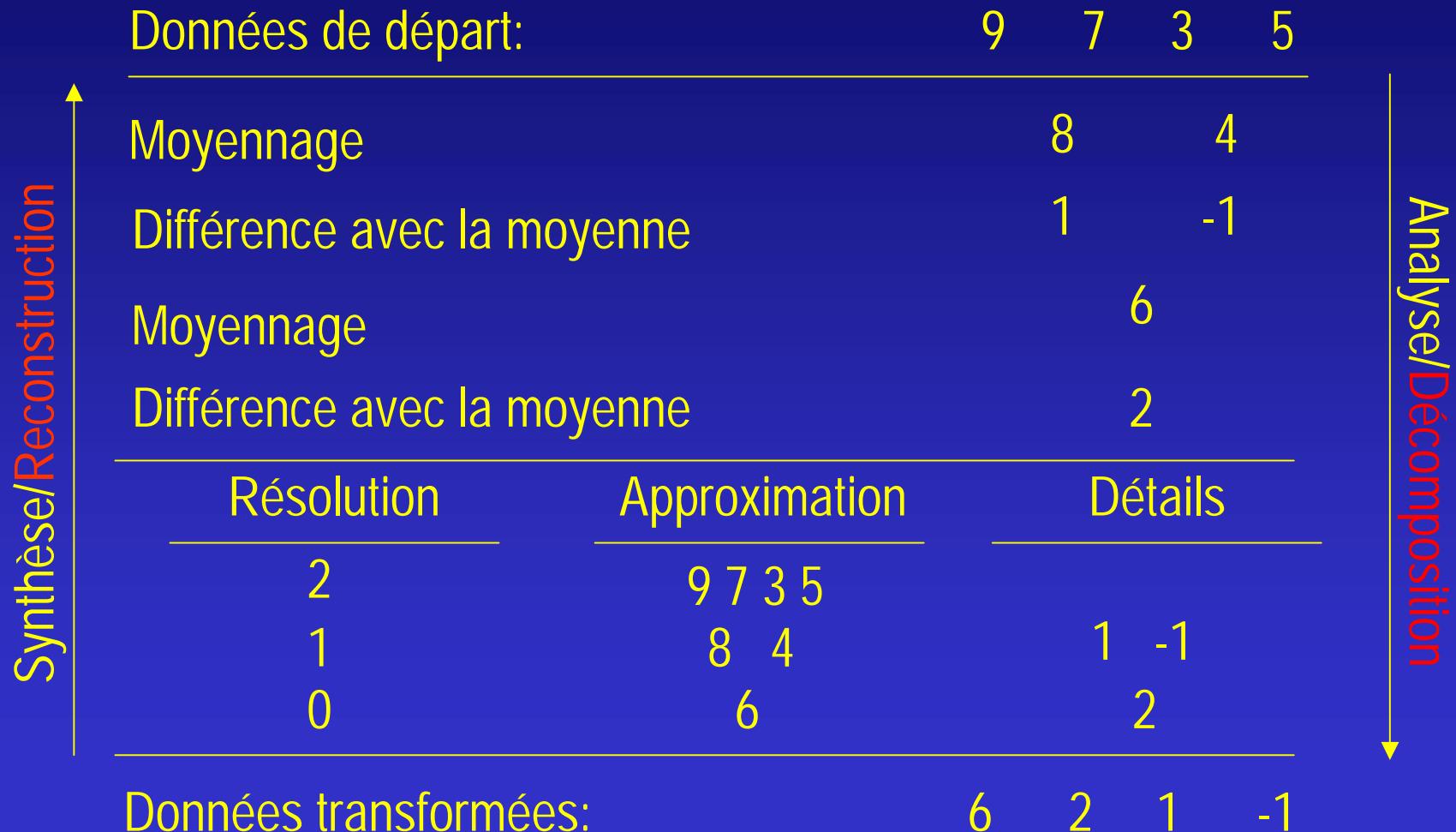


Multirésolution en Modélisation Géométrique

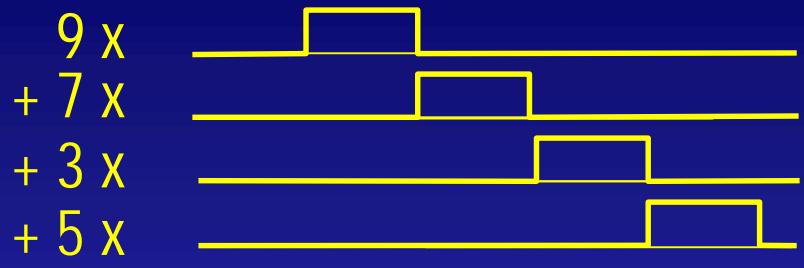
G.P. Bonneau, CR CNRS

- Ondelettes de Haar
- Cadre général de l'Analyse MR
- Courbes de subdivision, ex. de courbes MR
- Surfaces de subdivision
- Courbes MR
- Surfaces MR

