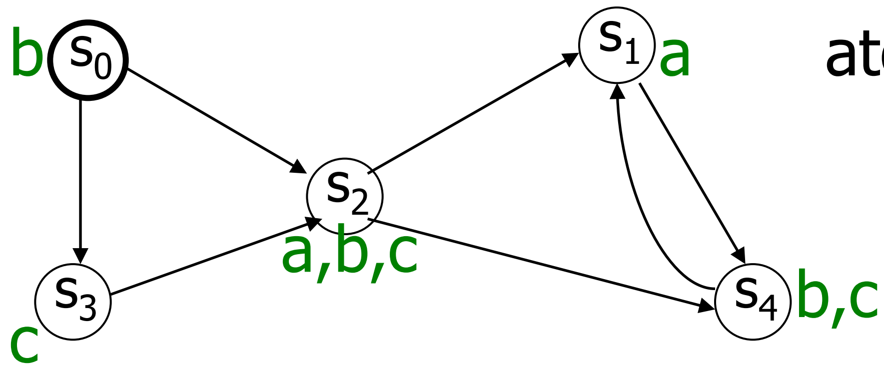
- Theorem:
  - f monotonic $\Rightarrow$ f has a lfp (resp glp)
  - $lfp(f) = \cup f^n(\emptyset)$
  - $gfp(f) = \cap f^n(S)$

Fixpoints are limits of approximations

Safety Properties Verification (4)

- We call Sat($\varnothing$) the set of states where $\varnothing$ is true.
- $\mathcal{M} \models \varnothing$ iff $s_{init} \in$ Sat($\varnothing$).
- Algorithm:
  - $Sat(\Phi) = \{ s \mid \Phi \models s\}$
  - $Sat(\text{not } \Phi) = S \setminus Sat(\Phi)$
  - $Sat(\Phi1 \text{ or } \Phi2) = Sat(\Phi1) \cup Sat(\Phi2)$
  - $Sat(EX\ \Phi) = \{s \mid \exists\, t \in Sat(\Phi), s \rightarrow t\}$ (Pre Sat(Φ))
  - $Sat(EG\ \Phi) = \mathit{gfp}\,(\Gamma(x) = Sat(\Phi) \cap Pre(x))$
  - $Sat(E(\Phi1\ U\ \Phi2)) = \mathit{lfp}\,(\Gamma(x) = Sat(\Phi2) \cup (Sat(\Phi1) \cap Pre(x))$

Example



atomic formulas: a, b, c

EG (a or b)

$$gfp\,(\Gamma(x) = \mathrm{Sat}(\Phi) \cap \mathrm{Pre}(x))$$

$$\Gamma(\{s_0, s_1, s_2, s_3, s_4\}) = \mathrm{Sat}\ (a \text{ or } b) \cap \mathrm{Pre}(\{s_0, s_1, s_2, s_3, s_4\})$$

$$\Gamma(\{s_0, s_1, s_2, s_3, s_4\}) = \{s_0, s_1, s_2, s_4\} \cap \{s_0, s_1, s_2, s_3, s_4\}$$

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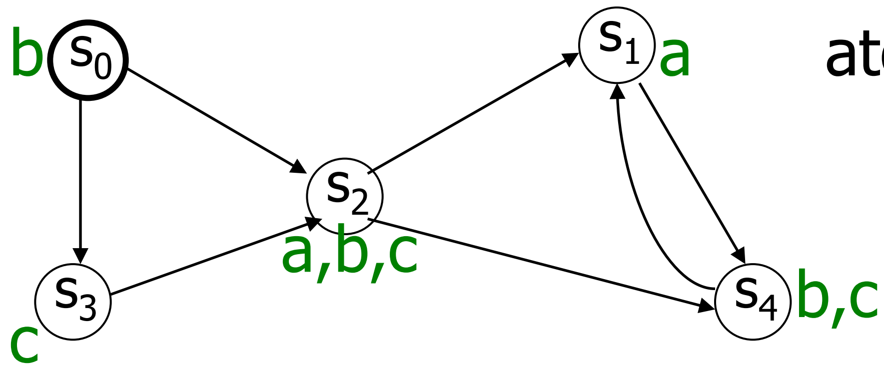
Example



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$\Gamma(\{s_0, s_1, s_2, s_4\})$ = Sat (a or b) $\cap$ Pre($\{s_0, s_1, s_2, s_4\}$)

$\Gamma(\{s_0, s_1, s_2, s_4\}) = \{s_0, s_1, s_2, s_4\}$

S_0 |= EG(a or b)

Model checking implementation

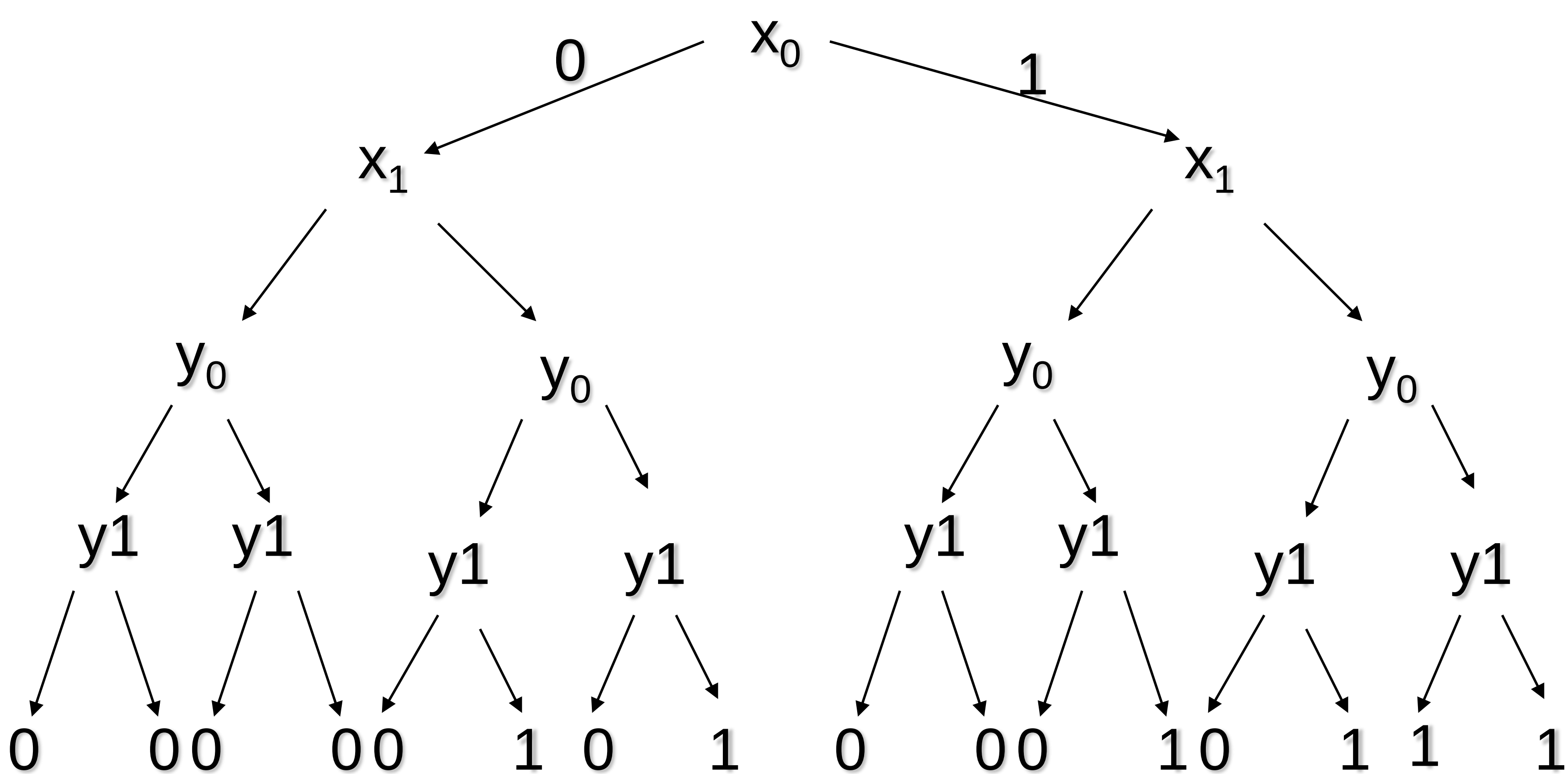
- Problem: the size of automata
- Solution: symbolic model checking
- Usage of BDD (Binary Decision Diagram) to encode both automata and formula.
- Each Boolean function has a unique representation
- Shannon decomposition:
  - $f(x_0,x_1,\ldots,x_n) = f(1, x_1,\ldots, x_n) \vee f(0, x_1,\ldots,x_n)$

Model Checking Implementation

- When applying recursively Shannon decomposition on all variables, we obtain a tree where leaves are either 1 or 0.
- BDD are:
  - A concise representation of the Shannon tree
  - no useless node (if x then g else g ⇔ g)
  - Share common sub graphs

Model Checking Implementation (2)

$(x_1 \wedge x_0) \vee ((x_1 \vee y_1) \wedge (x_0 \wedge y_0))$



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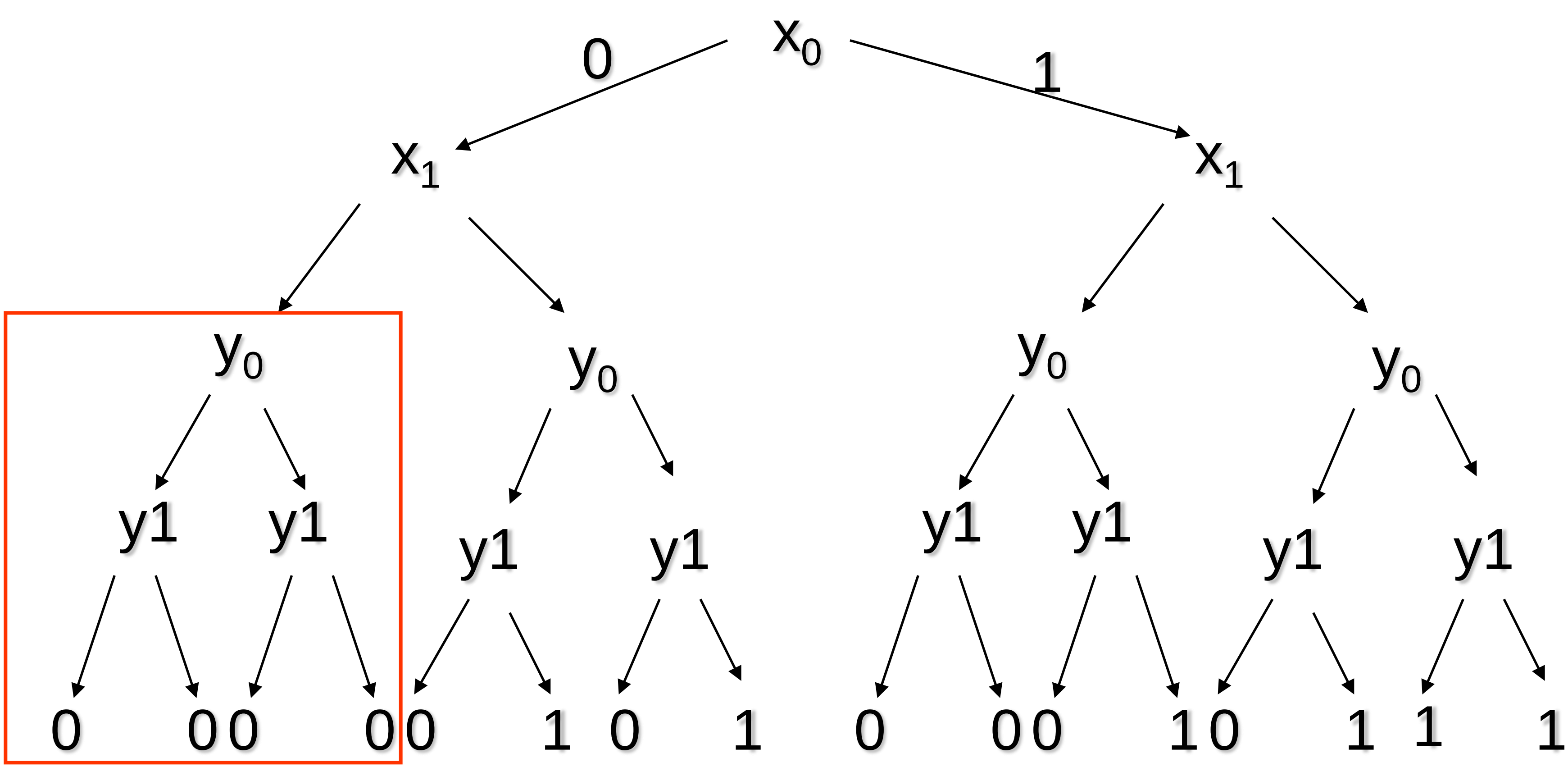

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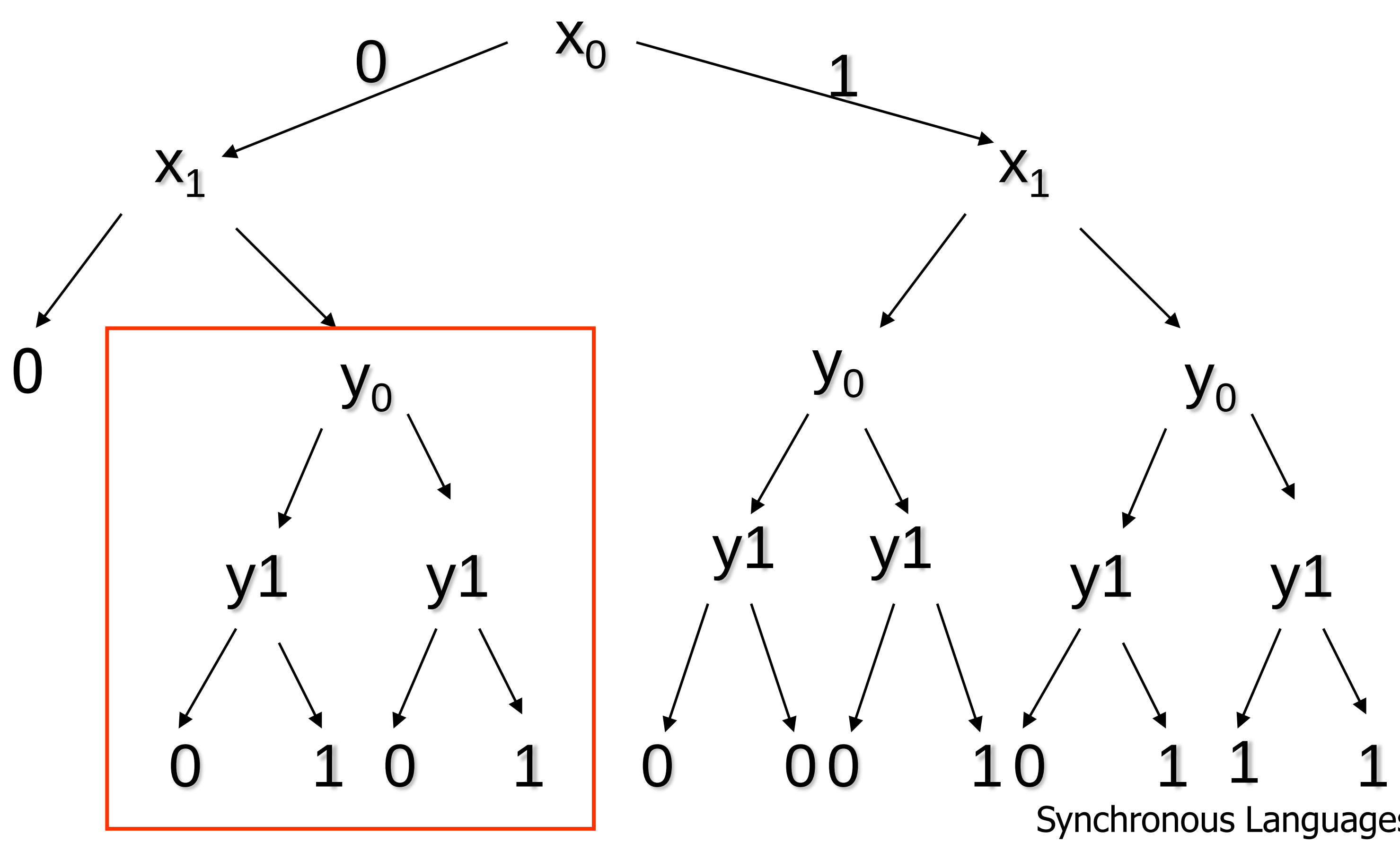

