

# Multirésolution en Modélisation Géométrique

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- Ondelettes de Haar
- Cadre général de l'Analyse MR
- Courbes de subdivision, ex. de courbes MR
- Surfaces de subdivision
- Courbes MR
- Surfaces MR

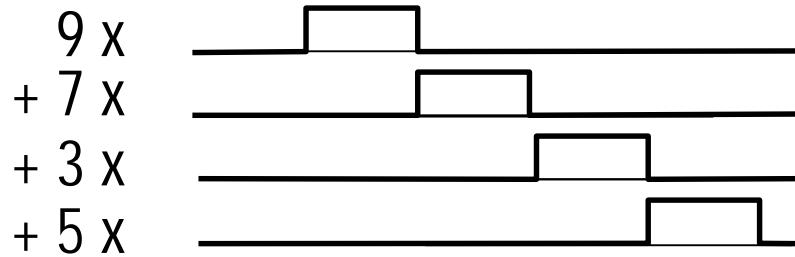
# Ondelettes de Haar: Analyse/Synthèse

↑  
Synthèse/Reconstruction  
↓

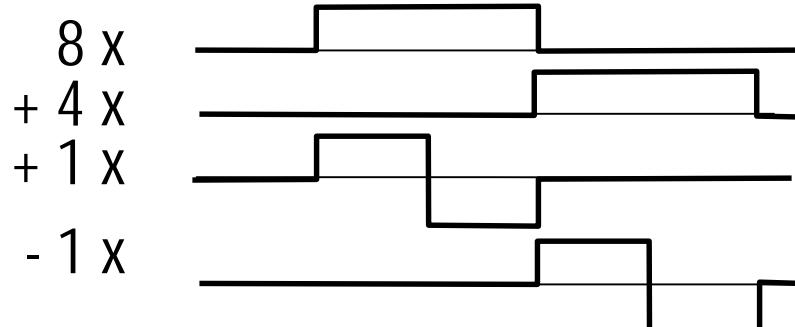
Analyse/Décomposition

Données de départ:	9	7	3	5
Moyennage	8		4	
Différence avec la moyenne	1		-1	
Moyennage			6	
Différence avec la moyenne			2	
Résolution	Approximation			Détails
2	9	7	3	5
1	8	4		1 -1
0	6			2
Données transformées:	6	2	1	-1

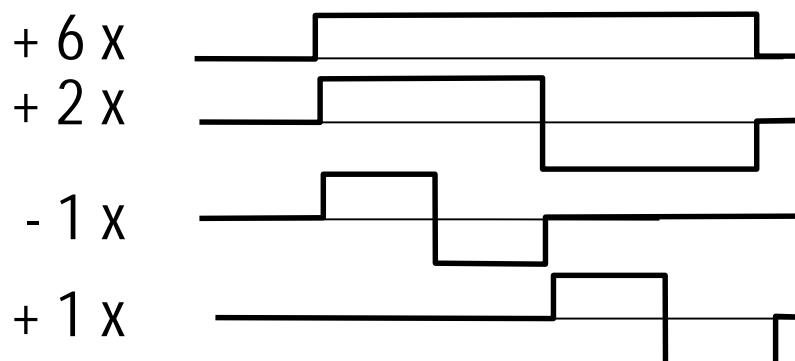
# Ondelettes de Haar: fonctions de base



$$9 \times \varphi_0^2 + 7 \times \varphi_1^2 + 3 \times \varphi_2^2 + 5 \times \varphi_3^2$$

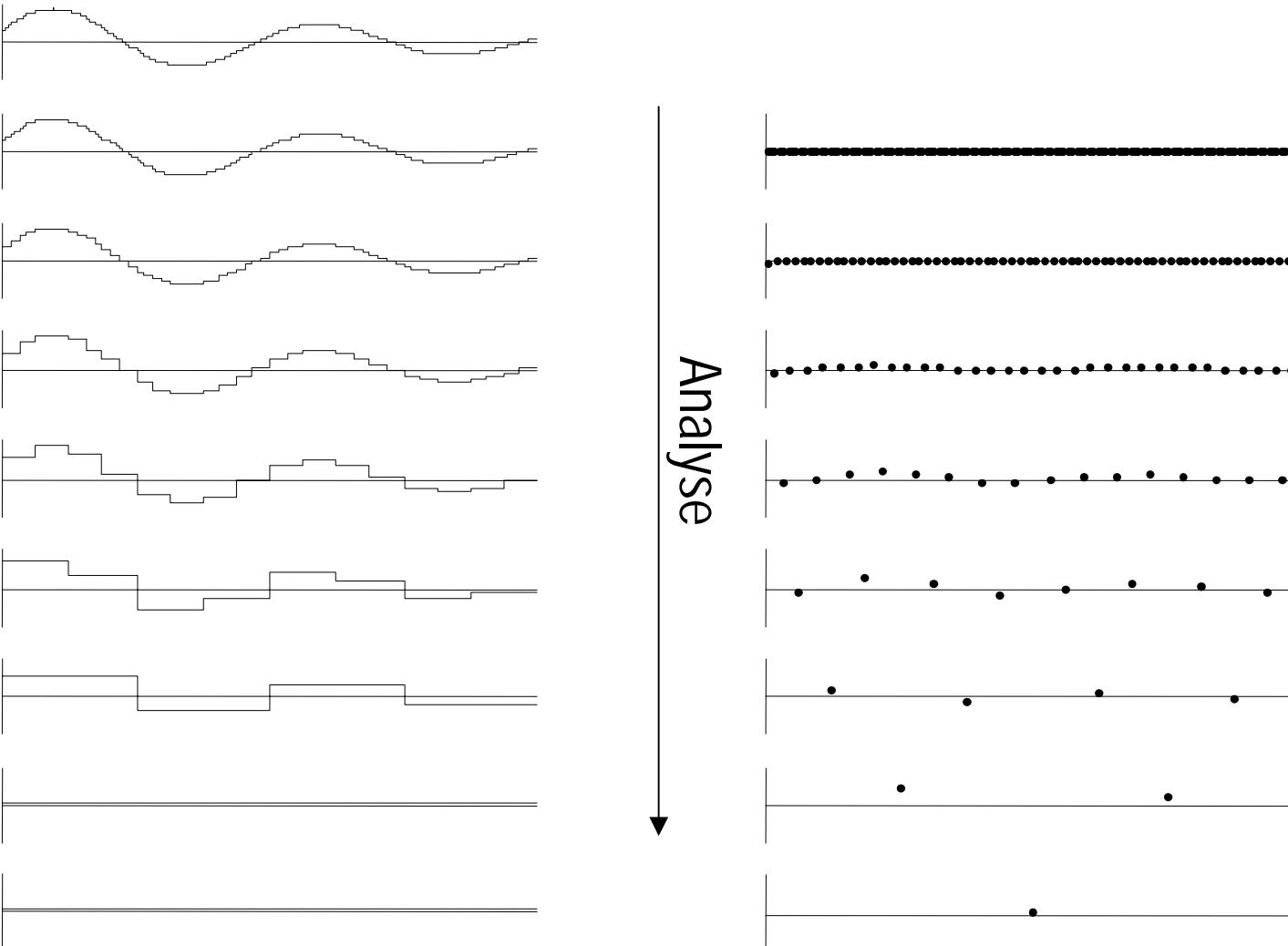


$$8 \times \varphi_0^1 + 4 \times \varphi_1^1 + 1 \times \psi_0^1 - 1 \times \psi_1^1$$

