3D Triangulations in CGAL

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Contents of the package

To appear in next (?) public release

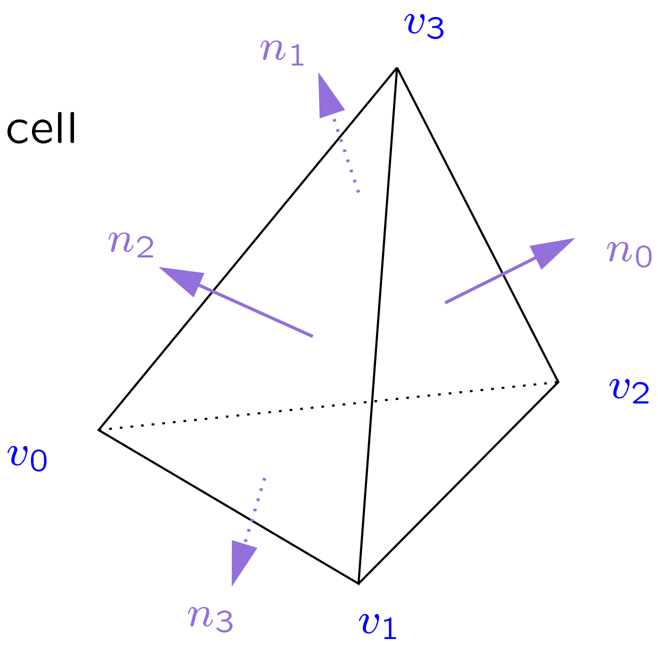
- Incremental triangulations
- Incremental Delaunay triangulations
- Delaunay hierarchy [Devillers]