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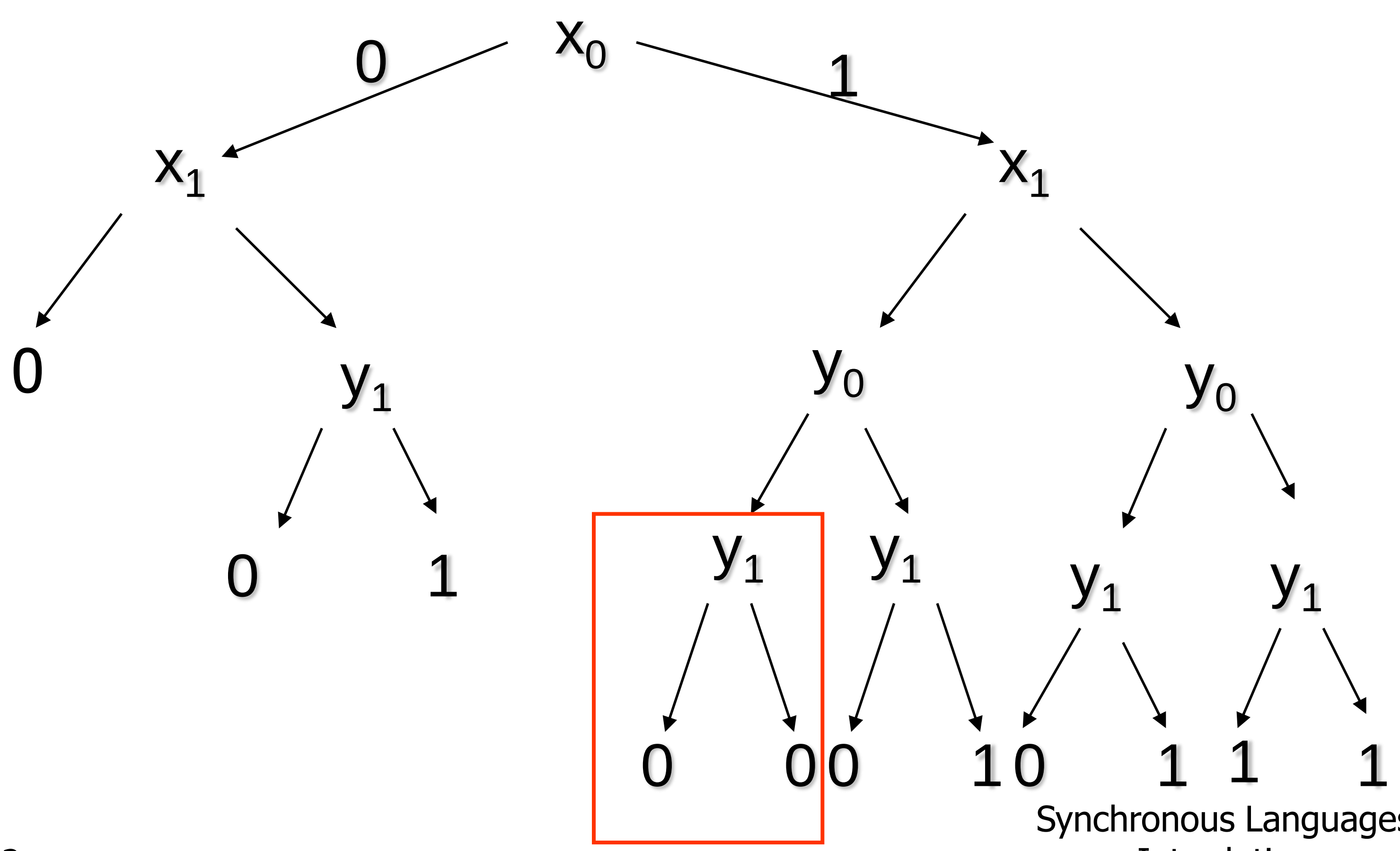

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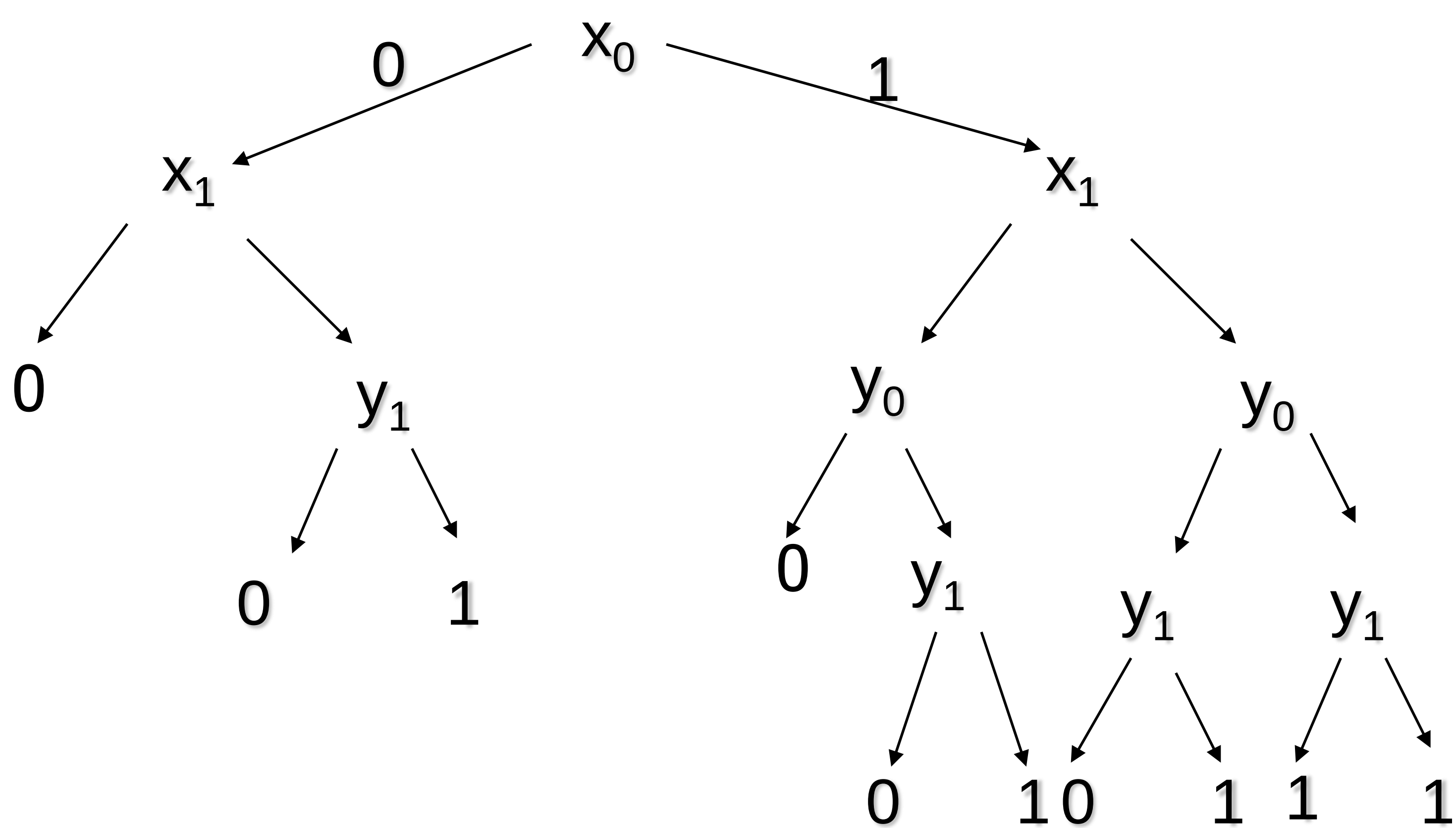

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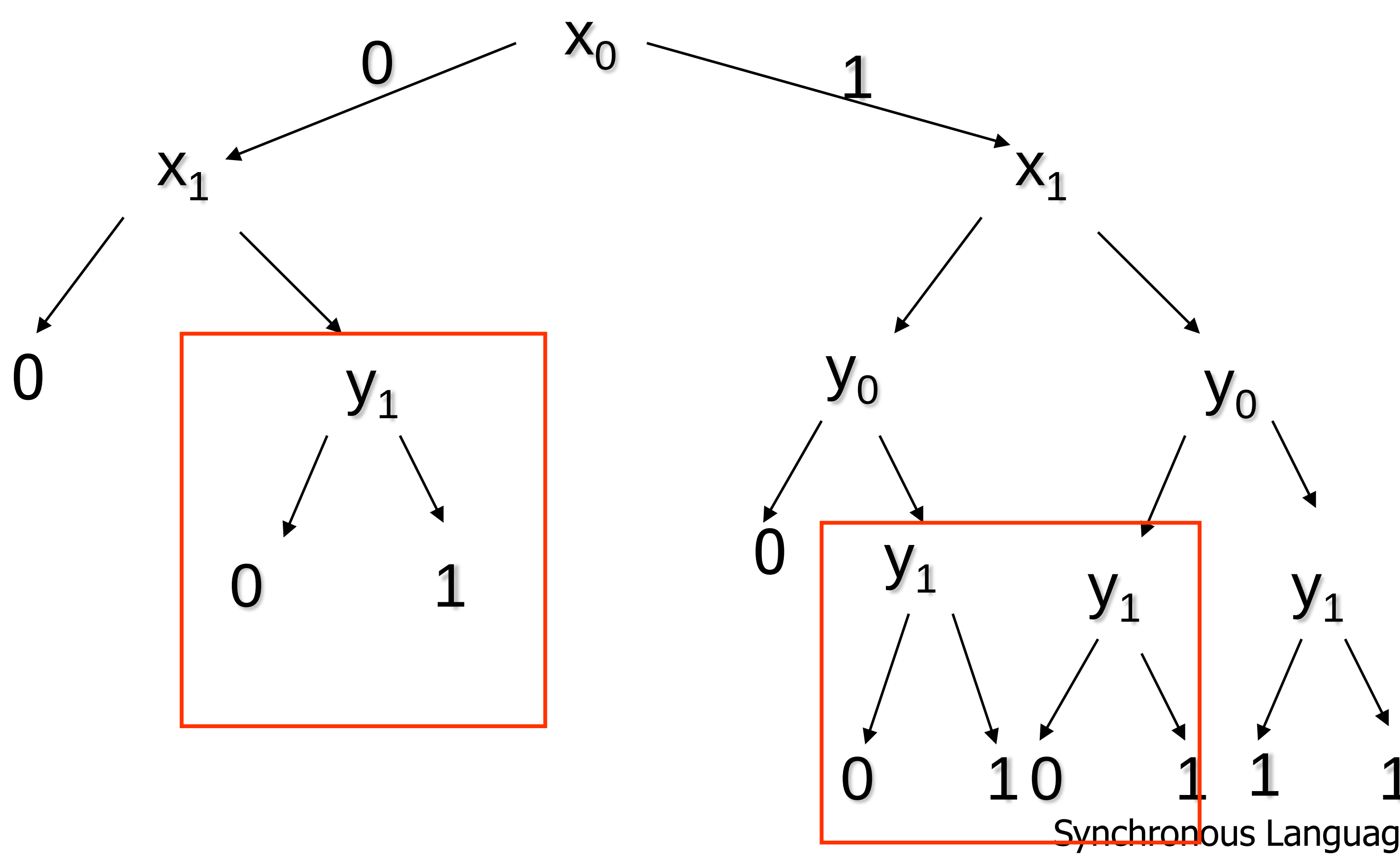

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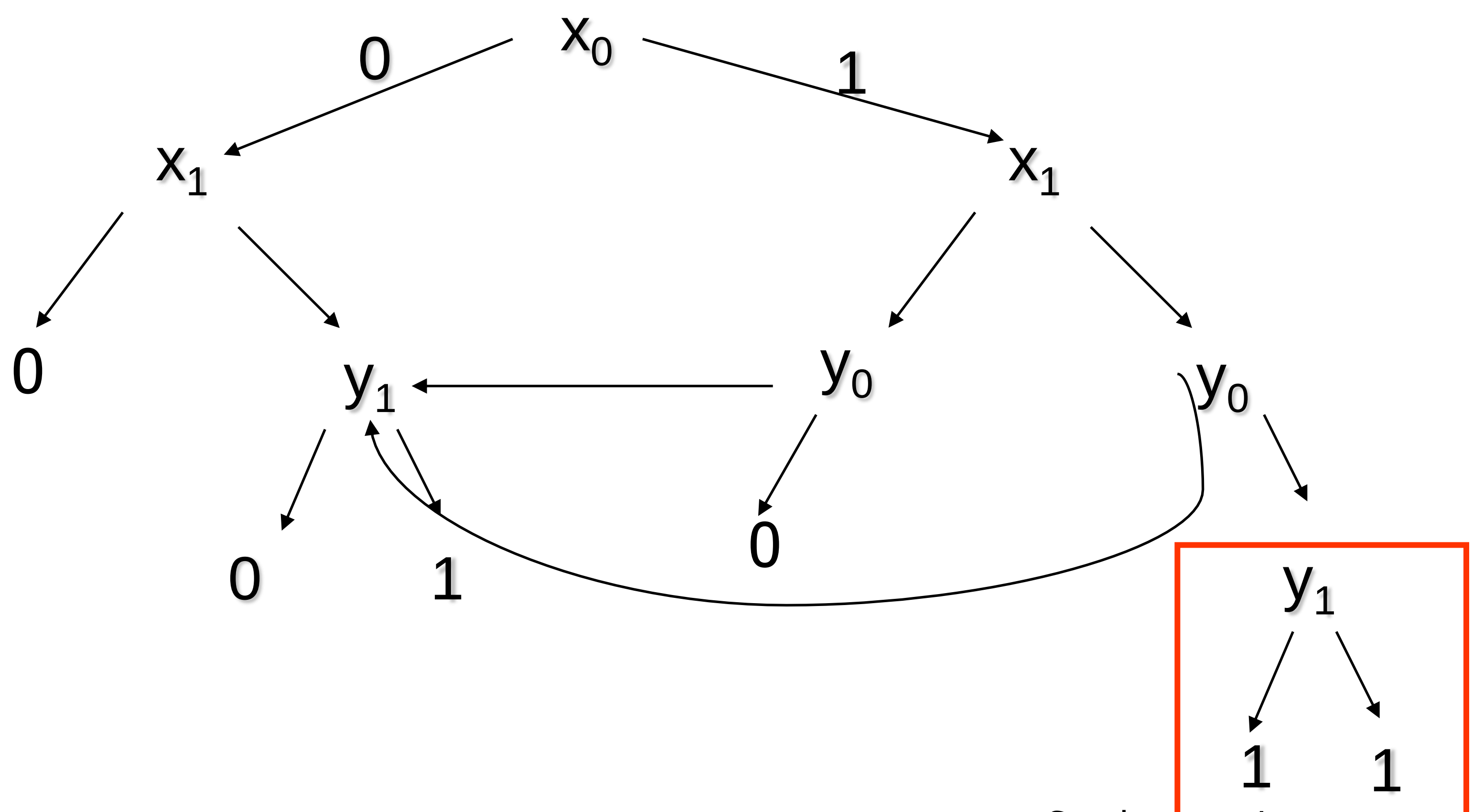