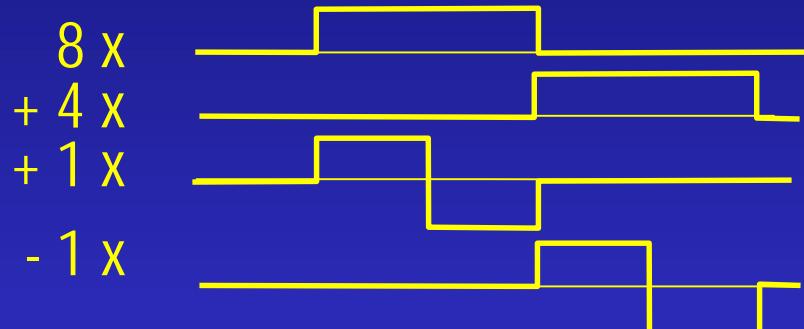
Ondelettes de Haar: Analyse/Synthèse



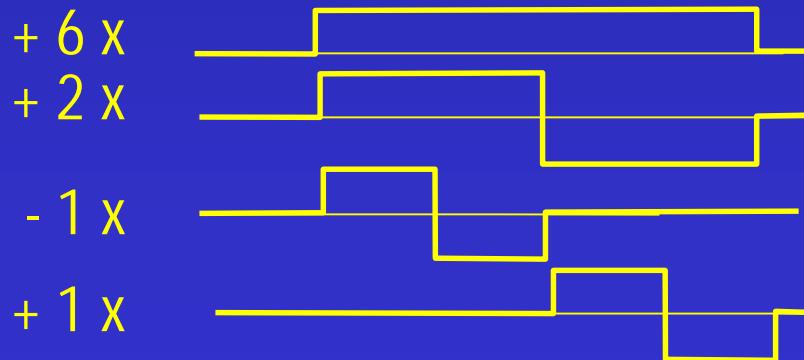
Ondelettes de Haar: fonctions de base



$$9 \times \varphi_0^2 + 7 \times \varphi_1^2 + 3 \times \varphi_2^2 + 5 \times \varphi_3^2$$

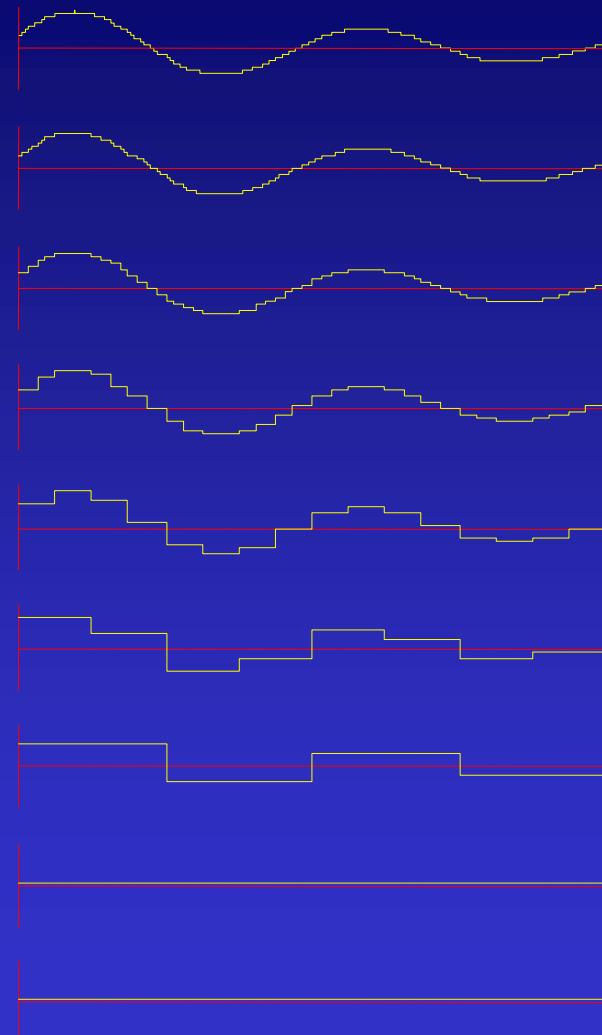


$$8 \times \varphi_0^1 + 4 \times \varphi_1^1 + 1 \times \psi_0^1 - 1 \times \psi_1^1$$