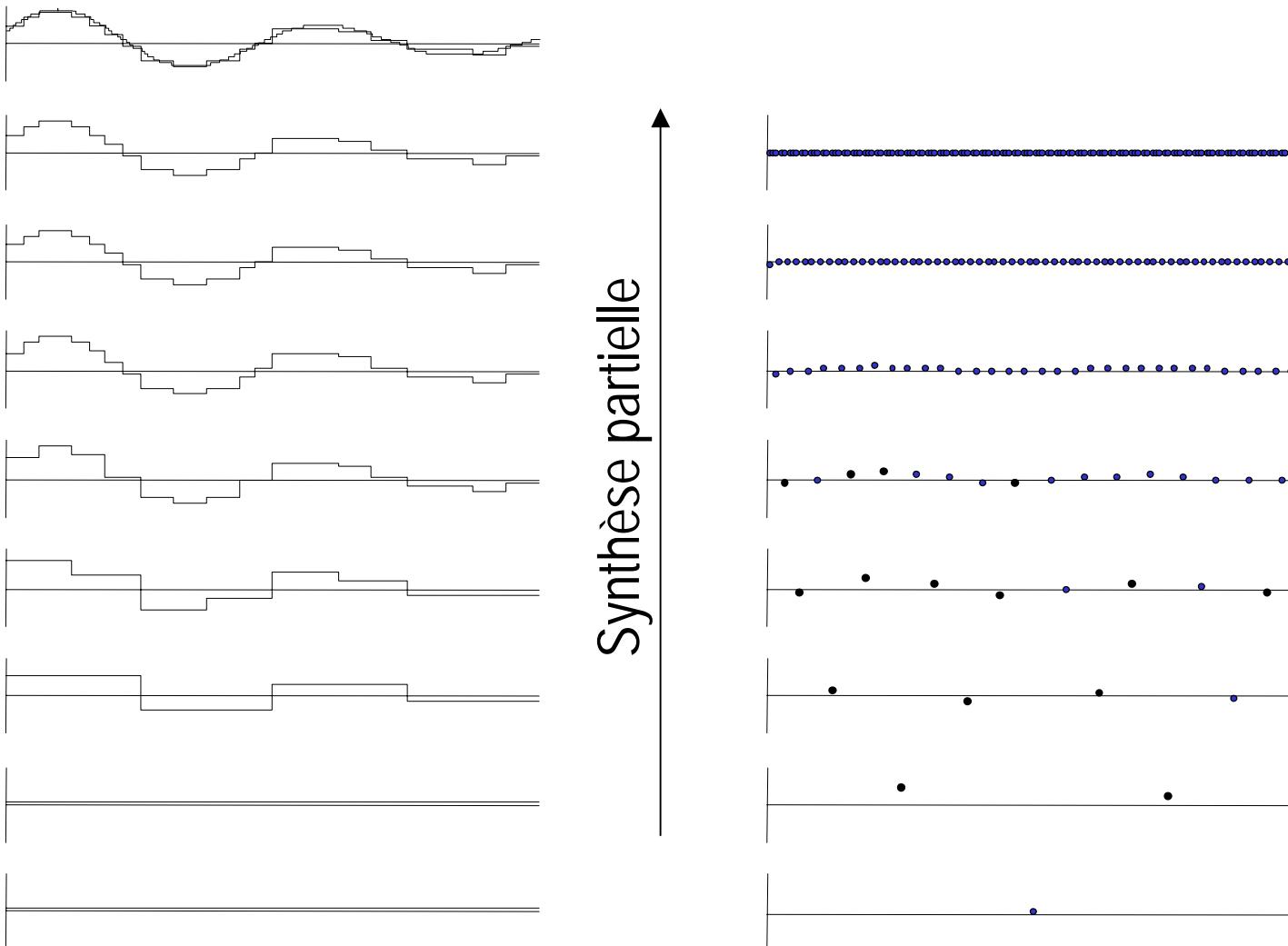


$$6 \times \varphi_0^0 + 2 \times \psi_0^0 + 1 \times \psi_0^1 - 1 \times \psi_1^1$$

# Ondelettes de Haar: Approximation



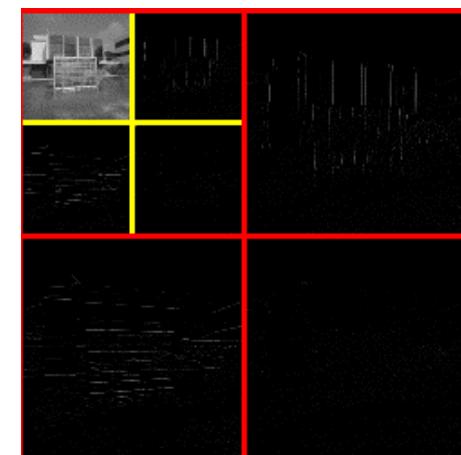
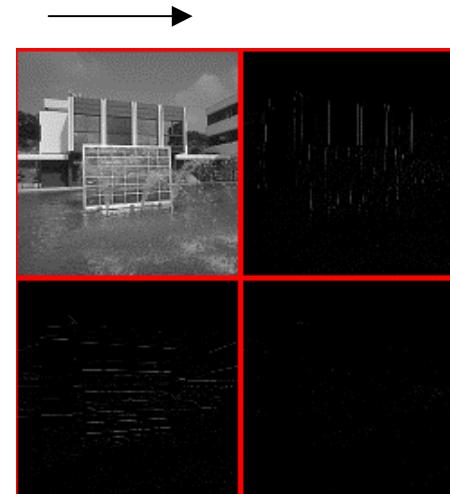
# Ondelettes de Haar: Approximation



# Ondelettes de Haar: Produit Tensoriel

Analyse sur les lignes

Analyse sur les colonnes



# Reconstruction progressive

99.9 %



99 %



95 %



90 %



Original



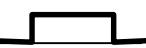
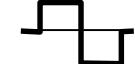
# Cadre général de l'Analyse MR

- Espace de fonctions  $E$
- Sous-espaces d'approximation  $V^n$
- $V^n \subset V^{n+1}$
- Espaces de détail  $W^n$

$$V^{n+1} = V^n \oplus W^n$$

Espaces

Fonctions de base

- Fonctions d'échelle  $(\varphi_i^n)$  
- Fonctions ondelettes  $(\psi_i^n)$  

# Équations de changement d'échelle

- Liens entre les fonctions échelles et les fonctions ondelettes de résolution successives

$$\left( \varphi_i^n \right) = \left[ {}^t P^n \right] \left( \varphi_i^{n+1} \right)$$