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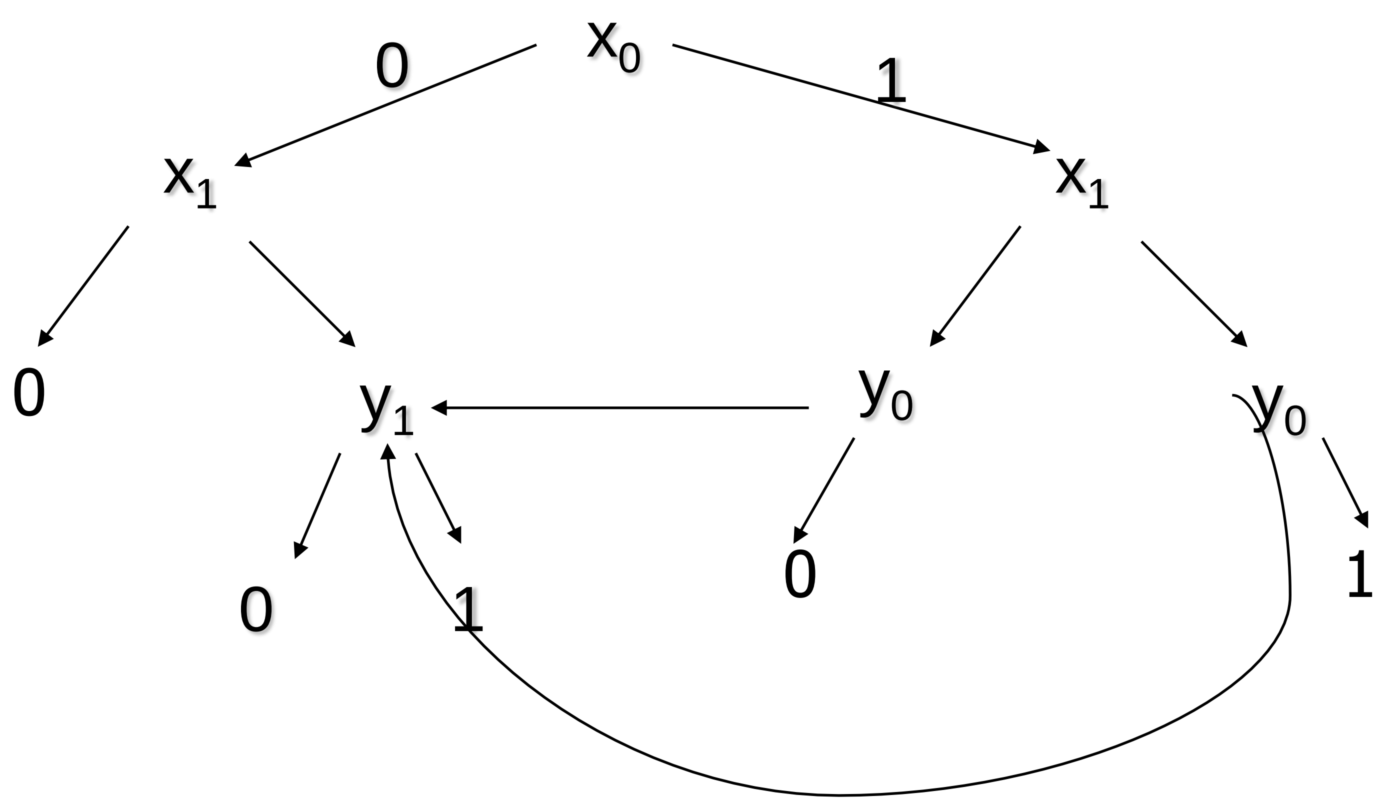

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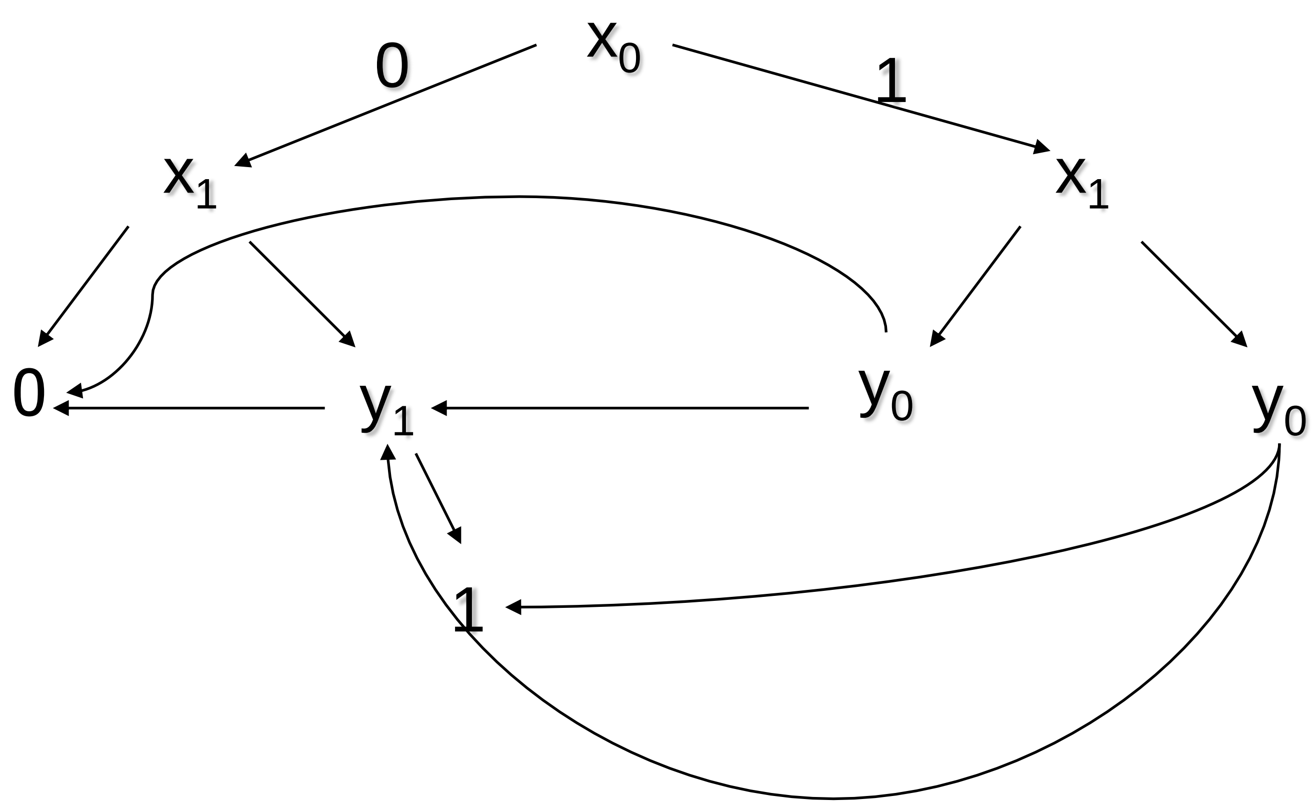


Model Checking Implementation(3)

- Implicit representation of the of states set and of the transition relation of automata with BDD.
- BDD allows
  - canonical representation
  - test of emptiness immediate (bdd =0)
  - complementarity immediate (1 = 0)
  - union and intersection not immediate
  - Pre immediate

Model Checking Implementation (4)

- But BDD efficiency depends on the number of variables
- Other method: SAT-Solver
  - Sat-solvers answer the question: given a propositional formula, is there exist a valuation of the formula variables such that this formula holds
  - first algorithm (DPLL) exponential (1960)

Model Checking Implementation (4)

- SAT-Solver algorithm:
  - formula ➔ CNF formula ➔ set of clauses
  - heuristics to choose variables
  - deduction engine:
    - propagation
    - specific reduction rule application (unit clause)
    - Others reduction rules
  - conflict analysis + learning

Model Checking Implementation (5)

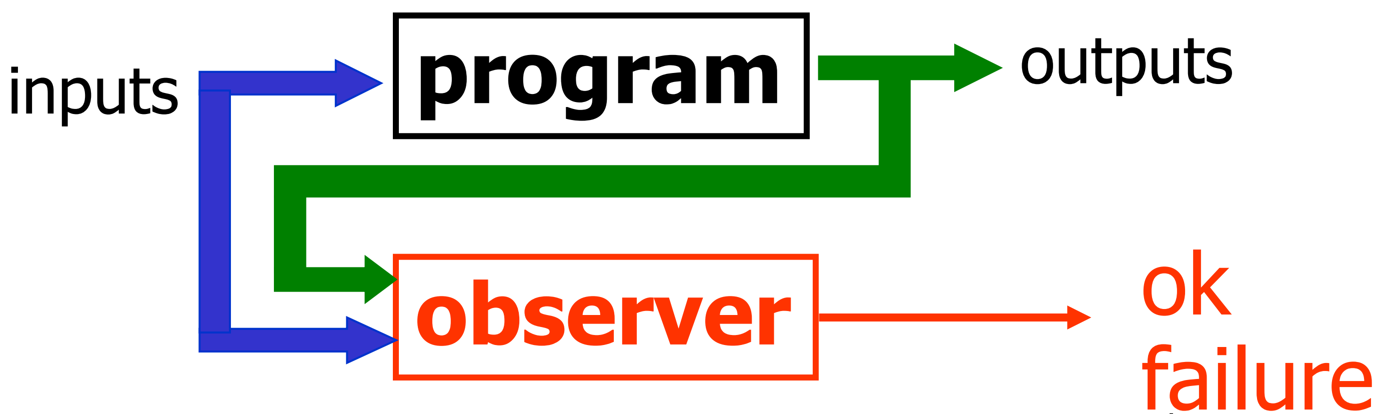
- SAT-Solver usage:
  - encoding of the paths of length k by propositional formulas
  - the existence of a path of length k (for a given k) where a temporal property Φ is true can be reduce to the satisfaction of a propositional formula
  - theorem: given Φ a temporal property and $\mathcal{M}$ a model, then $\mathcal{M} \models \Phi \Rightarrow \exists$ n such that $\mathcal{M} \models_n \Phi$ ($n < |S| \cdot 2^{|\Phi|}$)

Bounded Model Checking

- SAT-Solver are used in complement of implicit (BDD based) methods.
- $\mathcal{M} \models \Phi$
  - verify $\neg \Phi$ on all paths of length k (k bounded)
  - useful to quickly extract counter examples

Model Checking with Observers

- Express safety properties as observers.
- An observer is a program which observes the program and outputs ok when the property holds and failure when its fails

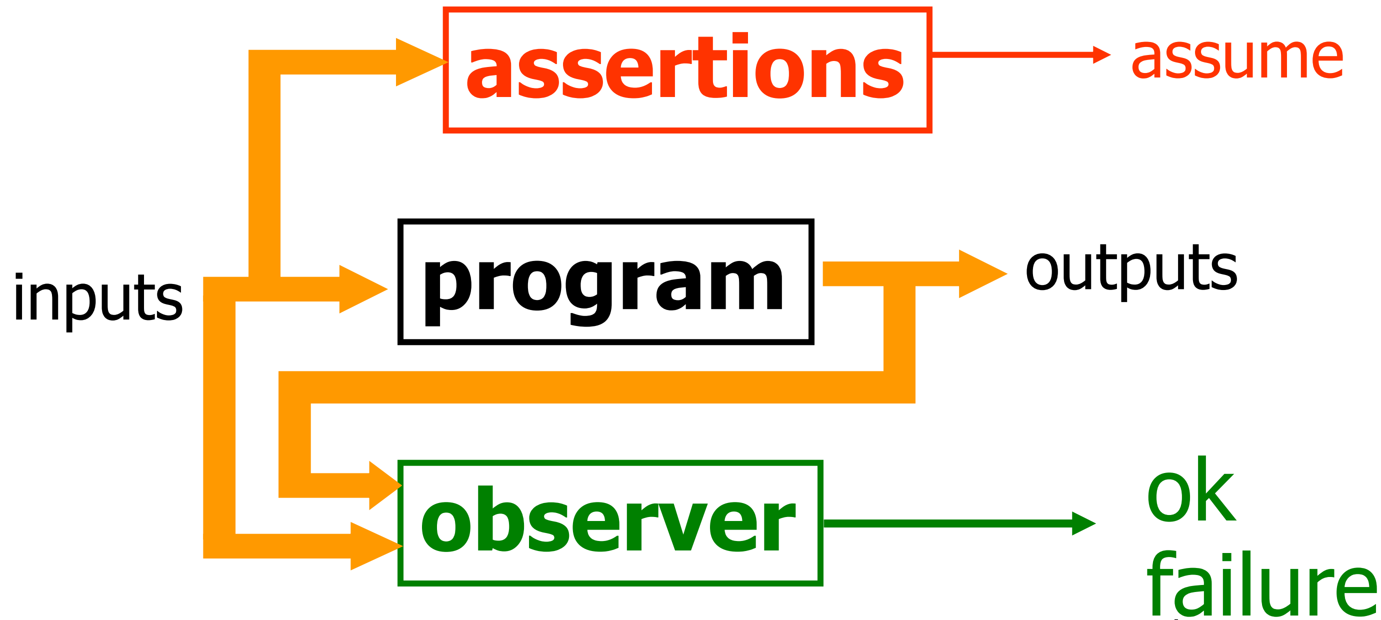


Properties Validation

- Taking into account the environment
  - without any assumption on the environment, proving properties is difficult
  - but the environment is indeterminist
    - Human presence no predictable
    - Fault occurrence
    - …
  - Solution: use assertion to make hypothesis on the environment and make it determinist