Representation

Based on cells and vertices

- **Cell :**
  four vertices
  four neighbors

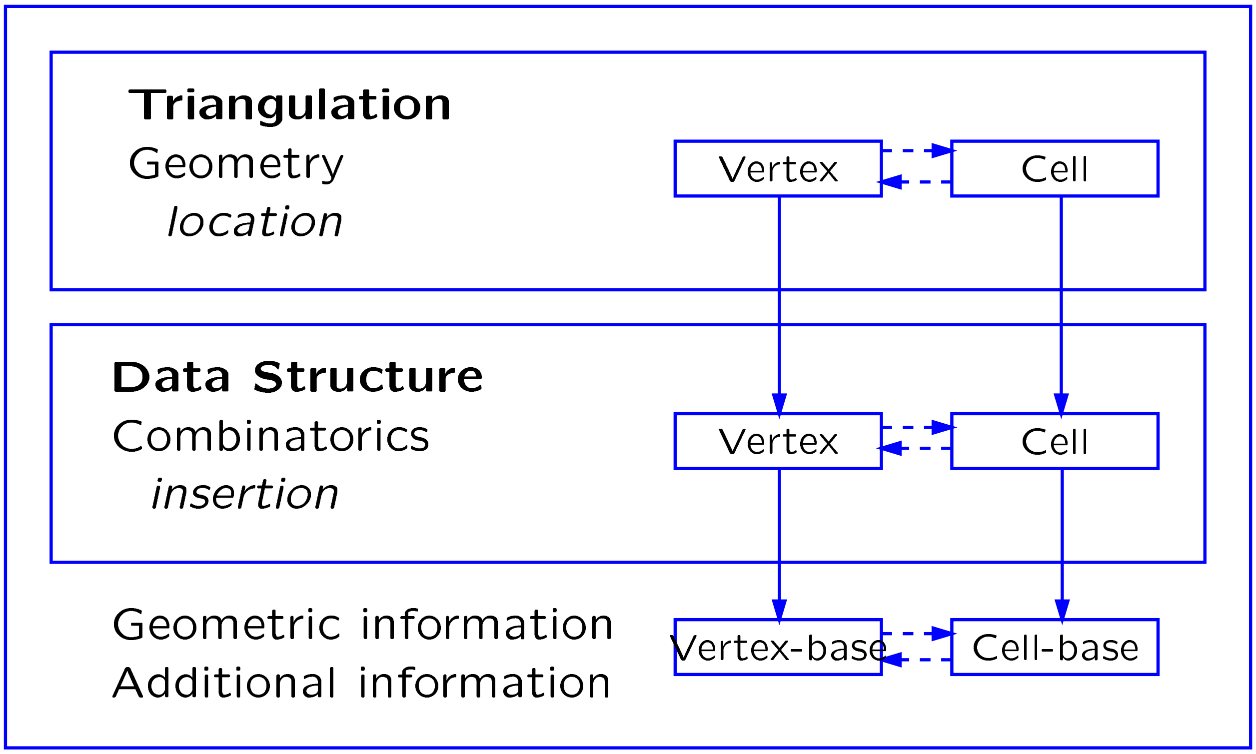
- **Vertex :**
  one incident cell



Adjacency relations
Incidence relations

Design Overview

Three layers



Traits class

```
CGAL_Triangulation_3< TDS, Traits >
```

Geometric definitions

- Objects : points, segments, triangles, tetrahedra. . .
- Predicates :
  orientation
  in_circle, in_sphere
  . . .

Traits class parameterized:

Cartesian or homogeneous coordinates

Number type

Geometric triangulation

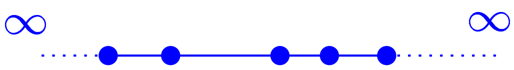
Triangulation data structure

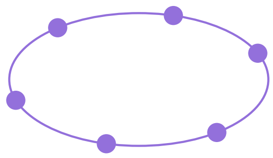
Triangulation of $\mathbb{R}^d$ $\rightleftarrows$ Triangulation of a sphere S^d in $\mathbb{R}^{d+1}$