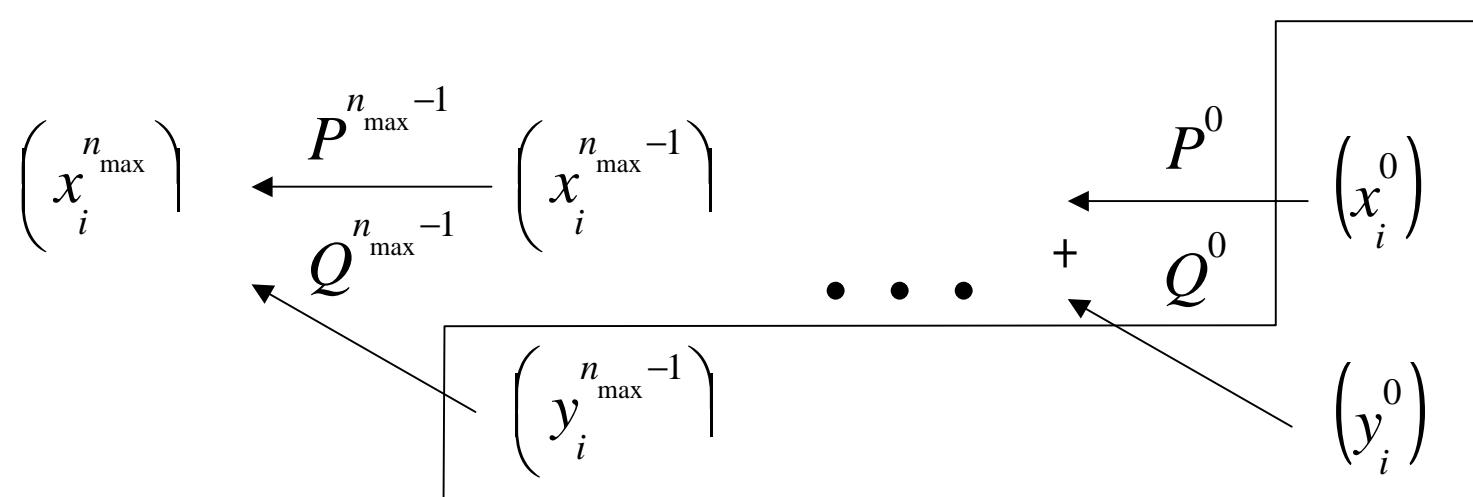
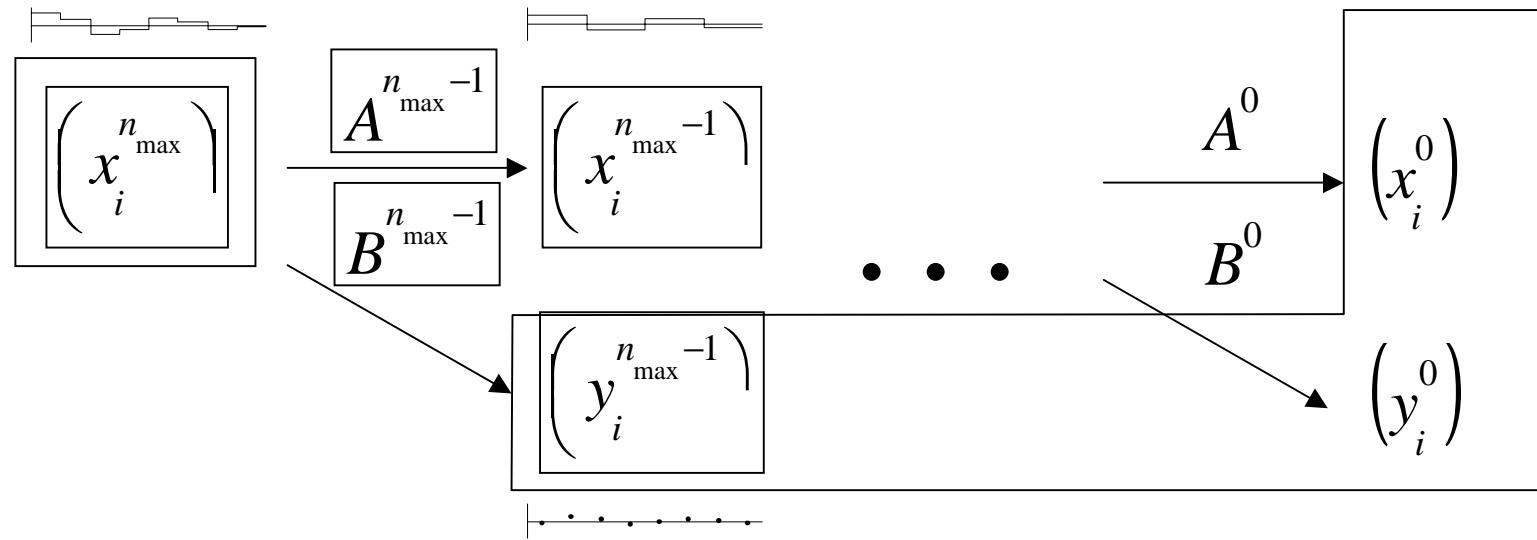
$$\left( \psi_i^n \right) = \left[ {}^t Q^n \right] \left( \varphi_i^{n+1} \right)$$

$$\left( \varphi_i^{n+1} \right) = \begin{bmatrix} {}^t A^n \\ {}^t B^n \end{bmatrix} \left( \varphi_i^n \right) + \left( \psi_i^n \right)$$

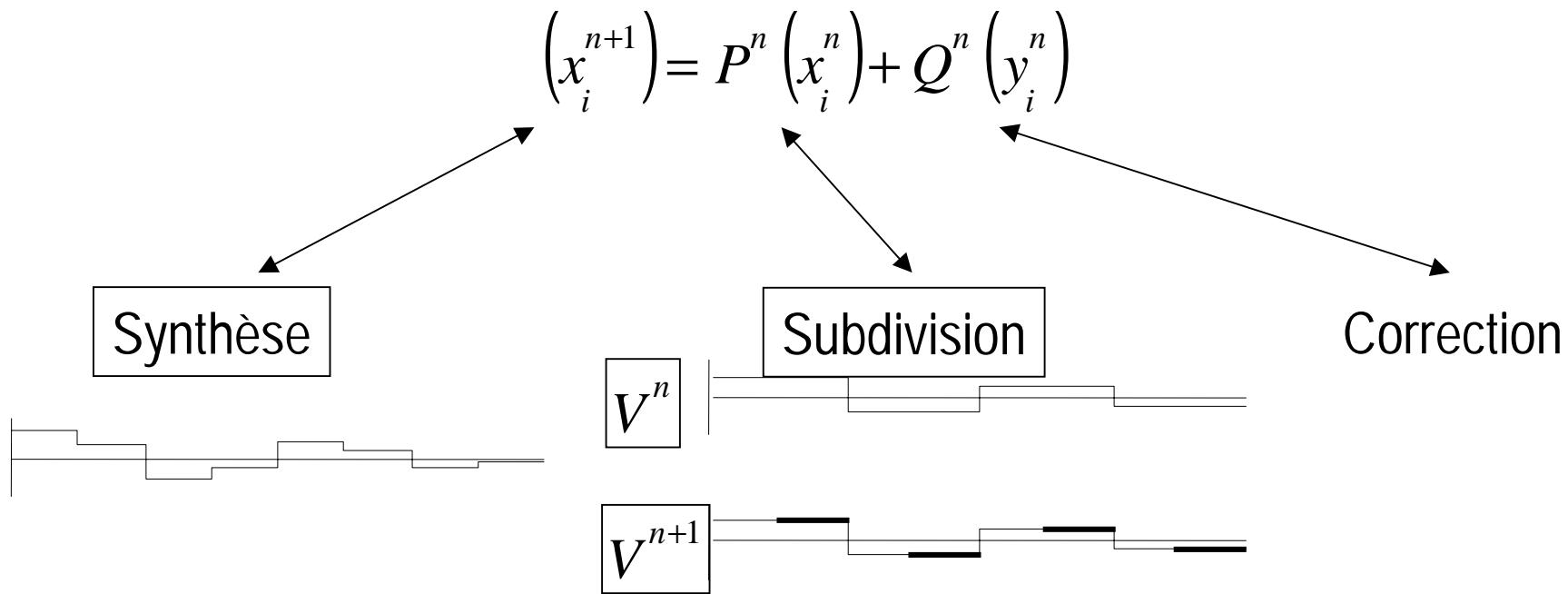
- Condition de reconstruction

$$\left[ P^n Q^n \right] \begin{bmatrix} {}^t A^n \\ {}^t B^n \end{bmatrix} = I$$