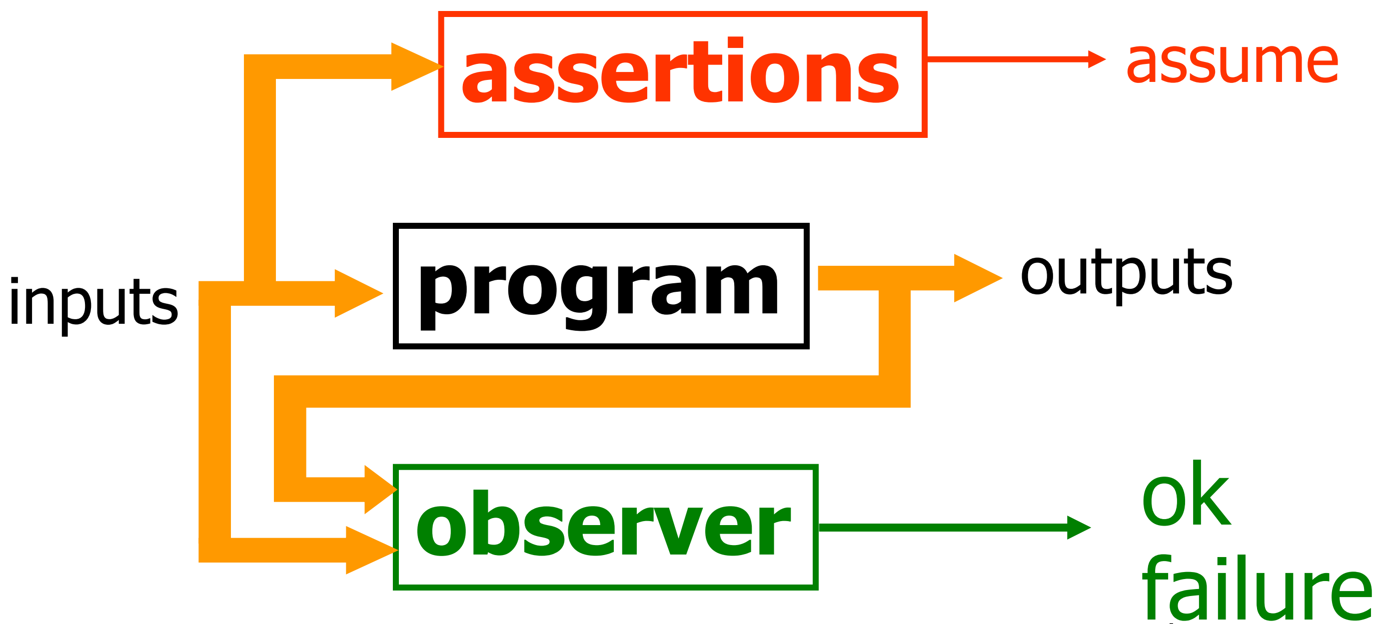
Properties Validation (2)

- Express safety properties as observers.
- Express constraints about the environment as assertions.



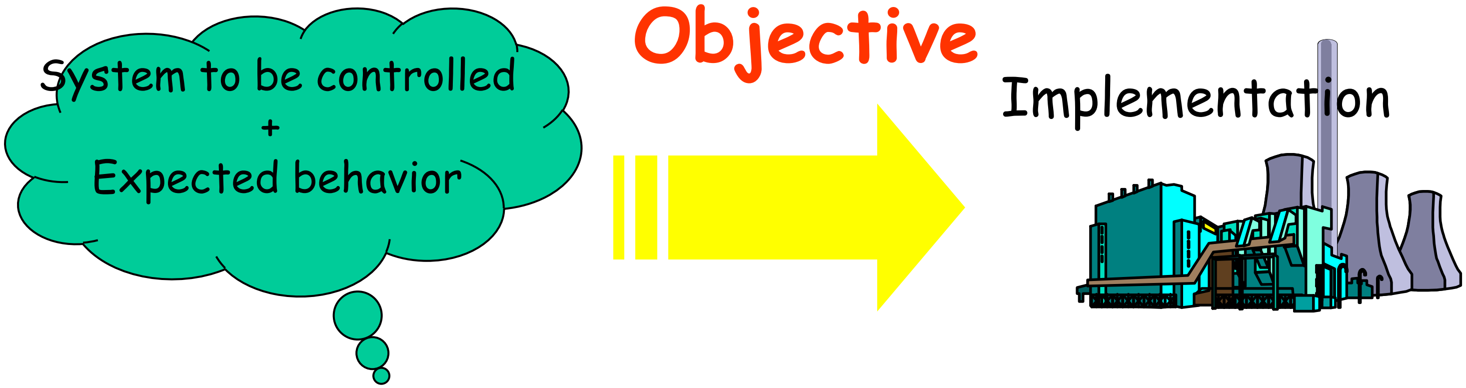
Properties Validation (3)

- if assume remains true, then ok also remains true (or failure false).