$d = -1$ ∞

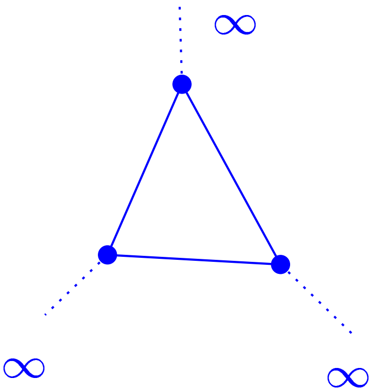
$d = 0$ ∞

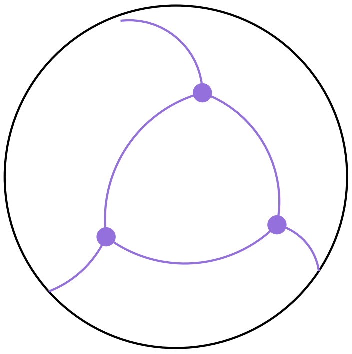
$d = 1$



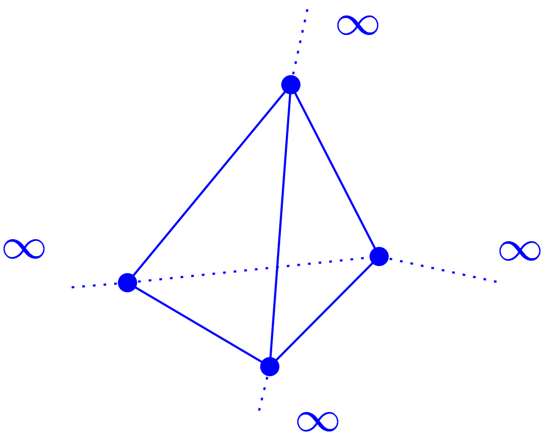


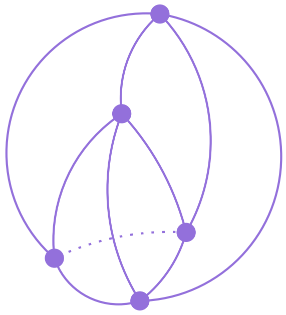
$d = 2$





$d = 3$

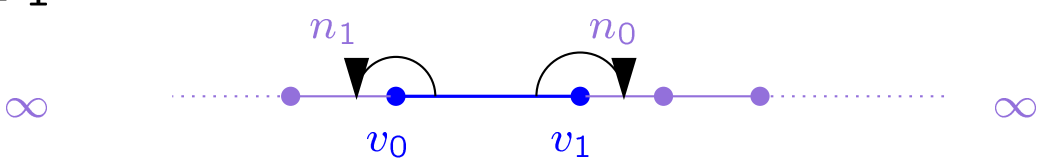




in $\mathbb{R}^4 \ldots$

Degenerate dimensions

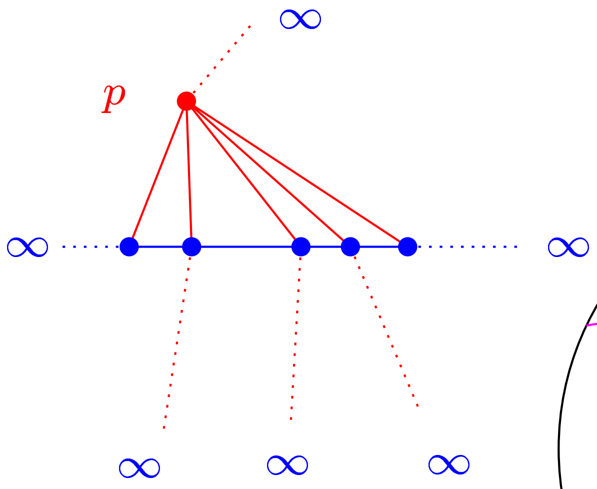
- $d = 1$



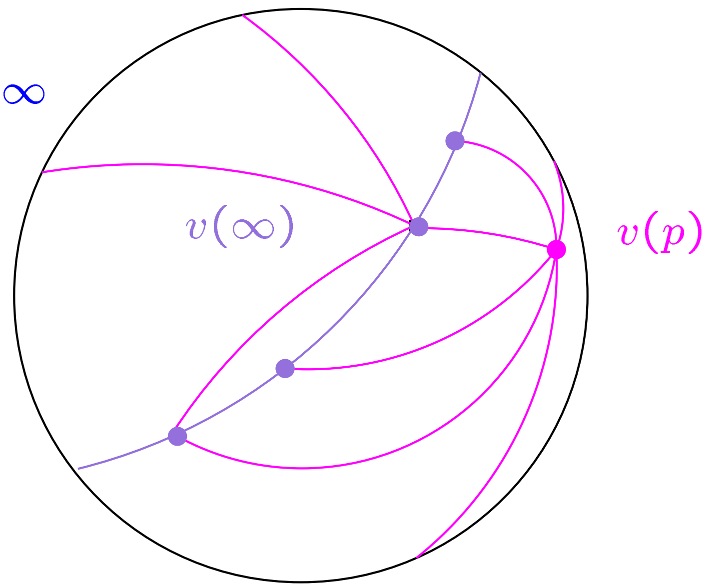
vertices and neighbors 2 and 3 not used

- from $d = 1$ to $d = 2$

Triangulation

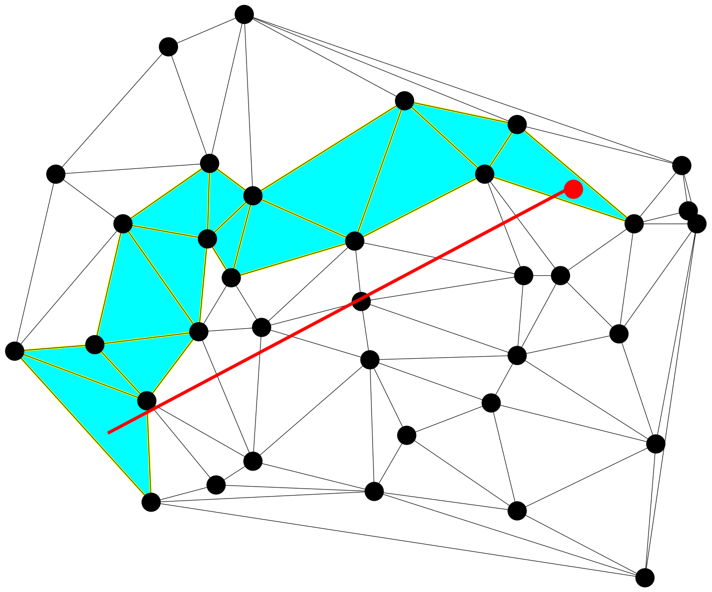


Triangulation data structure

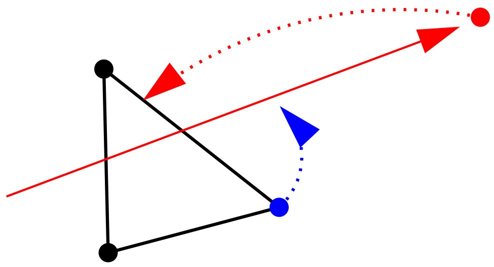


Algorithmic choices

Example : basic point location

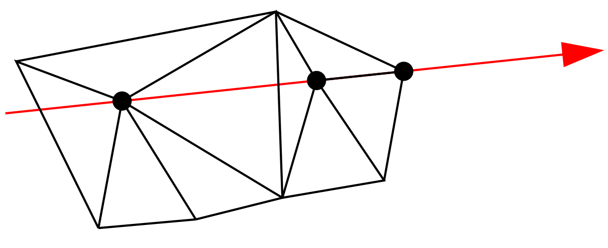