# Analyse/Synthèse



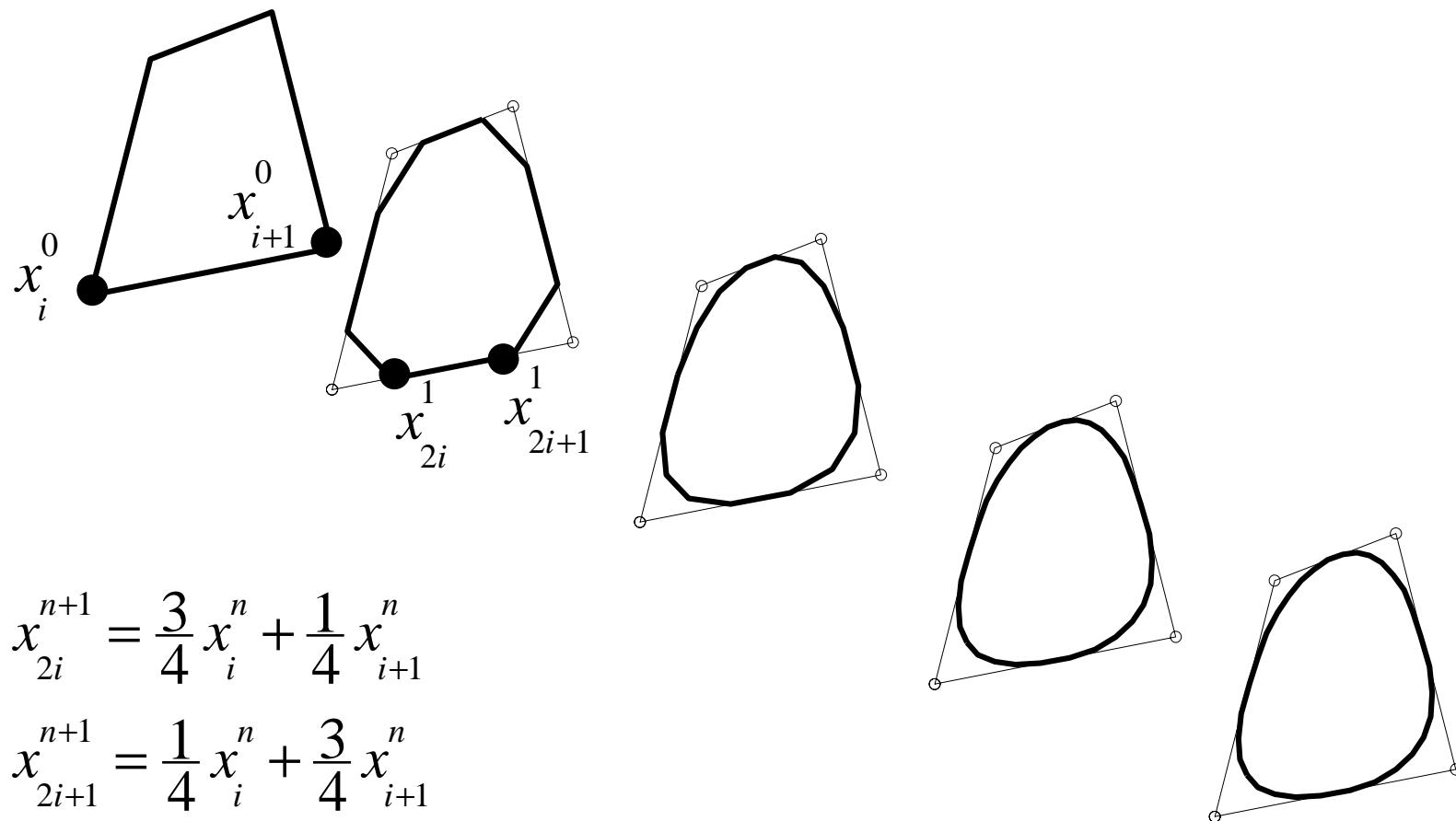
# Synthèse = Subdivision + Correction



- Applications en géométrie: la subdivision doit converger vers des formes lisses

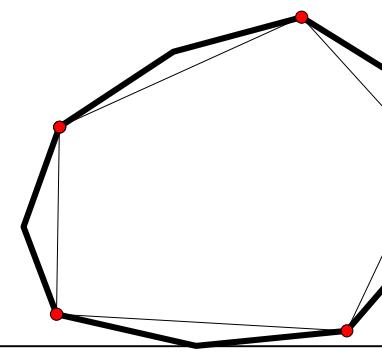
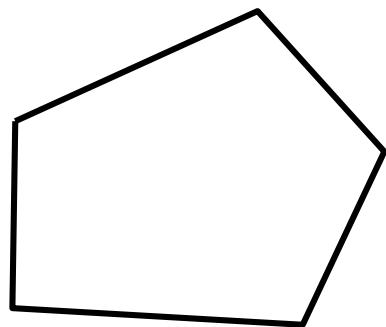
# Courbes de subdivision uniforme

Ex I: Chaikin (74)



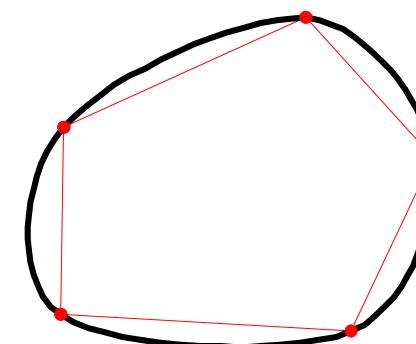
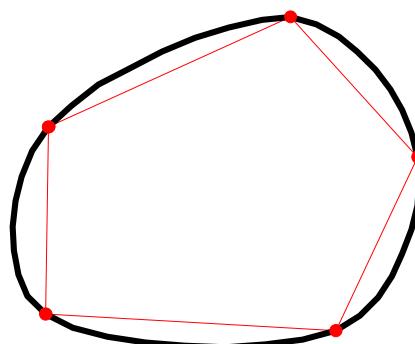
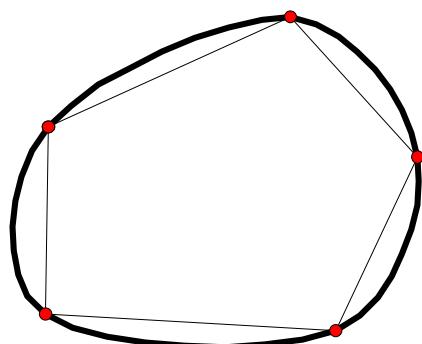
# Courbes de subdivision uniforme

Ex II: Dyn, Levin, Gregory (83)



$$x_{2i}^{n+1} = x_i^n$$

$$x_{2i+1}^{n+1} = -\frac{1}{16}x_{i-1}^n + \frac{9}{16}x_i^n + \frac{9}{16}x_{i+1}^n - \frac{1}{16}x_{i+2}^n$$



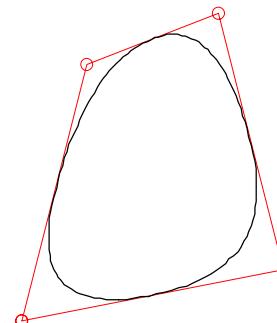
# Courbes de subdivision uniforme

Cas général:

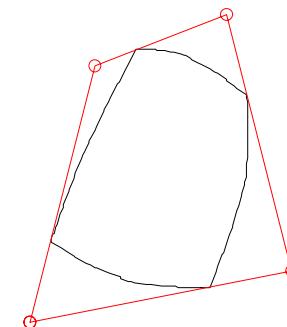
$$x_{2i}^{n+1} = a_j x_{i+j}^n$$

$$x_{2i+1}^{n+1} = b_j x_{i+j}^n$$

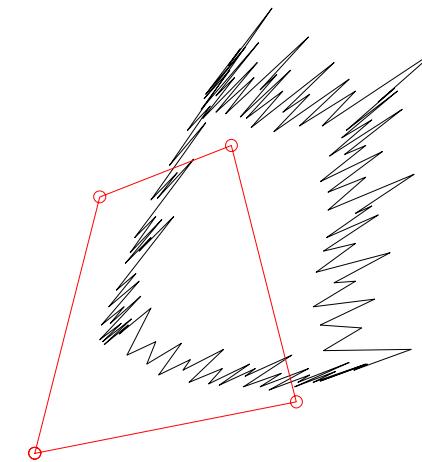
Pbm: Étude de la convergence



Convergence C1



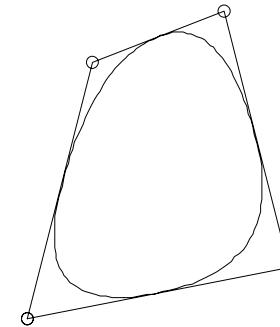
Convergence C0



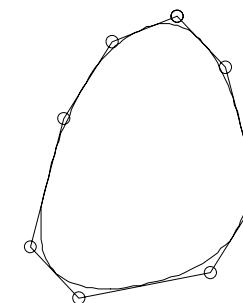
Divergence

# Chaikin: espaces d'approximation

$V^0$  Ensemble des courbes obtenues en partant de 4 points



$V^1$  Ensemble des courbes obtenues en partant de 8 points



$V^n$  Ensemble des courbes obtenues en partant de  $2^{n+2}$  points

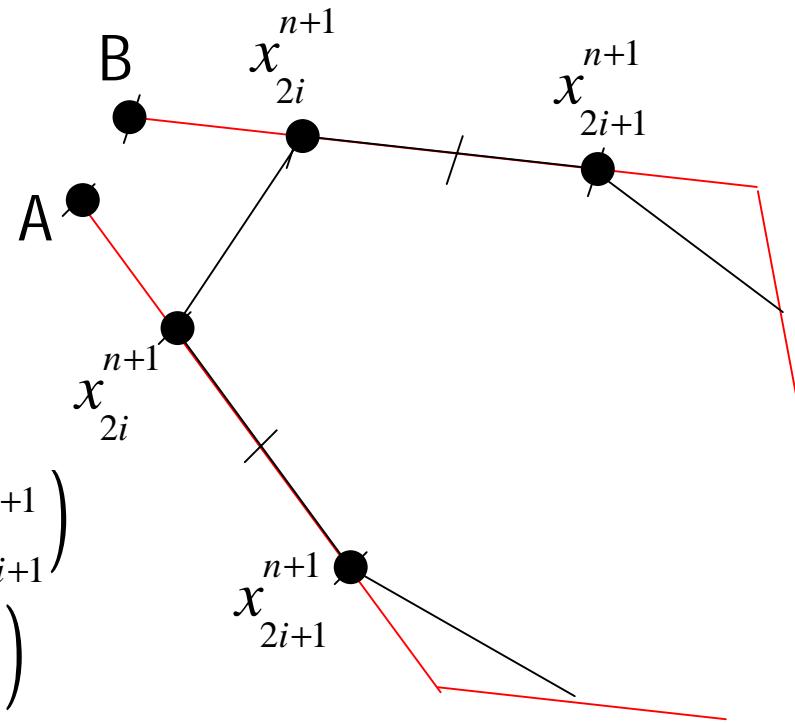
# Chaikin: Analyse

$$x_i^n = \frac{A + B}{2}$$

$$y_i^n = \frac{B - A}{2}$$

$$x_i^n = \frac{1}{4} \left( -x_{2i-2}^{n+1} + 3x_{2i-1}^{n+1} + 3x_{2i}^{n+1} - x_{2i+1}^{n+1} \right)$$