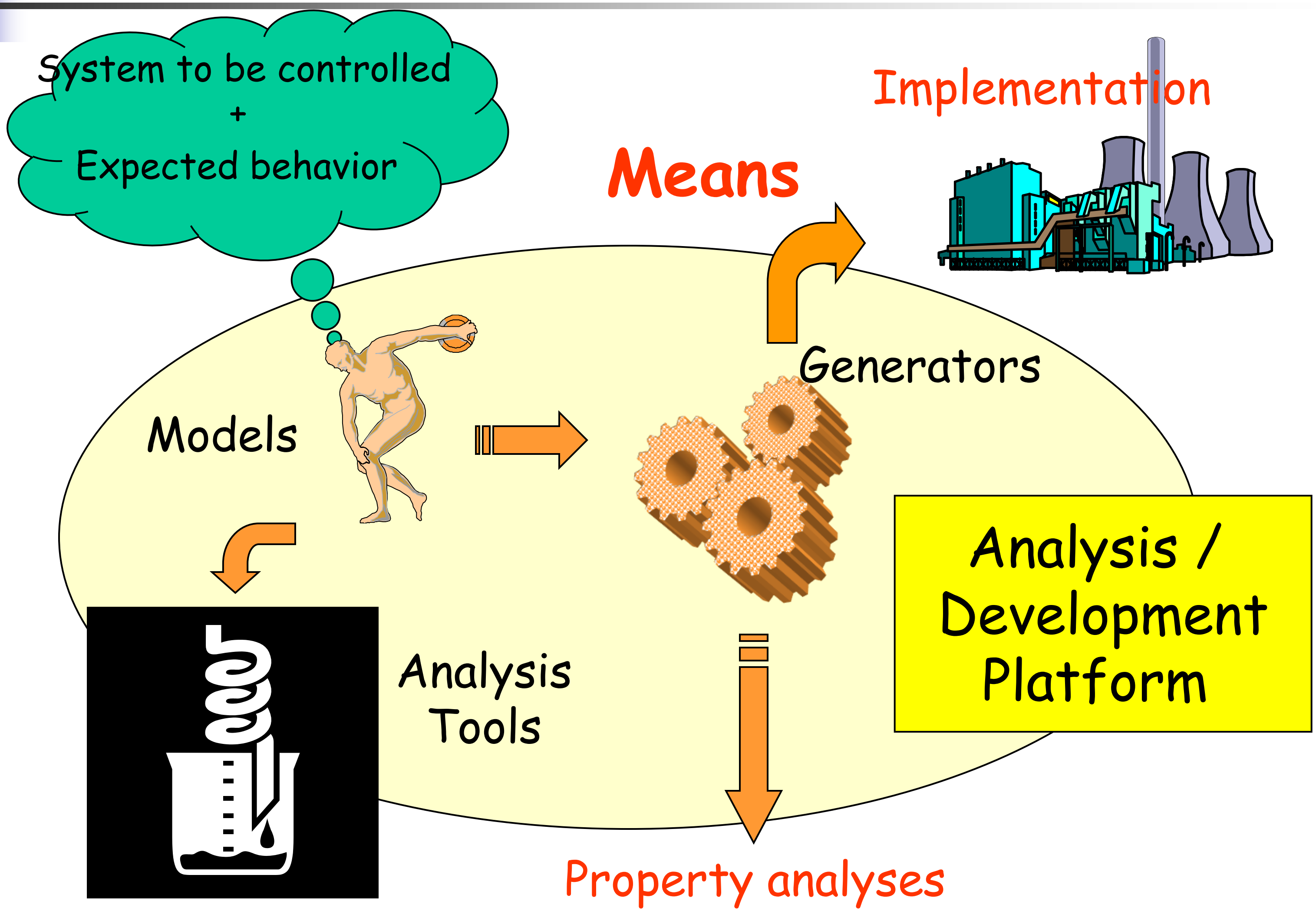


Reactive Program Model Specification

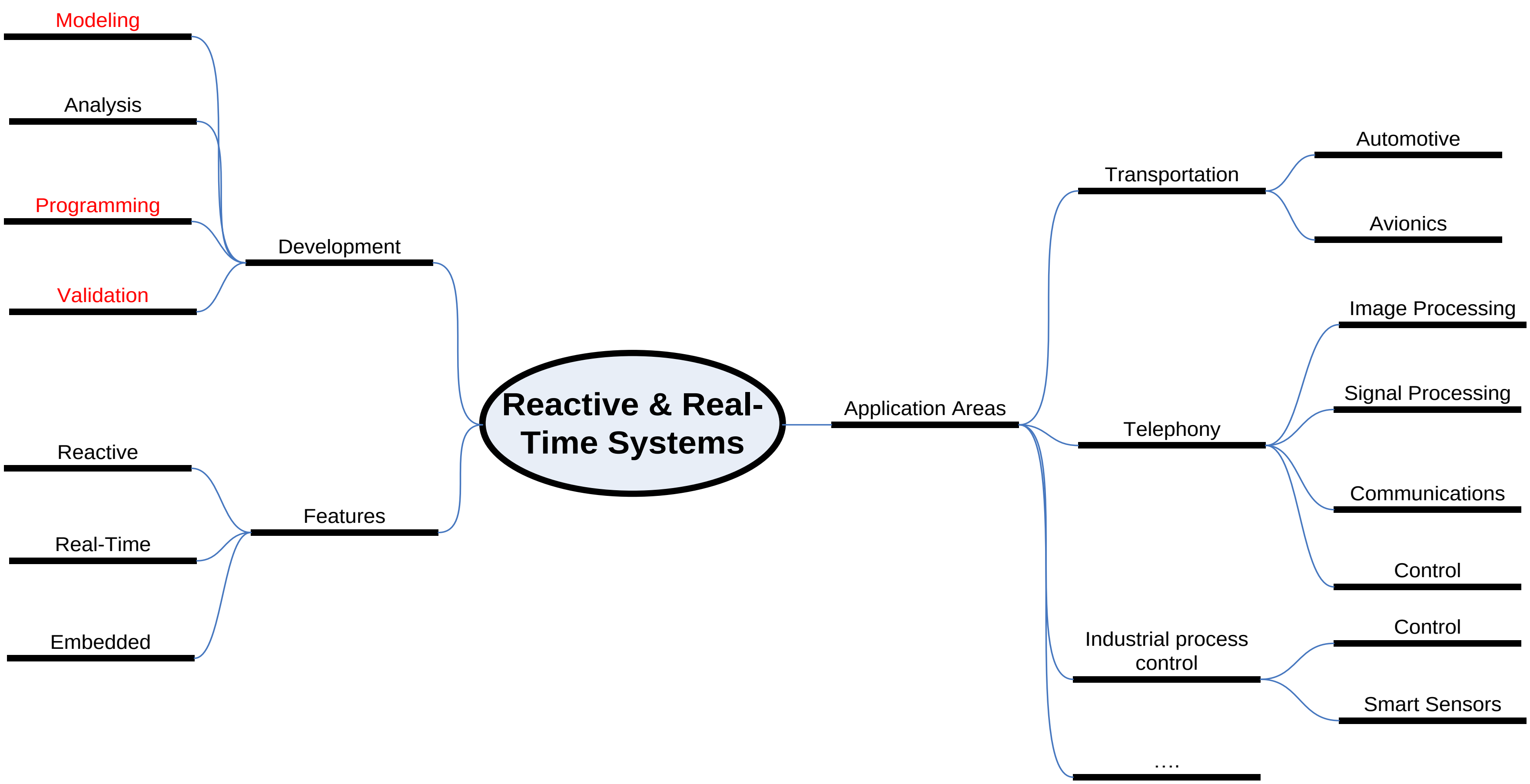
Reactive and Real-Time Systems



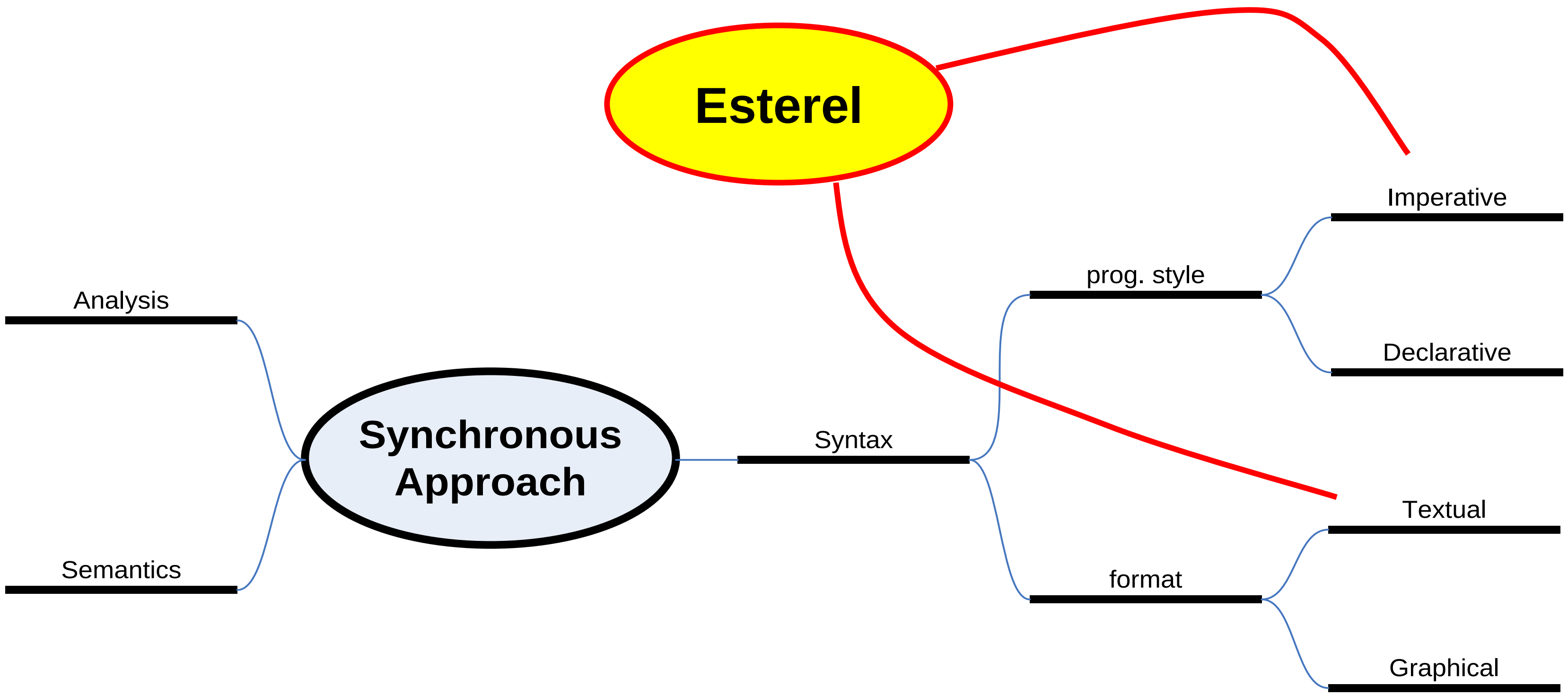
Reactive System Implementation



Reactive & Real-Time Systems



Synchronous Approach to Reactive System Programming



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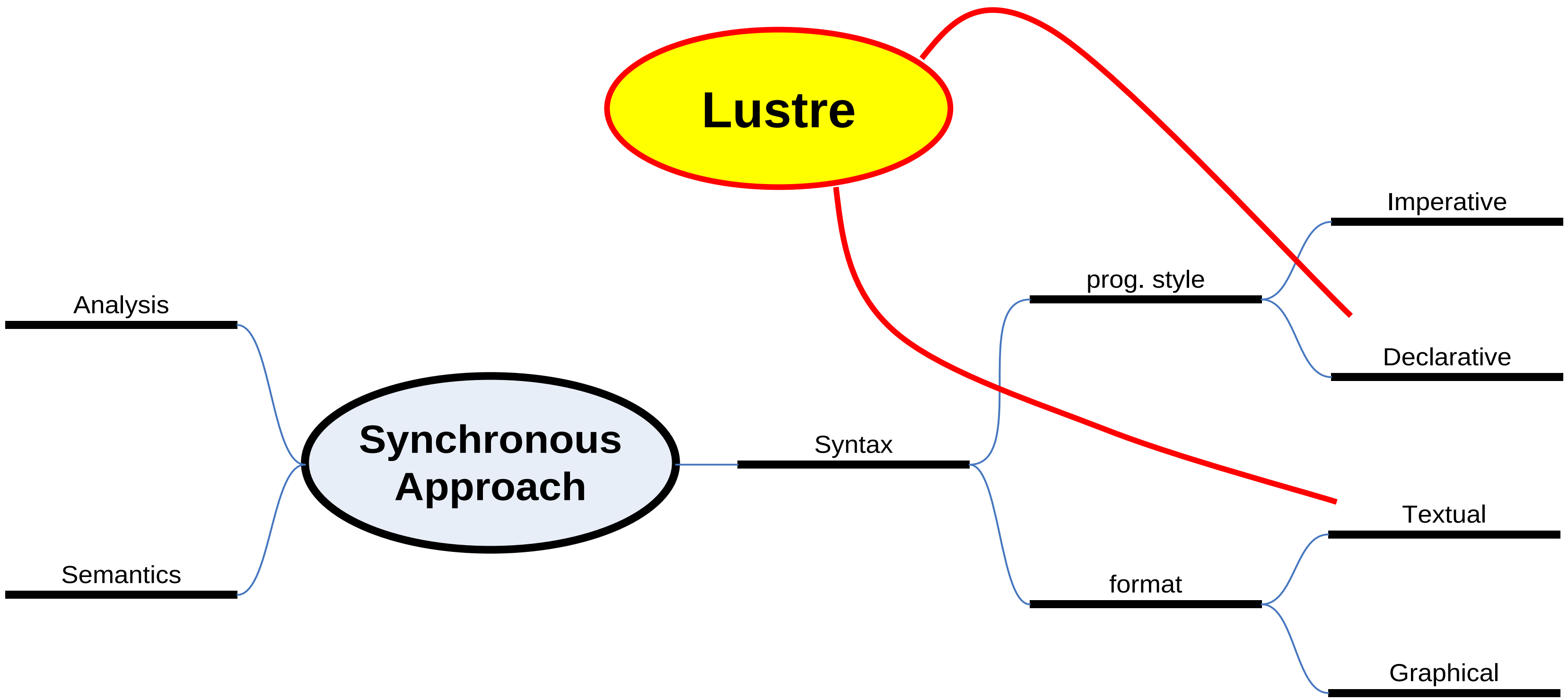

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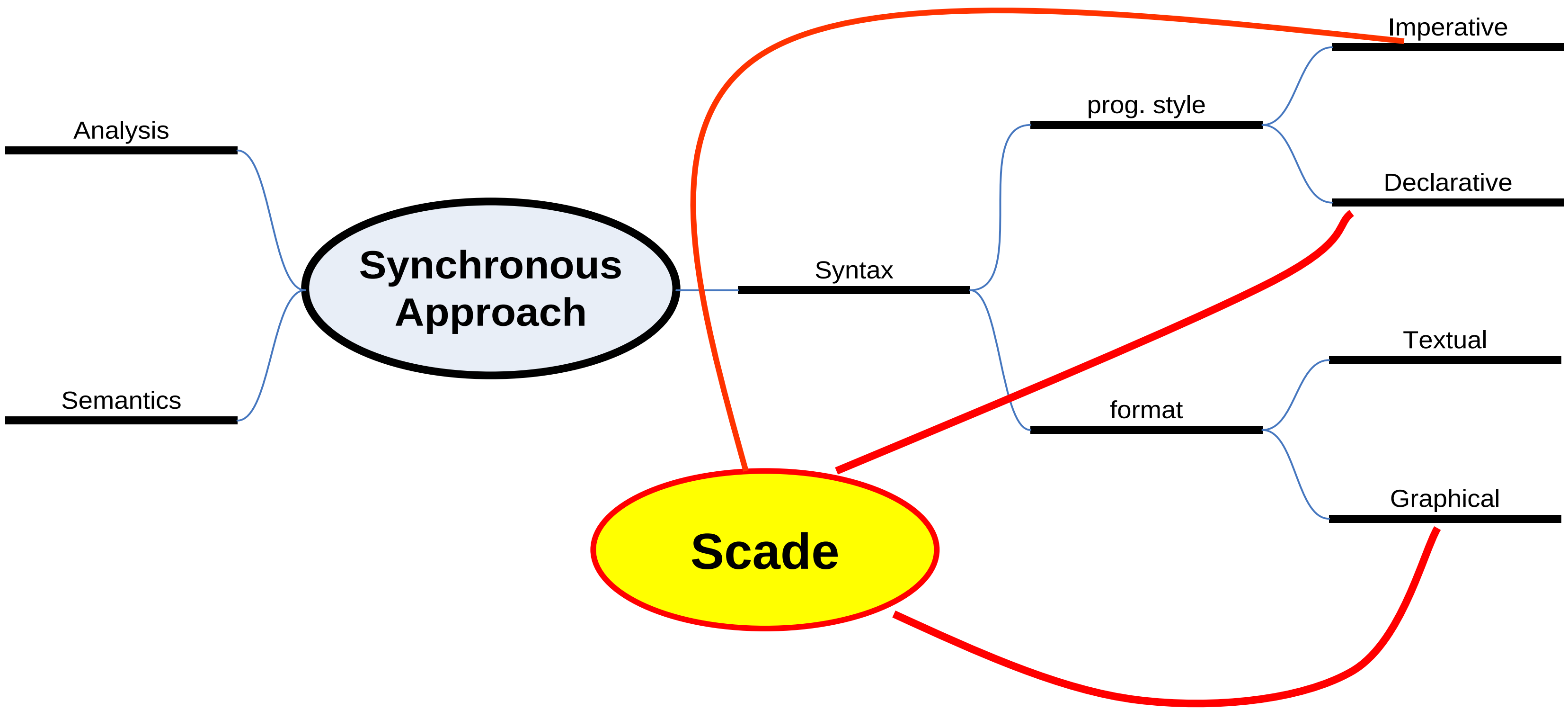

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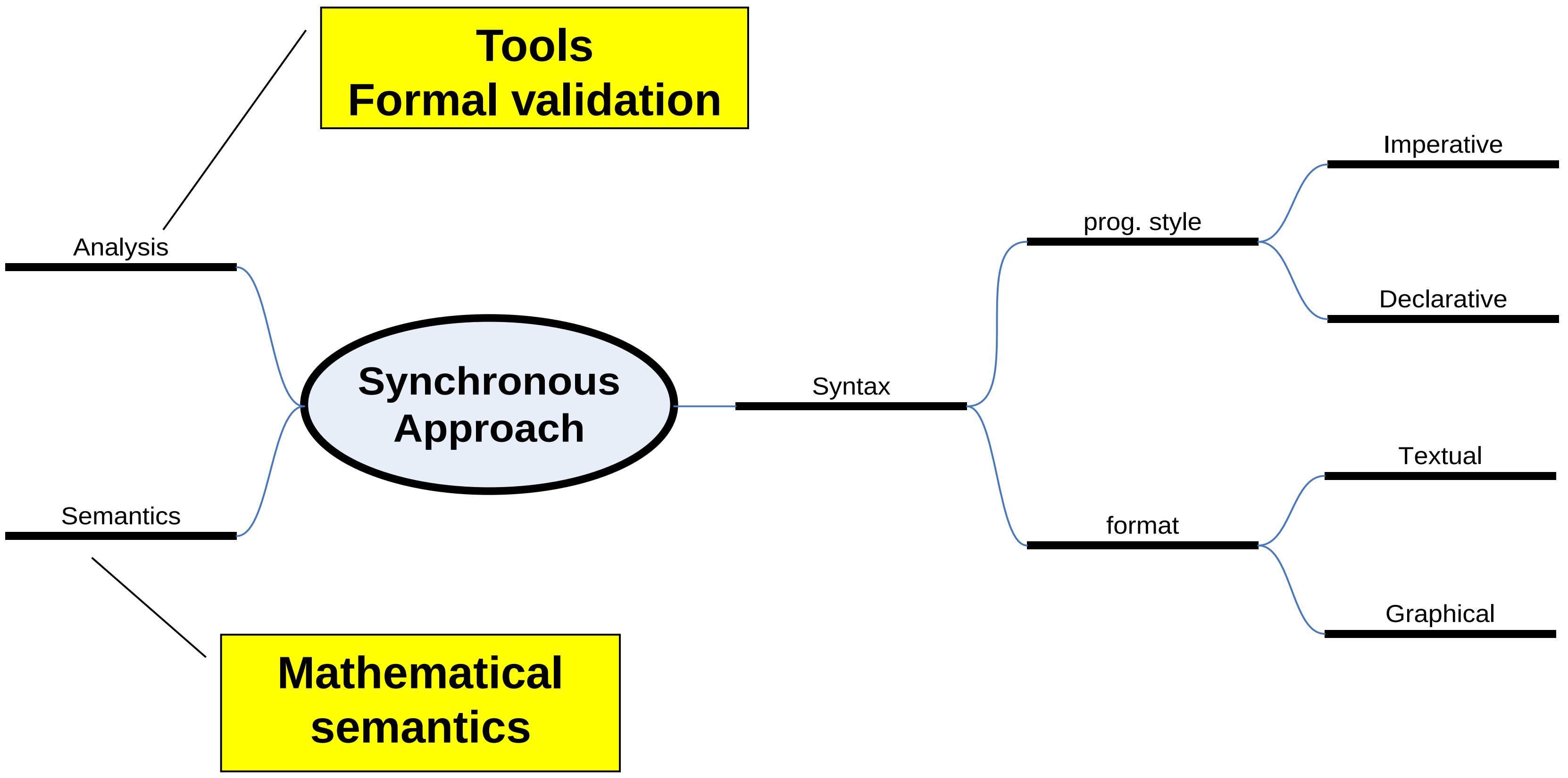

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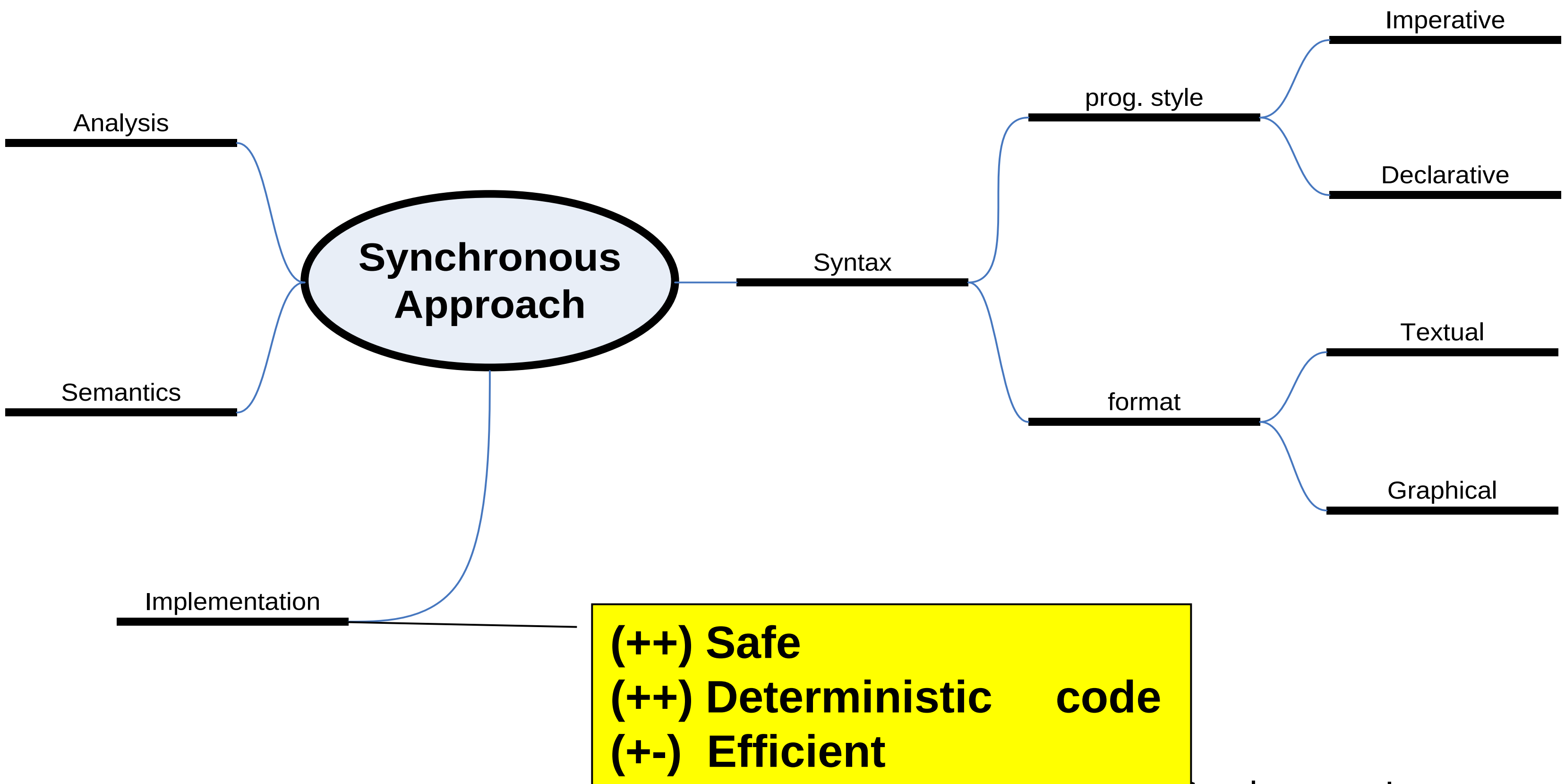