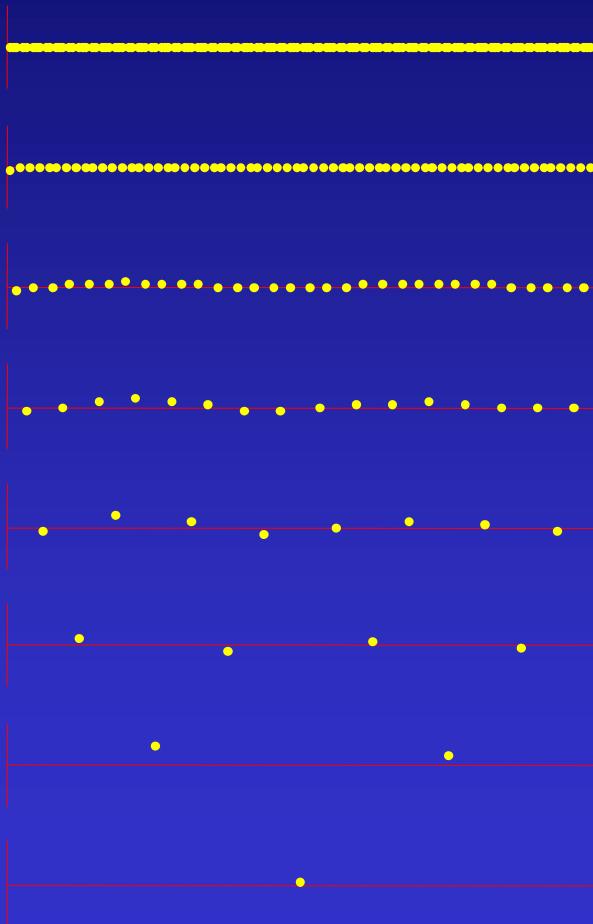


$$6 \times \varphi_0^0 + 2 \times \psi_0^0 + 1 \times \psi_0^1 - 1 \times \psi_1^1$$