$$y_i^n = \frac{1}{4} \left( x_{2i-2}^{n+1} - 3x_{2i-1}^{n+1} + 3x_{2i}^{n+1} - x_{2i+1}^{n+1} \right)$$

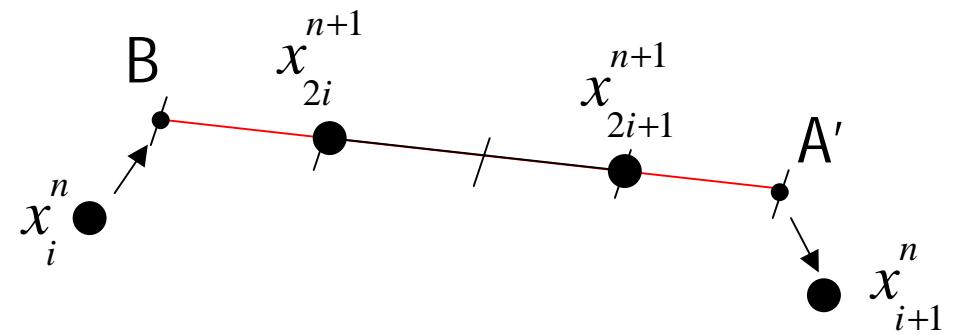


- Subdivision exacte => A=B  
=> détail = 0

# Chaikin: Synthèse

$$B = x_i^n + y_i^n$$

$$A' = x_{i+1}^n - y_{i+1}^n$$

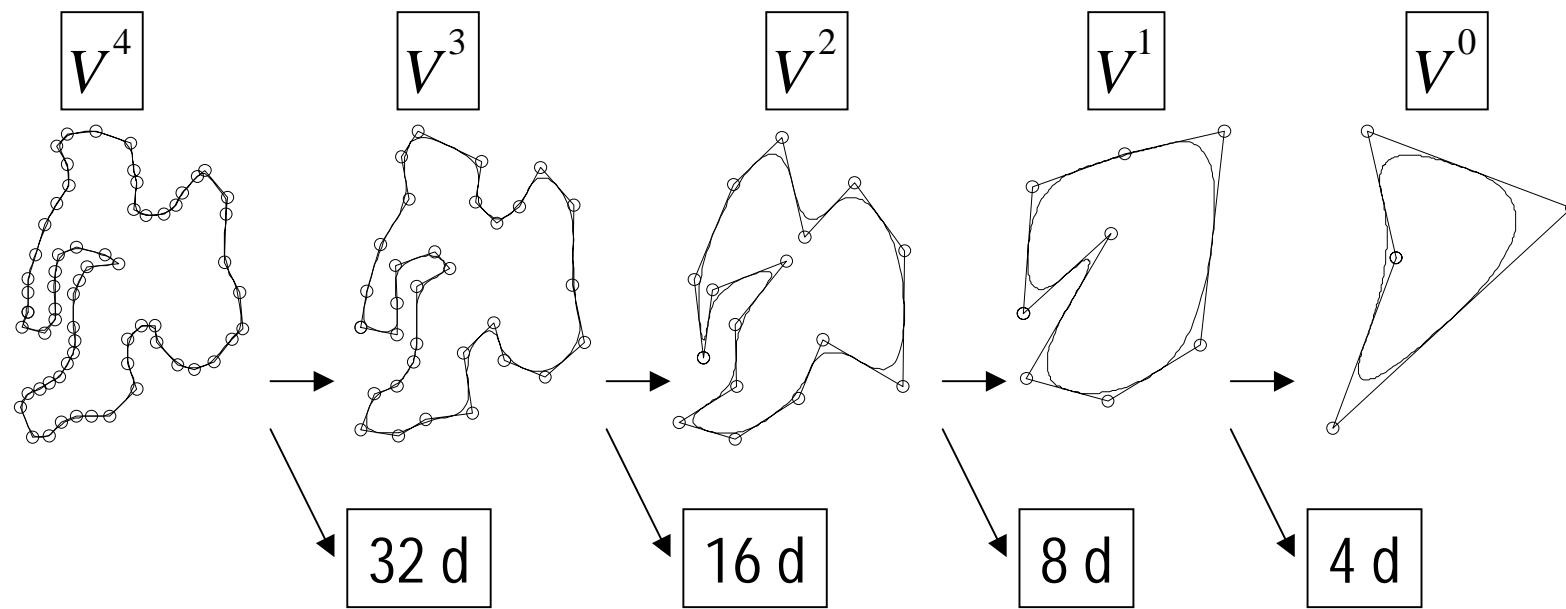


$$x_{2i}^{n+1} = \frac{3}{4} \left( x_i^n + y_i^n \right) + \frac{1}{4} \left( x_{i+1}^n - y_{i+1}^n \right)$$

$$x_{2i+1}^{n+1} = \frac{1}{4} \left( x_i^n + y_i^n \right) + \frac{3}{4} \left( x_{i+1}^n - y_{i+1}^n \right)$$