Determinism & Reactivity

- Determinism:

  The same input sequence always yields

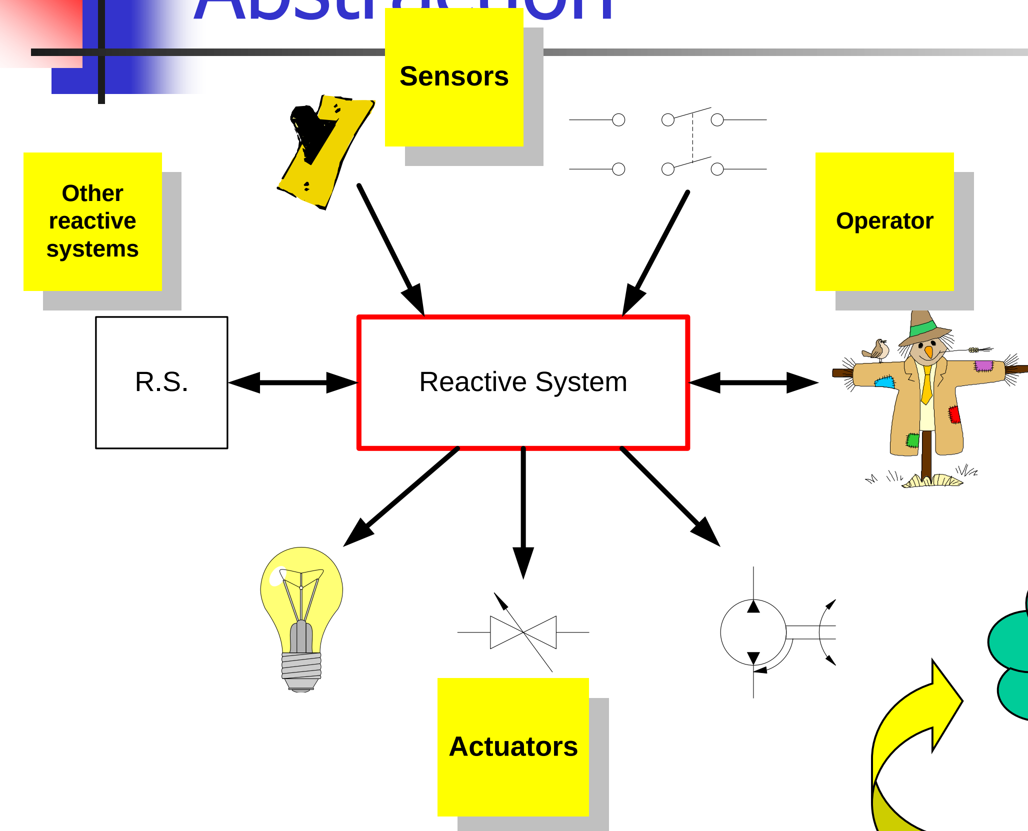
  The same output sequence
- Reactivity:

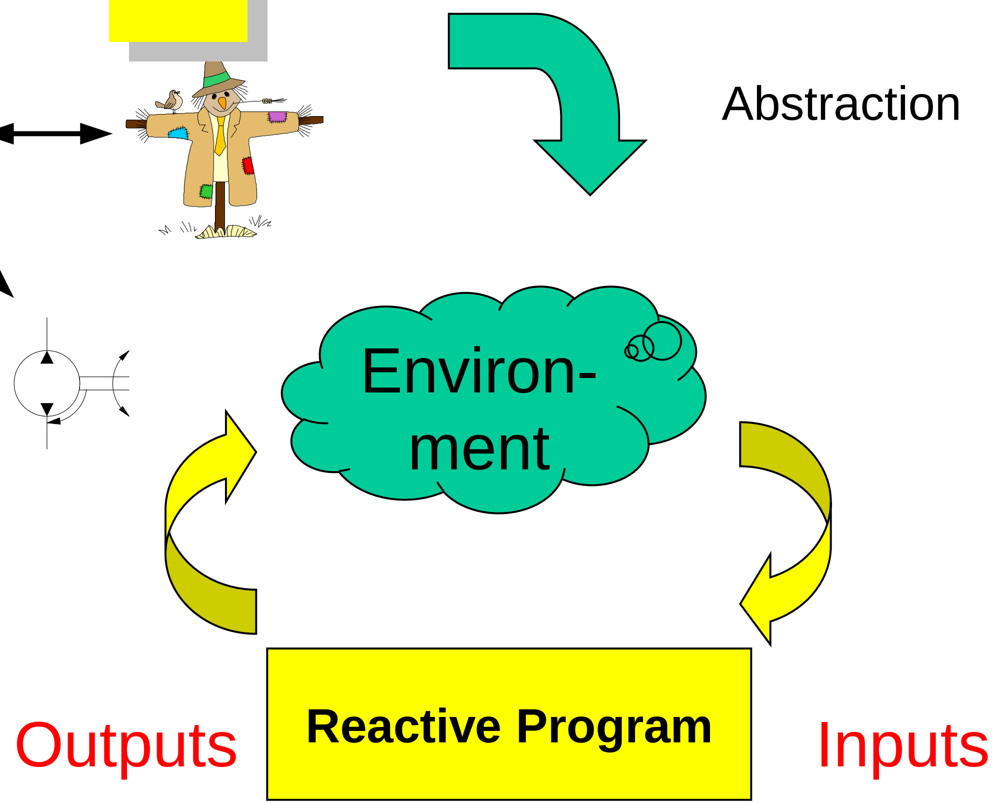
  The program must react[1] to any stimulus

  Implies absence of deadlock

(1) Does not necessary generate outputs, the reaction may change internal state only.

Abstraction





Signals or Flows = Abstraction of Communications

LUSTRE Declarative Synchronous Language

Languages

Declarative languages — Say what IS or what SHOULD BE

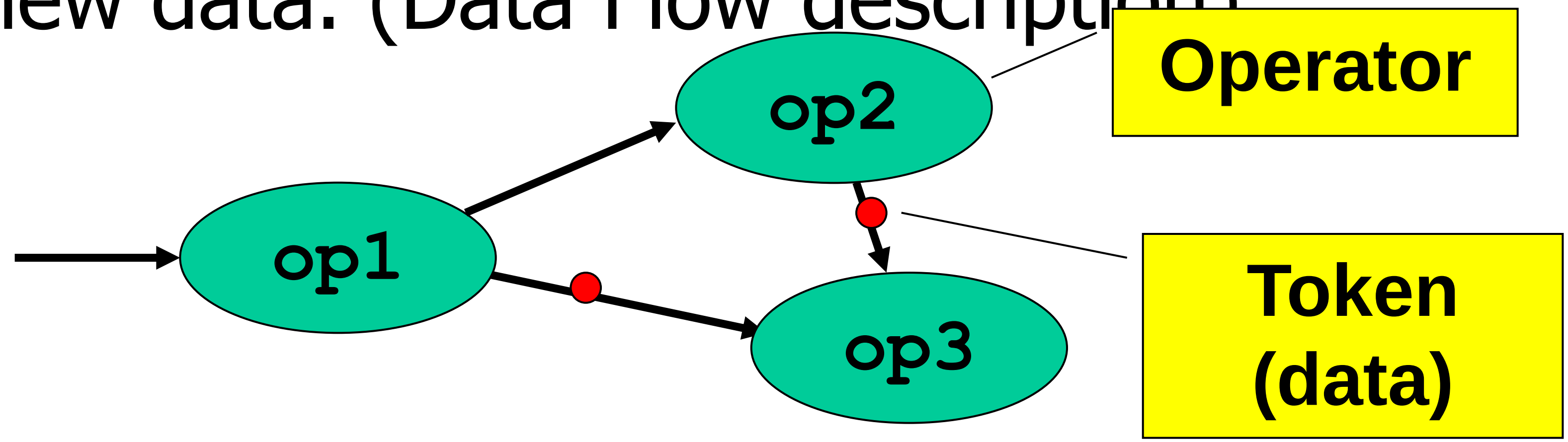
Imperative langages — Say what MUST BE DONE

LUSTRE

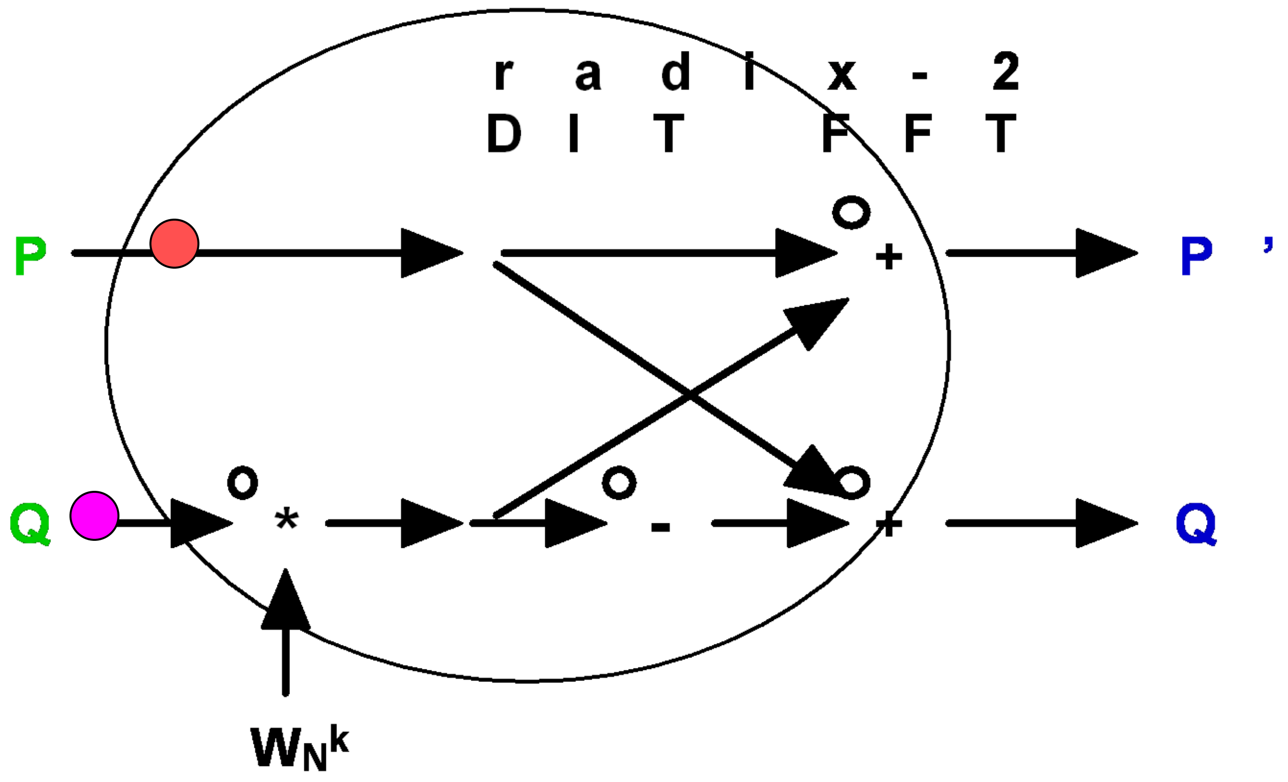
- LUSTRE
  - It is a very simple language (4 primitive operators to express reactions)
  - Relies on models familiar to engineers
    - Equation systems
    - Data flow network
  - Lends itself to formal verification (it is a kind of logical language)
  - Very simple (mathematical) semantics

Operator Networks

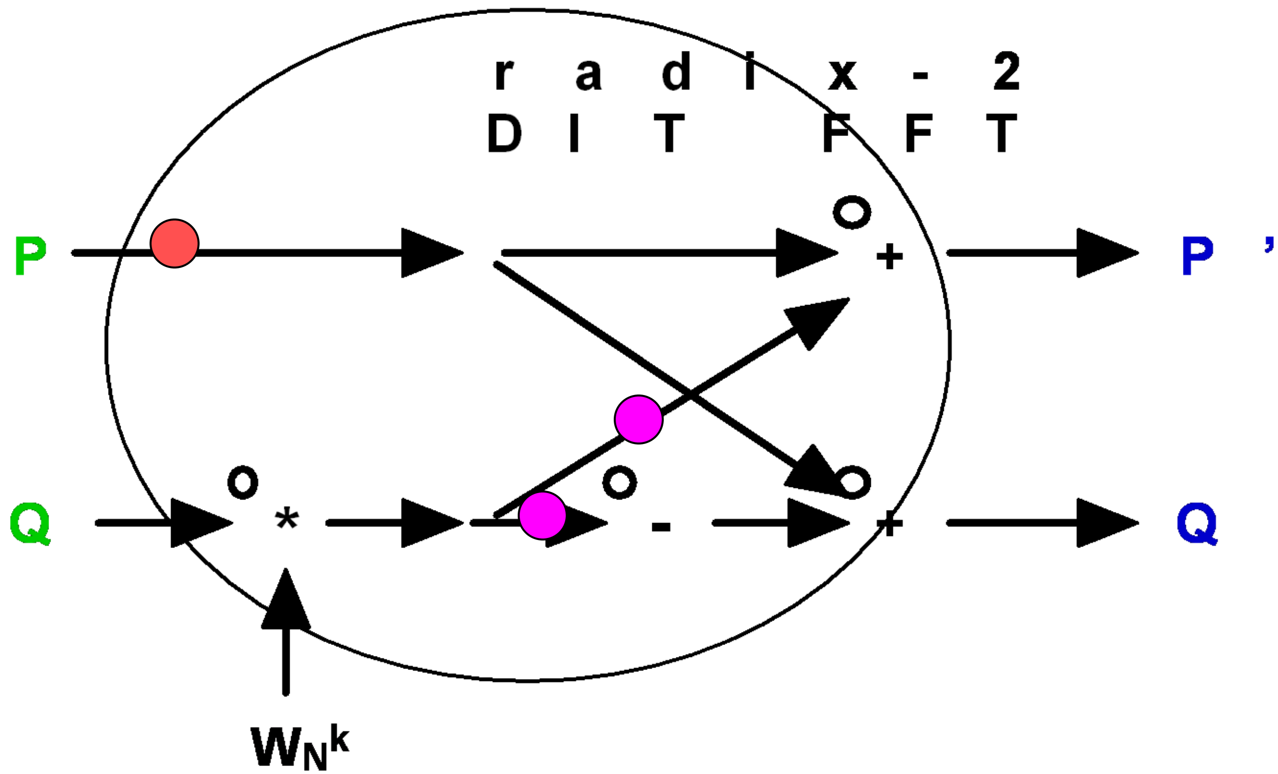
- LUSTRE programs can be interpreted as networks of operators.
- Data « flow » to operators where they are consumed. Then, the operators generate new data. (Data Flow description)