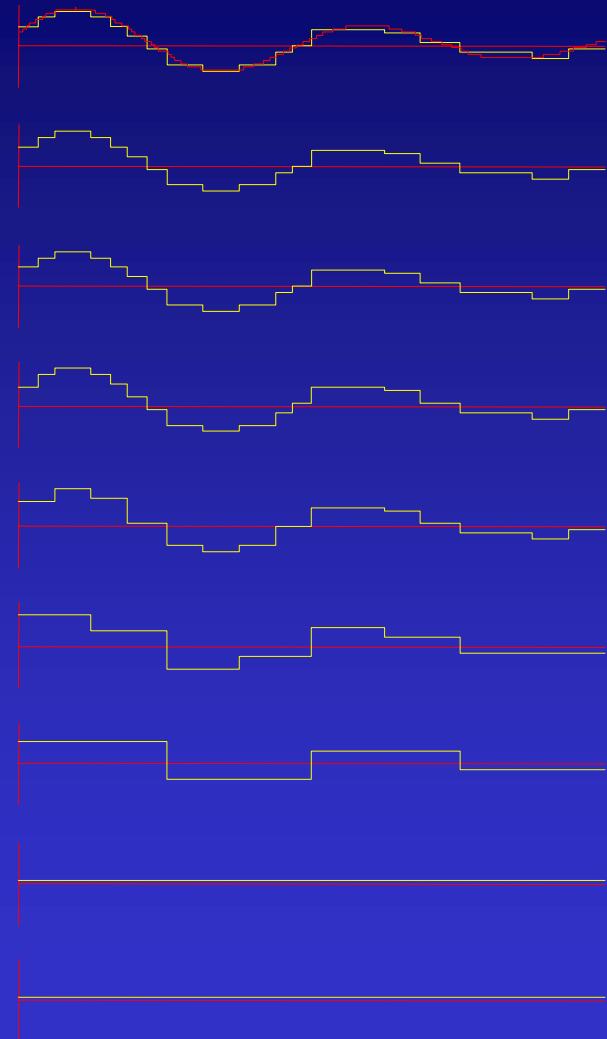
Ondelettes de Haar: Approximation



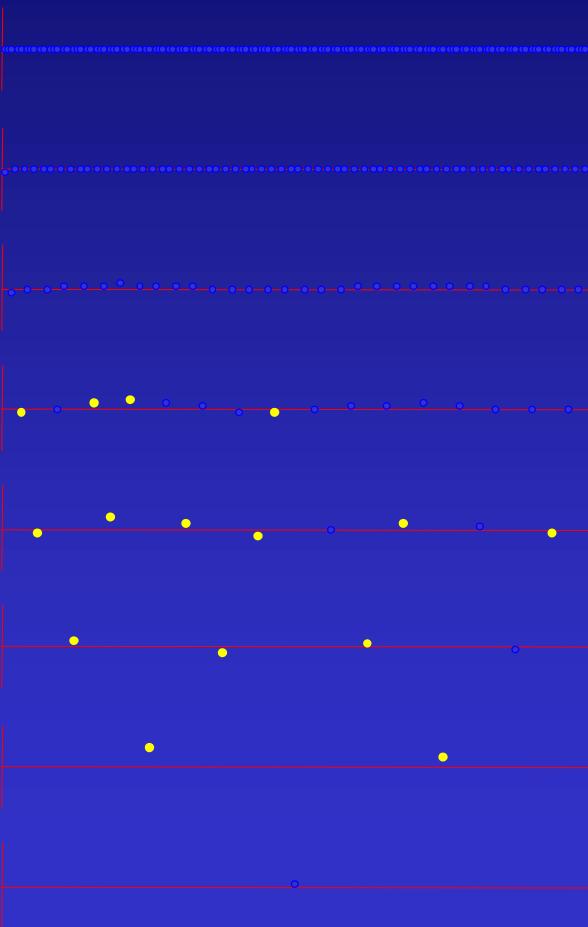
Analyse



Ondelettes de Haar: Approximation

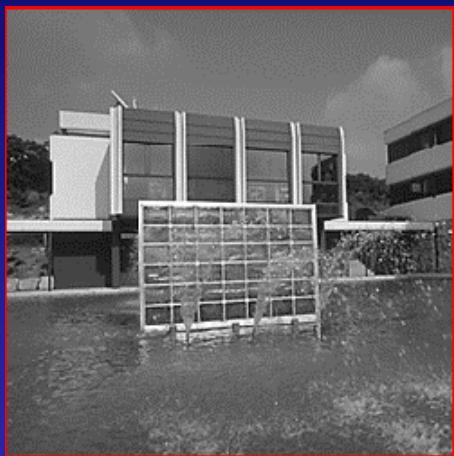


Synthèse partielle

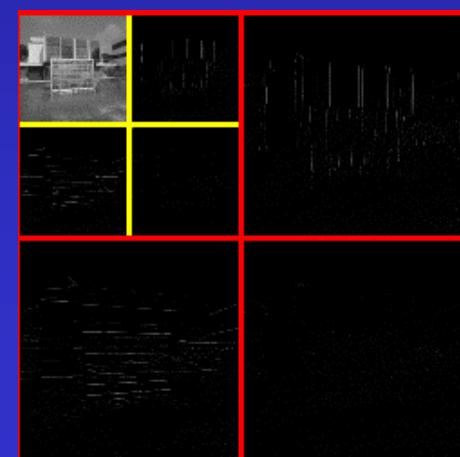
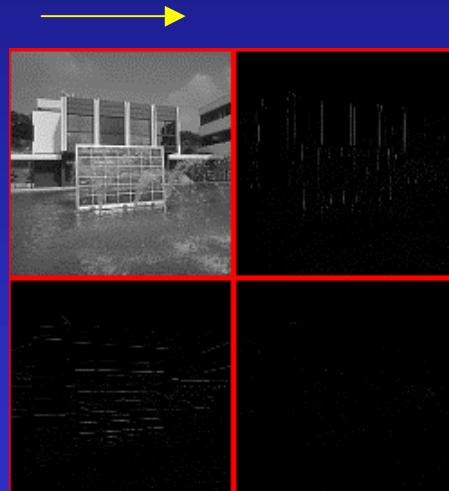