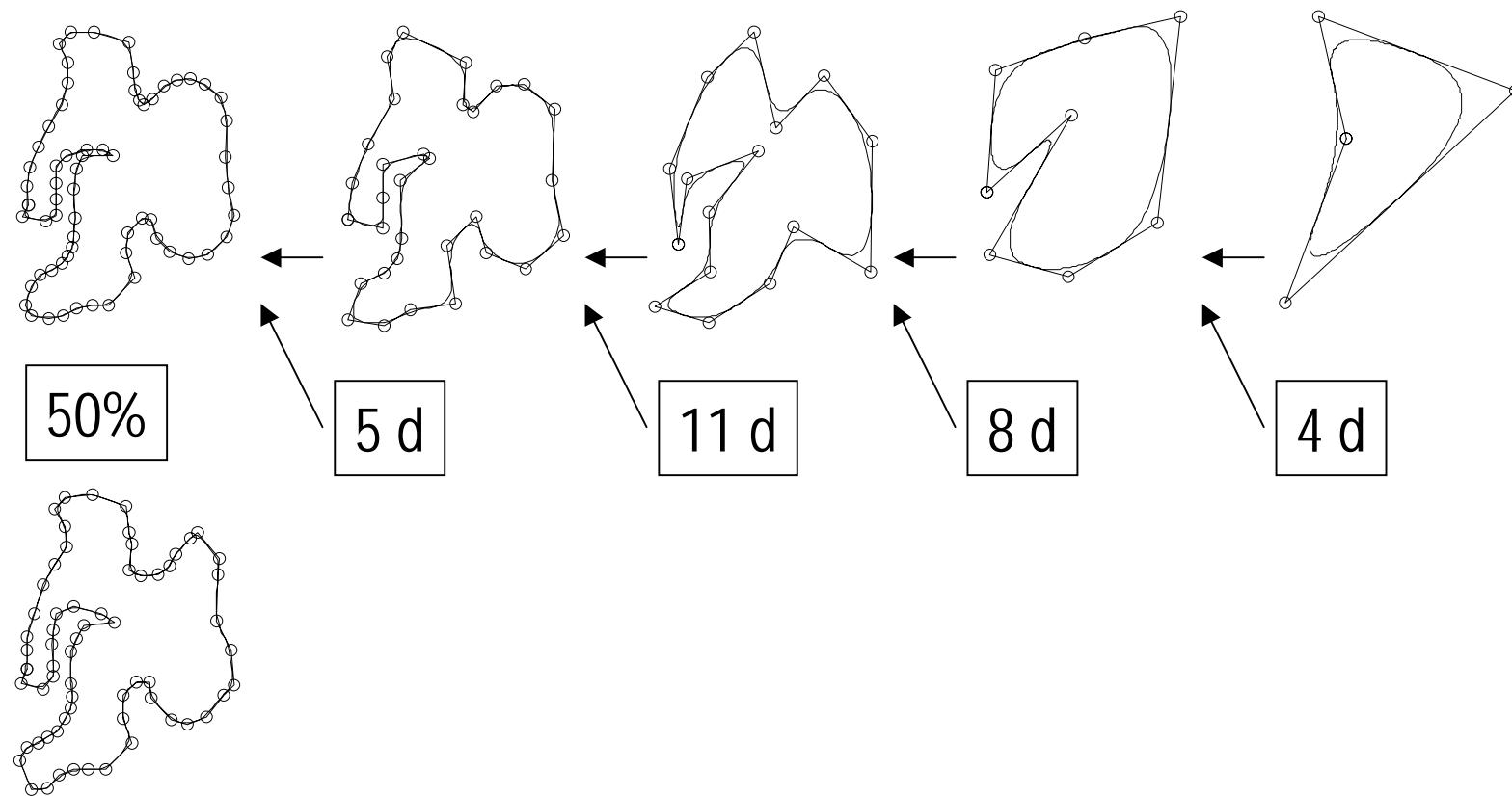
- Subdivision+Correction

# Chaikin: exemple d'analyse



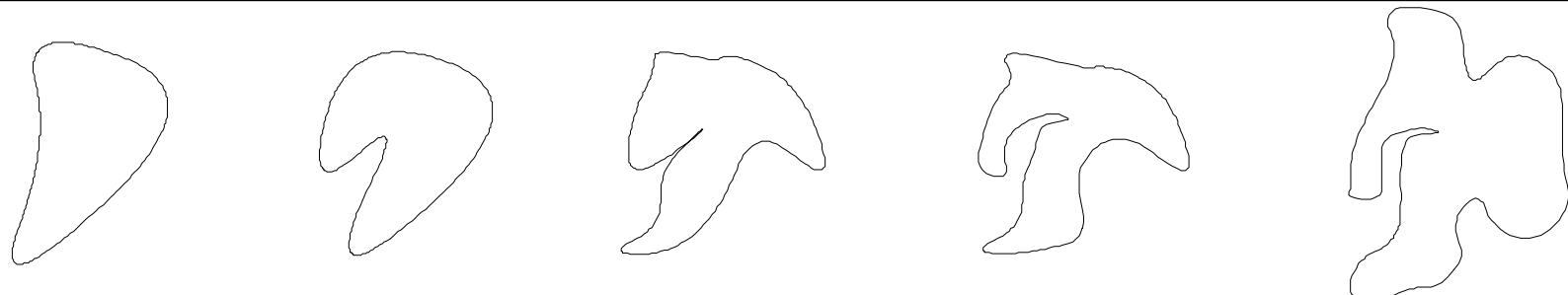
# Chaikin: exemple d'approximation

Reconstruction en utilisant les coefficients de détail de norme > 1.6

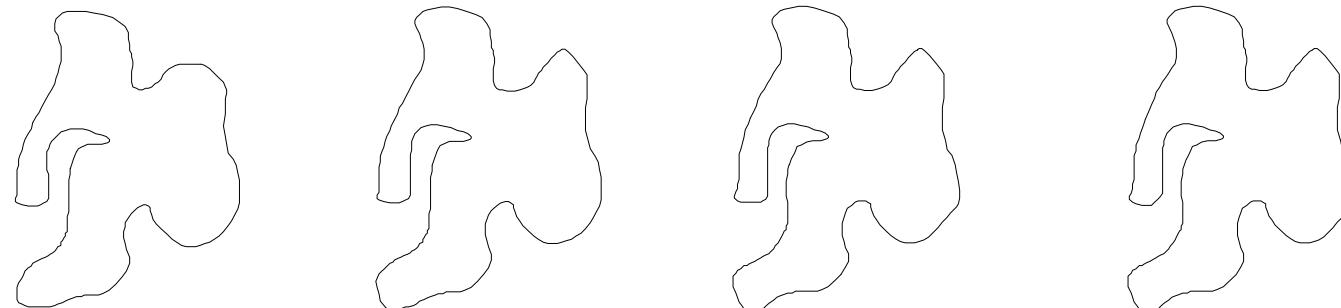


# Reconstruction progressive

Seuil	15	10	5	3	1.6
#détail	4	5	10	16	32
%coeff	6.25%	7.81%	15.62%	25%	50%

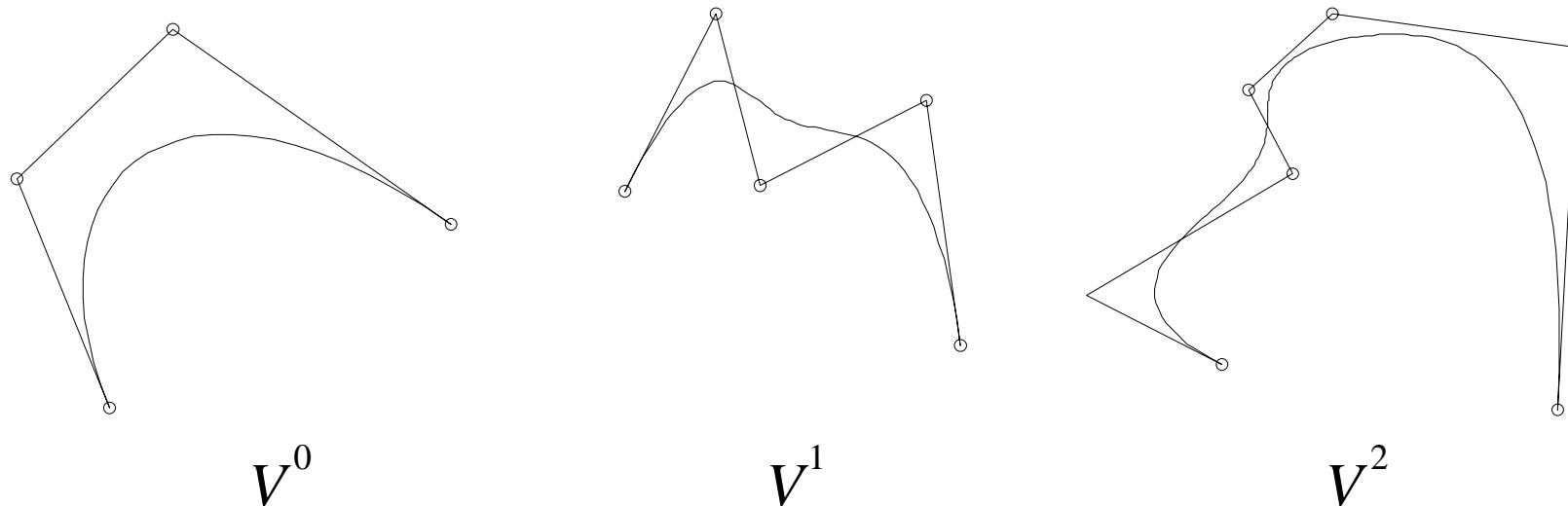


1	0.5	0.1	0
40	48	62	64
62.5%	75%	96.88%	100%



# Courbes de subdivision non uniforme

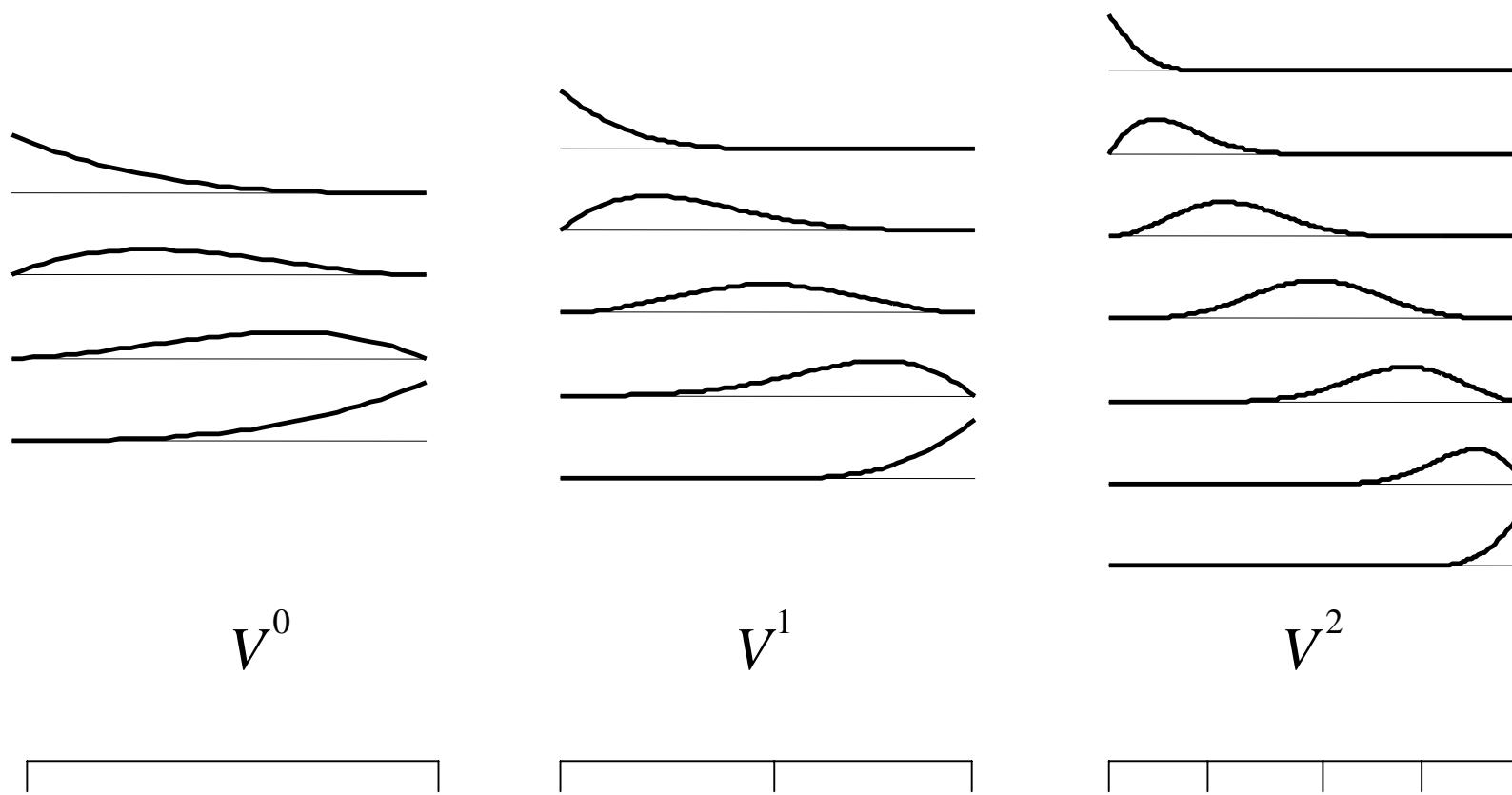
- Insertion récursive de nœuds dans l'intervalle de définition



• degré d =>  $V^n : 2^n + d$  points de contrôle