An example of Data Flow



Data Flow



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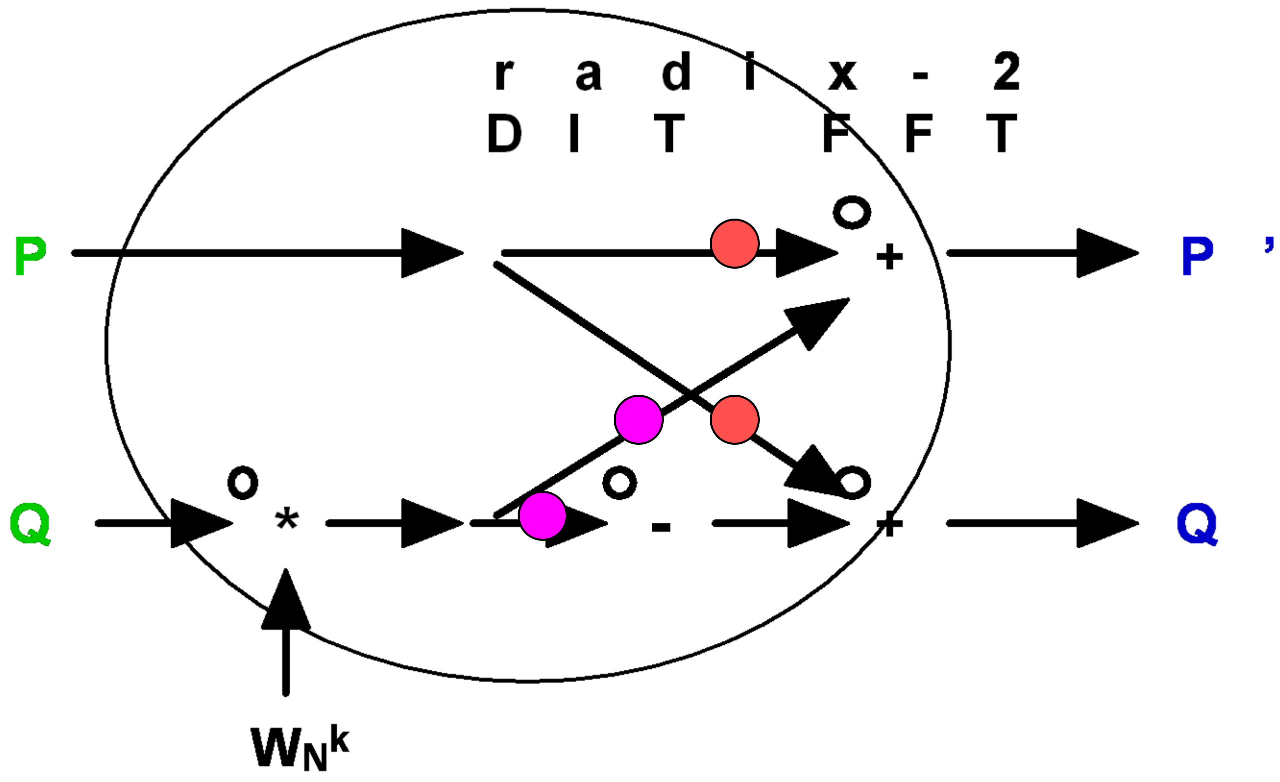
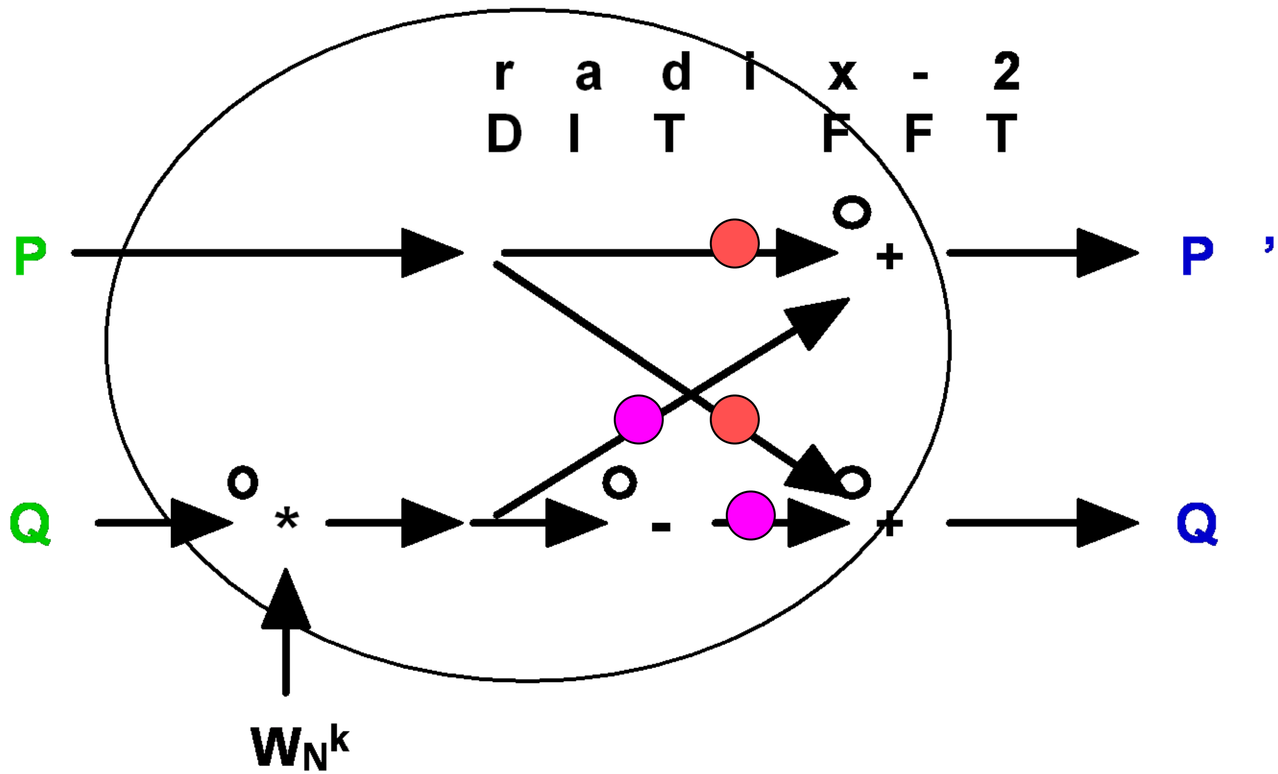
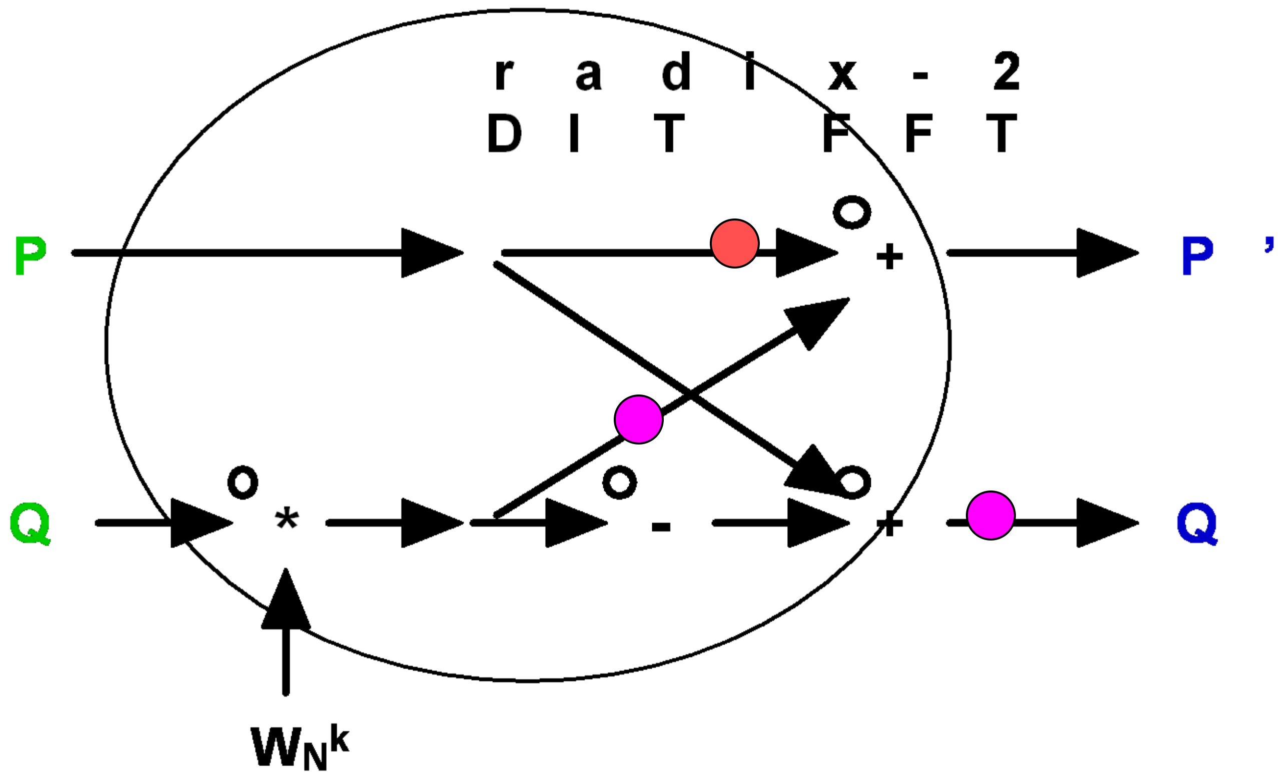


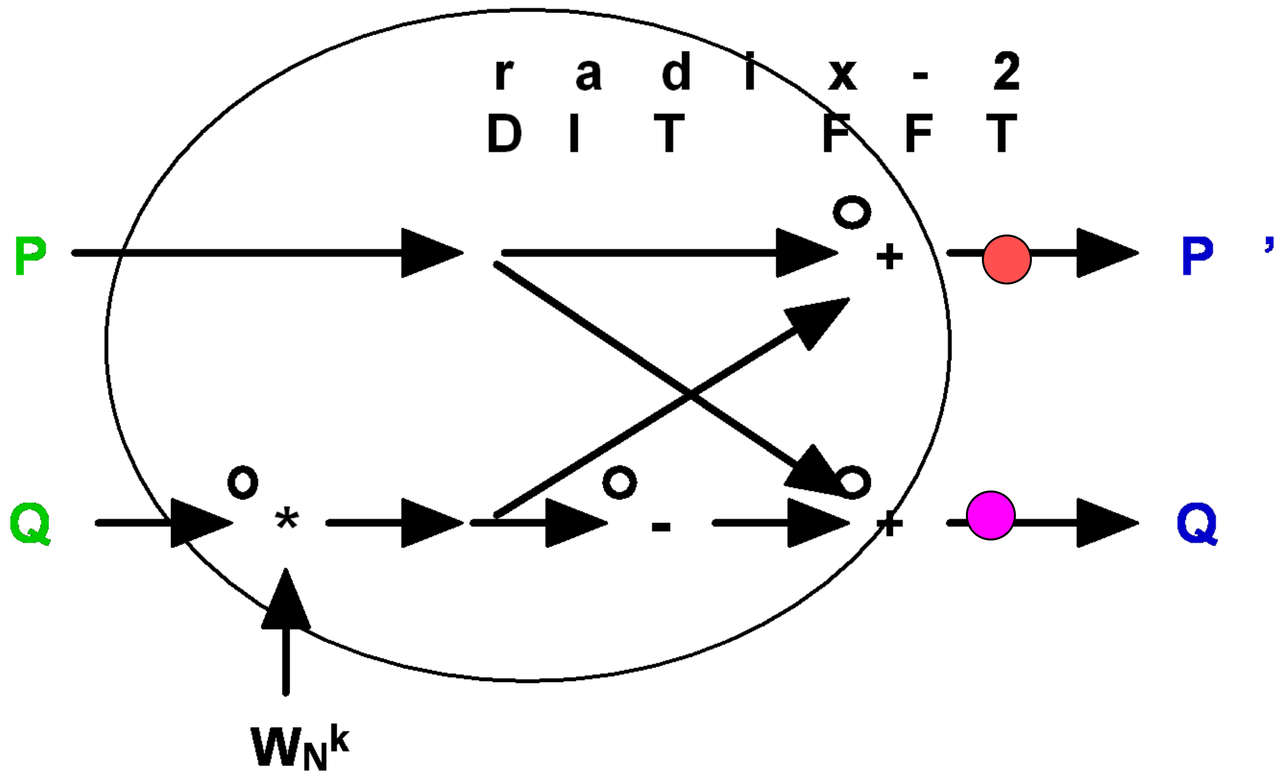
Data Flow



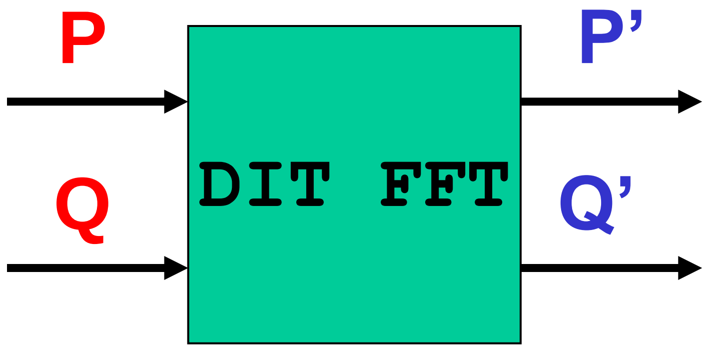
Data Flow



Data Flow



Functional Point of View



$$P' = P + W_N^k * Q$$

$$Q' = P - W_N^k * Q$$

Flows, Clocks

- A flow is a pair made of
  - A possibly infinite sequence of values of a given type
  - A clock representing a sequence of instants

$$\mathbf{X{:}T} \qquad (x_1, x_2, \ldots, x_n, \ldots)$$

Language (1)

Variable :

- typed
- If not an input variable, defined by 1 and only 1 equation
- Predefined types: **`int`, `bool`, `real`**
- tuples: **`(a,b,c)`**

Equation :

`X = E` means $\forall k, x_k = e_k$

Assertion :

Boolean expression that should be always **`true`** at each instant of its clock.