- traversing the triangulation along a line :

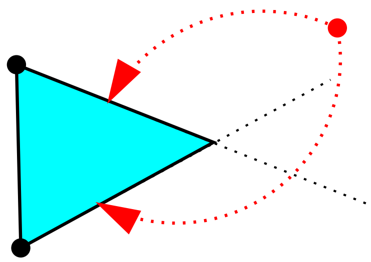


2 orientation tests per triangle
3 per tetrahedron in 3D

degenerate cases :

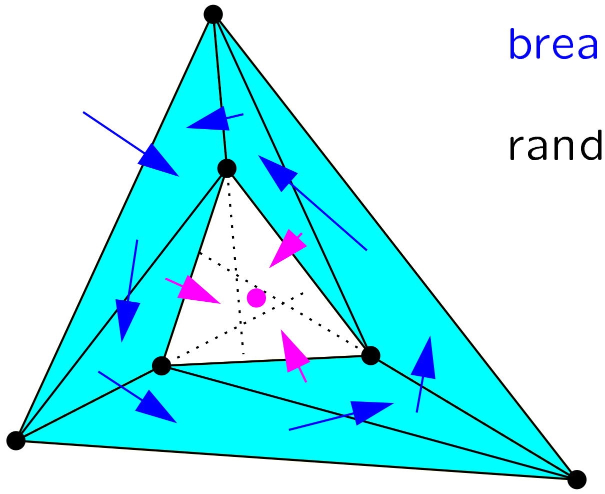


■ by orientation :



< 1.5 tests per triangle

< 2 per tetrahedron in 3D



breaking cycles :

random choice of the neighbor

→ Number of tetrahedra traversed by each method ?

Future

- (Benchmarks, optimization,. . .)

- Deletion of a vertex
- . . .

- Regular triangulations
- Constrained triangulations
- Mesh generation
- . . .

Demo

- by François Rebufat
  (also the test-suite for 3D triangulations)

  3D-visualization : MESA (OpenGL)

- Future : CGAL on the web
  `http://cgal.inria.fr/`