# Fonctions échelles

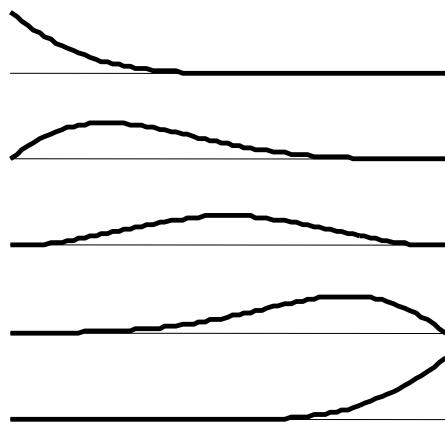
- Fonctions échelles cubiques



# Fonctions ondelettes

• Semi-orthogonalité:  $V^{n+1} = V^n \overset{\perp}{\oplus} W^n$

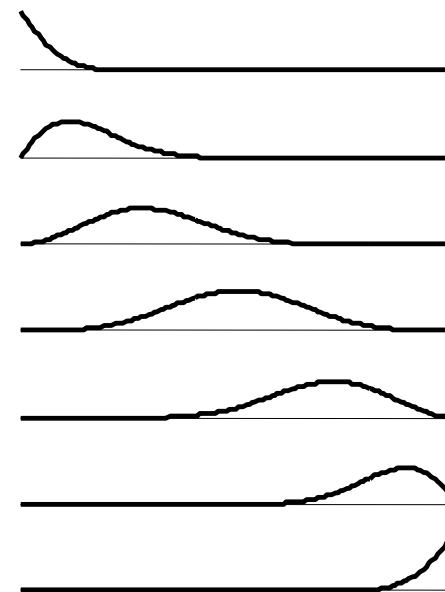
=> meilleure approximation entre  $V^{n+1}$  et  $V^n$



$$V^n, \phi_i^n$$

$$q_{ki} \phi_k^{n+1}$$

$$W^n, \psi_i^n$$



$$V^{n+1}, \phi_i^{n+1}$$

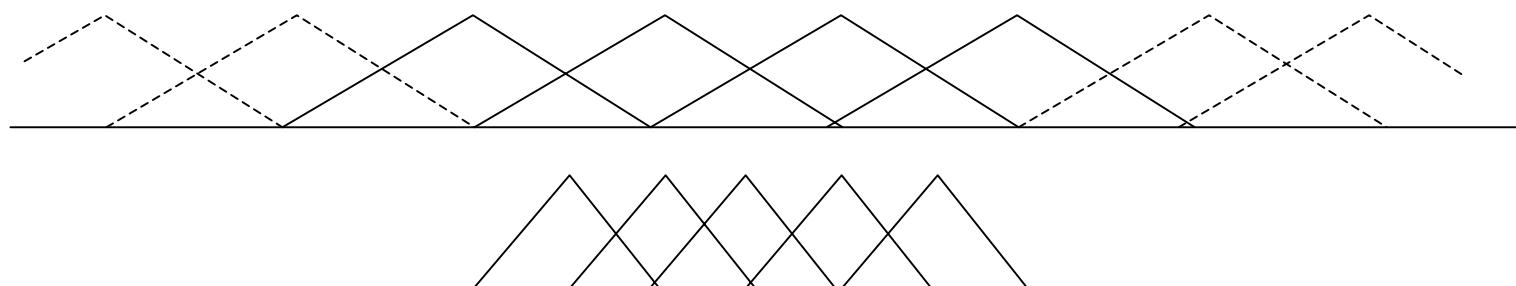
# Fonctions ondelettes

$$\psi_i^n = \sum_k q_{ki} \varphi_k^{n+1}$$

Inconnues:  $q_{ki}$

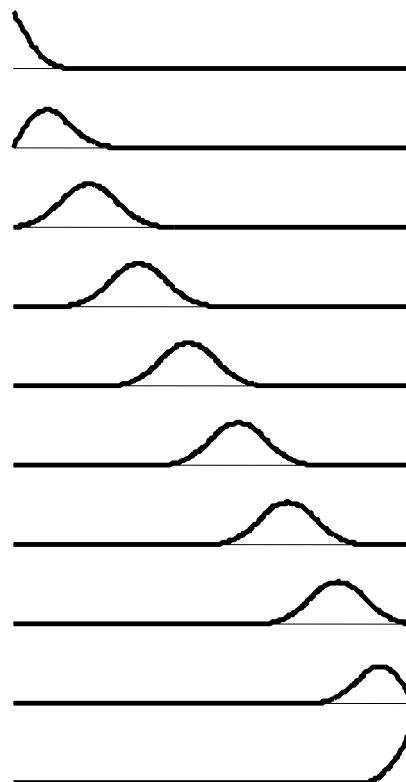
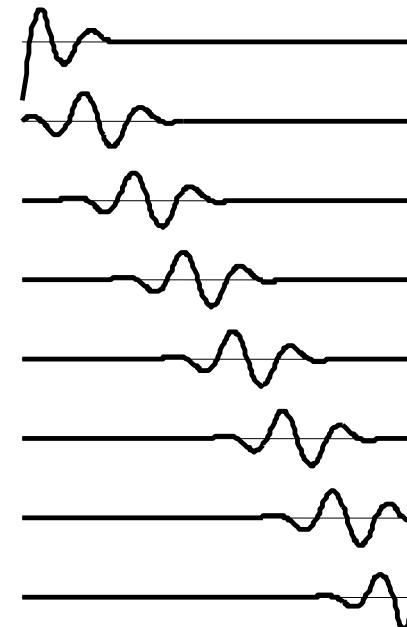
$$\langle \psi_i^n | \varphi_j^n \rangle = 0 \text{ : conditions linéaires en les inconnues}$$