Language (2)

Substitution principle:

if **`X = E`** then **`E`** can be substituted for **`X`** anywhere in the program and conversely

Definition principle:

A variable is fully defined by its declaration and the equation in which it appears as a left-hand side term

Expressions

Constants

`0, 1, …, true, false, …, 1.52, …`

real

int

bool

\+
Imported types and operators

$$c : \alpha \Leftrightarrow \forall k \in \mathbb{N}, c_k = c$$

« Combinational » Lustre

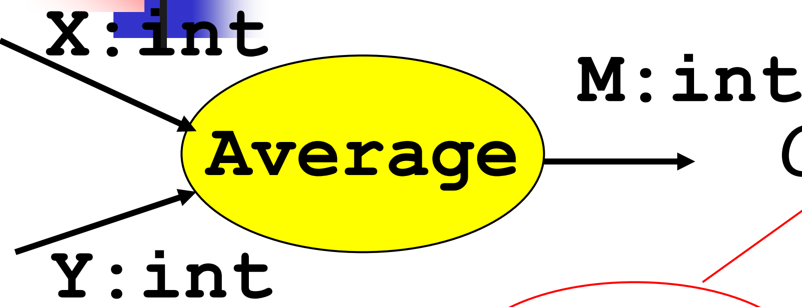
Data operators

Arithmetical: **+**, **−**, *****, **/**, **div**, **mod**
Logical: **and**, **or**, **not**, **xor**, **=>**
Conditional: **if** … **then** … **else** …
Casts: **int**, **real**

« Point-wise » operators

$$X \; op \; Y \Leftrightarrow \forall k, (X \; op \; Y)_k = X_k \; op \; Y_k$$

« Combinational » Example




```
node Average (X,Y:int)
  returns (M:int);
let
  M = (X + Y) / 2;
tel
```

Operator

Flows

Result

Definition

$$\forall k \in \mathbb{N}, M_k = (X_k + Y_k) / 2_k$$

Example (suite)

```
node Average (X,Y:int)
   returns (M:int);
var S:int;  -- local variable
let
   S = X + Y; -- non significant order
   M = S / 2;
tel
```

By substitution, the behavior is the same

« Combinational » Example (2)

- if operator

```
 node Max (a,b : real) returns (m: real)
let
    m = if (a >= b) then a else b;
tel
```

functional «if then else »; it is not a statement

« Combinational » Example (2)

- if operator

```
node Max (a,b : real) returns (m: real)
let
    m = if (a >= b) then a else b;
tel
```

~~let~~
~~if (a >= b) then m = a ;~~
~~else m = b;~~
~~tel~~

Memorizing

Take the past into account!

`pre` (previous):

$$X = (x_1, x_2, \cdots, x_n, \cdots) \ : \ pre(X) = \ \text{nil}, x_1, \cdots, x_{n-1}, \cdots$$

Undefined value denoting uninitialized memory: nil

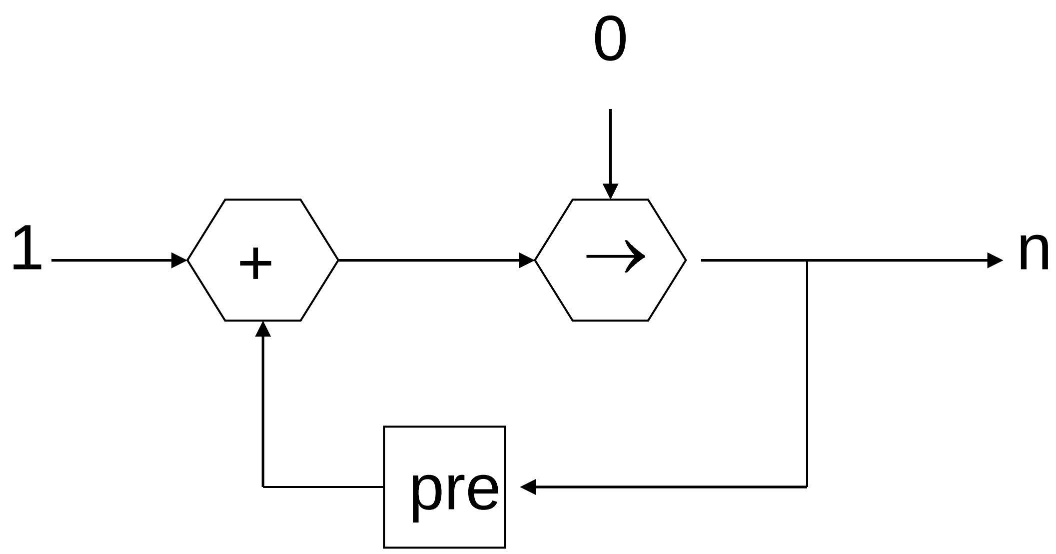
`->` (initialize): sometimes call “followed by”

$$X = (x_1, x_2, \cdots, x_n, \ldots) \ , \ Y = (y_1, y_2, \cdots, y_n, \ldots) \ :$$

$$(X -> Y) = (x_1, y_2, \cdots, y_n, \ldots)$$

« Sequential » Examples

$$n = 0 \rightarrow pre(n) + 1$$



Sequential » Examples

```
node MinMax (X:int) returns (min,max:int);
let
   min = X -> if (X < pre min) then X else
   pre min;
   max = X -> if (X > pre max) then X else
   pre max;
tel
```

« Review » Example

```
node Count (init:int) returns (c:int);
 let c = init -> pre c + 2; tel

 node DoubleCall (even:bool) returns (n:int);
 let
    n = if even then Count(0) else
          Count(1);
 tel
```

Doublecall(ff ff tt tt ff ff tt tt ff) = ?

Recursive definitions

Temporal recursion

Usual. Use **pre** and **->**

e.g.: nat = 1 **->** **pre** nat + 1

Instantaneous recursion

e.g.: X = 1.0 / (2.0 – X)

Forbidden in Lustre, even if a solution exists!

Be carefull with cross-recursion.

Clocks

Basic clock

Discrete time induced by the input sequence

Derived clocks (slower)

when (filter operator):

`E when C` is the sub-sequence of **E** obtained by keeping only the values of indexes e_k for which c_k=true

Examples of clocks

Basic cycles	1	2	3	4	5	6	7	8
C1	true	false	true	true	false	true	false	true
Cycles of C1	1		2	3		4		5
C2	false		true	false		true		true
Cycles of C2			1			2		3

Example of sampling

nat,odd:int

halfBaseClock:bool

nat = 0 -> pre nat +1;

halfBaseClock =

true -> not pre halfBaseClock;

odd = nat **when** **halfBaseClock**;

`nat` is a flow on the basic clock;

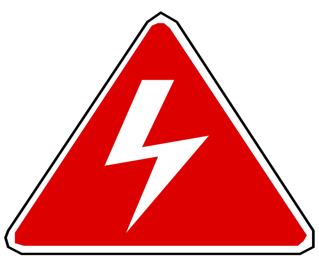
`odd` is a flow on `halfBaseClock`

Exercice: write `even`

Interpolation operator

« converse » of sampling

current (interpolation) :
Let **E** be an expression whose clock is **C**, **current(E)** is an expression on the clock of **C**, and its value at any instant of this clock is the value of **E** at the last time when **c** was **true**.



`current (X when C) ≠ X`

`current` can yield **`nil`**

Example of current

Basic cycles	1	2	3	4	5	6	7
C	ff	tt	ff	tt	ff	ff	tt
X	x1	x2	x3	x4	x5	x6	x7
Y = X when C		x2		x4			x7
Z = current(Y)	nil	x2	x2	x4	x4	x4	x7

Other examples of current

X	1	2	3	4	5	6	7
Y	t	f	t	t	t	f	f
C	t	t	f	t	t	f	t
Z=X when C							
H=Y when C							
T=Z when H							
current T							
current (current T)							

Other examples of current

X	1	2	3	4	5	6	7
Y	t	f	t	t	t	f	f
C	t	t	f	t	t	f	t
Z=X when C	1	2		4	5		7
H=Y when C							
T=Z when H							
current T							
current (current T)							

Other examples of current

X	1	2	3	4	5	6	7
Y	t	f	t	t	t	f	f
C	t	t	f	t	t	f	t
Z=X when C	1	2		4	5		7
H=Y when C	t	f		t	t		f
T=Z when H							
current T							
current (current T)							

Other examples of current

X	1	2	3	4	5	6	7
Y	t	f	t	t	t	f	f
C	t	t	f	t	t	f	t
Z=X when C	1	2		4	5		7
H=Y when C	t	f		t	t		f
T=Z when H	1			4	5		
current T							
current (current T)							

Other examples of current

X	1	2	3	4	5	6	7
Y	t	f	t	t	t	f	f
C	t	t	f	t	t	f	t
Z=X when C	1	2		4	5		7
H=Y when C	t	f		t	t		f
T=Z when H	1			4	5		
current T	1	1		4	5		5
current (current T)							

Other examples of current

X	1	2	3	4	5	6	7
Y	t	f	t	t	t	f	f
C	t	t	f	t	t	f	t
Z=X when C	1	2		4	5		7
H=Y when C	t	f		t	t		f
T=Z when H	1			4	5		
current T	1	1		4	5		5
current (current T)	1	1	1	4	5	5	5

The initialization issue

Y=current(X when C) where C_1 = false is erroneous.

Possible solutions:

- Strict discipline: ensure that C_1 is always true.
- Force the clock to true at the first instant:
  CC = true -> C; Y=current(X when CC);
- Provide a default value D :
  Y = if C then current(X when C) else
  D -> pre Y;

First programs

Bistable

- Node Switch (on,off:bool) returns (s:bool); such that:
  - S raises (false to true) if on, and falls (true to false) if off
  - must work even off and on are the same

```
node Switch (on,off:bool) returns (s:bool)
let
   s = if (false→pre s) then not off else on;
tel
```

Count

- A node Count (reset, x: bool) returns (c:int) such that:
  - c is reset to 0 if reset, otherwise it is incremented if x

```
node Count (reset, x: bool) returns (c:int)
let
  c = if reset then 0
      else if x then (0 -> pre c) + 1
      else (0 -> pre c)
tel
```

Counters

```
node COUNTER (init, incr:int; reset:bool)
        returns (n:int);
let
   n = init -> if reset then init else
                    pre(n) + incr;
tel;
```

C	tt	ff	tt	tt	ff	ff	tt
COUNTER(0,2,false)	0	2	4	6	8	10	12
COUNTER((0,2,false)when C)	0		2	4			6
COUNTER(0,2,false)when C	0		4	6			12

Edges

```
node Edge (b:bool) returns (f:bool);
-- detection of a rising edge
let
    f = false -> (b and not pre(b));
tel;
```

initial

Undefined at the first instant

```
Falling_Edge = Edge(not c);
```

A Stopwatch

- 1 integer output : time
- 3 input buttons: on_off, reset, freeze
  - on_off starts and stops the watch
  - reset resets the stopwatch (if not running)
  - freeze freezes the displayed time (if running)
- Local variables
  - running, freezed : bool (Switch instances)
  - cpt : int (Count instance)

A stopwatch

```
node Stopwatch (on_off, reset, freeze: bool)
                        returns (time:int)
 var running, freezed: bool; cpt:int
let
  running = Switch(on_off, on_off);
  freezed = Switch(freeze and running,
                        freeze or on_off);
  cpt = Count (reset and not running, running);
  time = if freezed then (0 -> pre time) else cpt;
tel
```

A Stopwatch with Clocks

```
node Stopwatch (on_off, reset, freeze: bool)
                          returns (time:int)
var running, freezed : bool;
      cpt_clock, time_clock : bool;
      (cpt : int) when cpt_clock;
let
   running = Switch(on_off, on_off);
   freezed = Switch ( freeze and running,
                                    freeze or on_off);
   cpt_clock = true -> reset or running;
   cpt = Count ((not running, true) when cpt_clock);
   time_clock = true -> not freezed;
   time = current(current(cpt) when time_clock);
tel
```

Modulo Counter

```
node Counter (incr:bool, modulo : int)
                    returns (cpt:int)

  let
     cpt = 0 -> if incr
                then MOD(pre (cpt) +1, modulo)
                else pre (cpt);
  tel
```

Modulo Counter with Clock

```
node ModuloCounter (incr:bool, modulo : int)
                    returns (cpt:int,
                             modulo_clock: bool)
  let
    cpt = 0 -> if incr
                    then MOD(pre (cpt) +1, modulo)
                    else pre (cpt);
    modulo_clock = false ->
                       pre(cpt) <> MOD(pre(cpt)+1);
  tel
```

Timer

```
node Timer (dummy:bool)
            returns (hour, minute, second:bool)
var hour_clock, minute_clock, day_clock;
let
  (second, minute_clock) = ModuloCounter(true, 60);
  (minute, hour_clock) =
                   ModuloCounter(minute_clock,60);
  (hour, day_clock) =
                   ModuloCounter(hour_clock, 24);
tel
```

Numerical Examples

- Integrator node:
  - f : real function and Y its integrated value using the trapezoid method:
  - F, STEP : 2 real such that:

$$F_n = f(x_n) \text{ and } x_{n+1} = x_n + STEP_{n+1}$$

$$Y_{n+1} = Y_n + (F_n + F_{n+1}) * STEP_{n+1}/2$$

Numerical Examples

```
node integrator (F, STEP, init : real)
        returns (Y : real);
let
  Y =  init ->pre(Y) + ((F + pre(F))*STEP)/2.0
tel
```

Numerical Examples

```
node sincos (omega : real)
     returns (sin, cos : real);
let
  sin = omega * integrator(cos, 0.1, 0.0);
  cos = 1 – omega * integrator(sin, 0.1, 0.0);
tel
```

Numerical Examples

```
node sincos (omega : real)
     returns (sin, cos : real);
let
  sin = omega * integrator(cos, 0.1, 0.0);
  cos = 1 – omega * integrator(    , 0.1, 0.0);
tel
```

(0.0 ->pre(sin))

Safety and Liveness Properties

- Example: the beacon counter in a train:
  - Count the difference between beacons and seconds
  - Decide when the train is ontime, late, early

```
node train (sec, bea : bool) returns (ontime, early, late: bool)
  let
      diff = (0 ->pre diff) + (if bea then 1 else 0) + (if sec then -1 else 0);
      early = (true -> pre ontime) and (diff > 3) or
                  (false -> pre early) and (diff > 1);
      late  = (true -> pre ontime) and (diff < -3)  or
                  (false -> pre late) and (diff < -1);
      ontime = not (early or late);
  tel
```

Train Safety Properties

- It is impossible to be late and early;
  - ok = not (late and early)
- It is impossible to directly pass from late to early;
  - ok = true -> (not early and pre late);
- It is impossible to remain late only one instant;
  - Plate = false -> pre late;
    PPlate = false -> pre Plate;
    ok = not (not late and Plate and not PPlate);

Train Assumptions

- property = assumption + observer: *" if the train keeps the right speed, it remains on time"*
- observer = ok = ontime
- assumption:
  - naïve: assume = (bea = sec);
  - more precise : bea and sec alternate:
    - SF = Switch (sec and not bea, bea and not sec);
      BF = Switch (bea and not sec, sec and not bea);
      assume = (SF => not sec) and (BF => not bea);

Model Checking with observers

Observers in Scade

P: aircraft autopilot and security system