- Quand on augmente le nombre d'inconnues, le nombre d'équations augmente moins vite => existence de solutions
- Recherche de la solution de support minimal

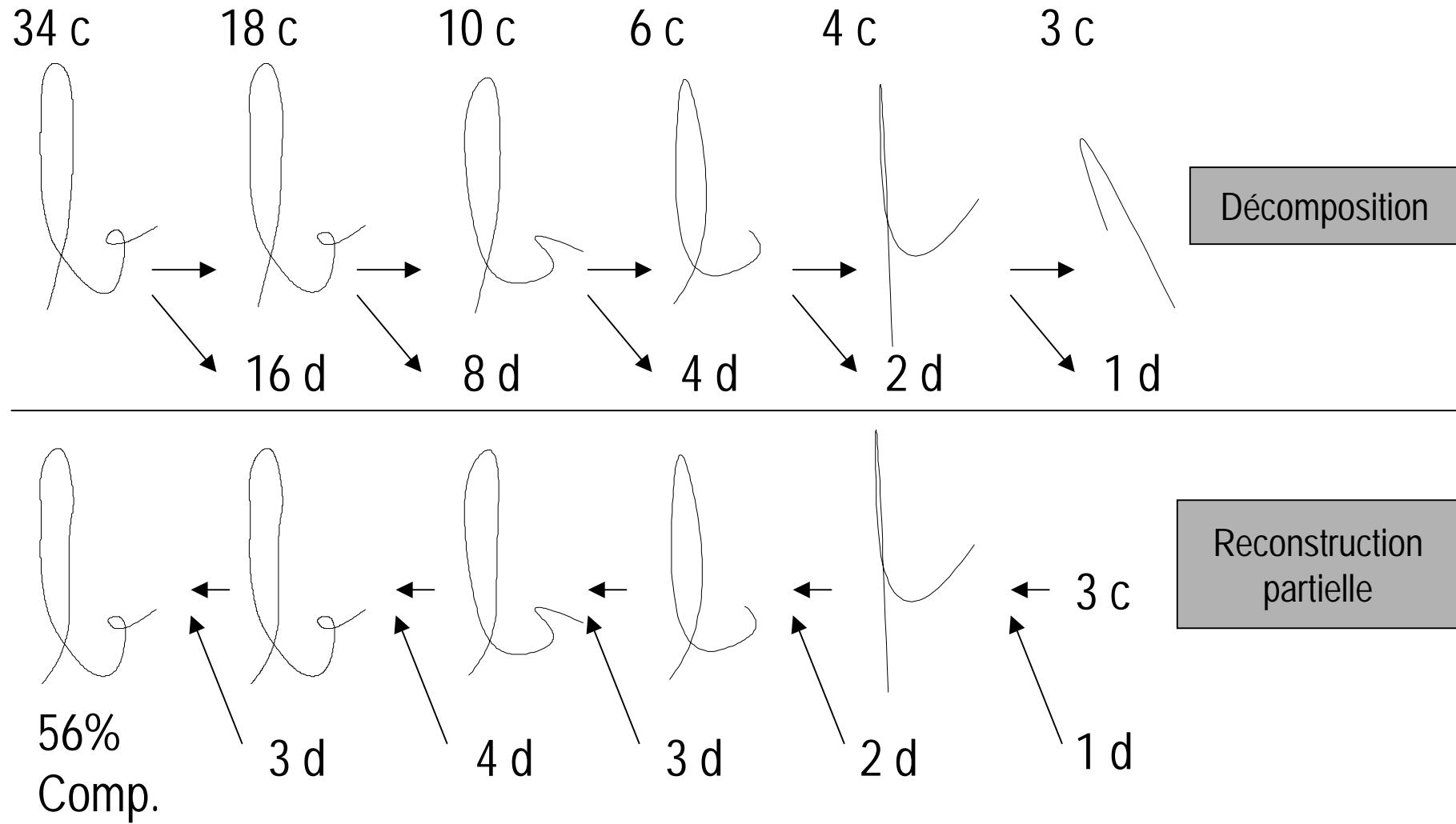


# Fonctions ondelettes

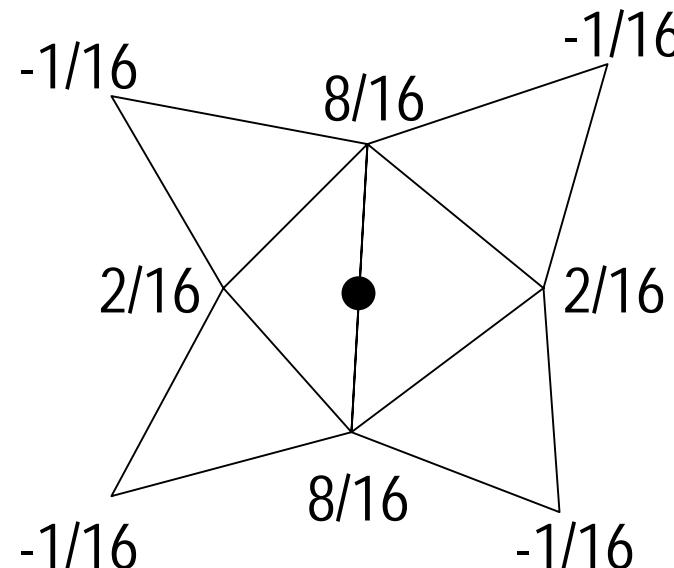
- Degré 2, niveau 3 de subdivision

 $V^3$  $W^3$

# Exemple de reconstruction partielle

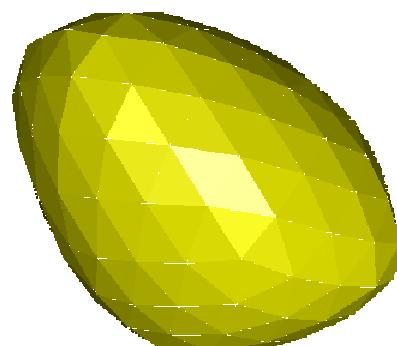


# Surfaces de subdivision uniforme

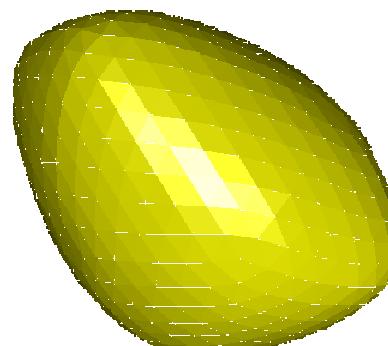


Butterfly Scheme (Dyn, Levin, Gregory)

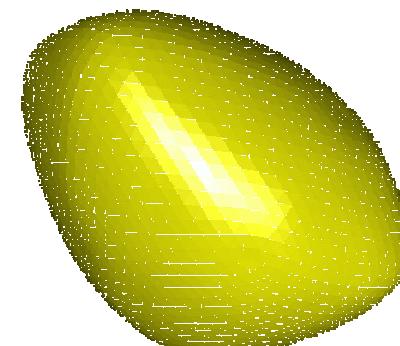
320 t



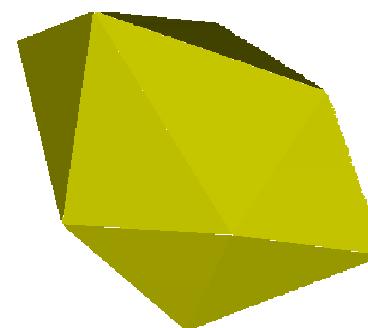
1280 t



5120 t



20 t



80 t