Ondelettes de Haar: Produit Tensoriel

Analyse sur les lignes

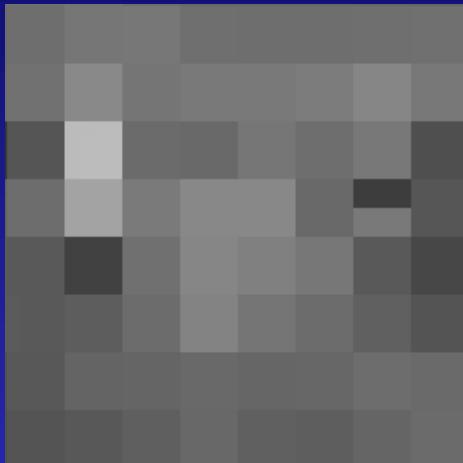


Analyse sur les colonnes



Reconstruction progressive

99.9 %



99 %



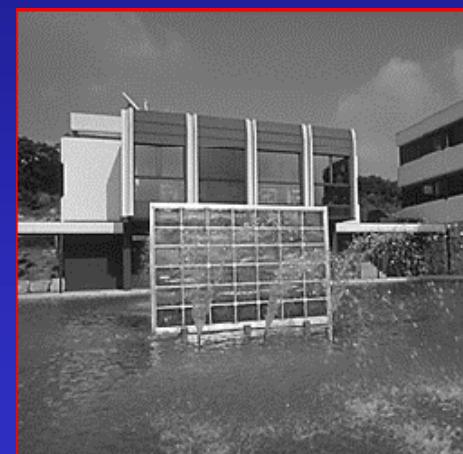
95 %



90 %



Original



Cadre général de l'Analyse MR

- Espace de fonctions E
- Sous-espaces d'approximation V^n
- $V^n \subset V^{n+1}$
- Espaces de détail W^n

$$V^{n+1} = V^n \oplus W^n$$

Espaces

Fonctions de base

- Fonctions d'échelle (φ_i^n) 
- Fonctions ondelettes (ψ_i^n) 

Équations de changement d'échelle

- Liens entre les fonctions échelles et les fonctions ondelettes de résolution successives

$$\left(\varphi_i^n \right) = \left[{}^t P^n \right] \left(\varphi_i^{n+1} \right)$$

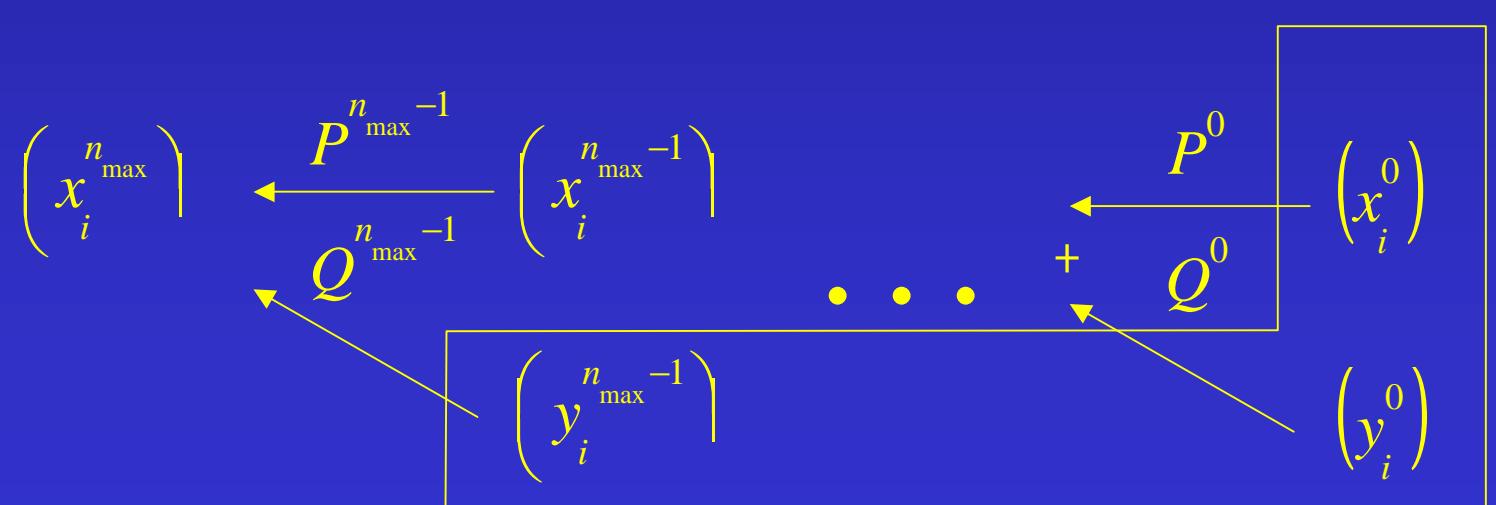
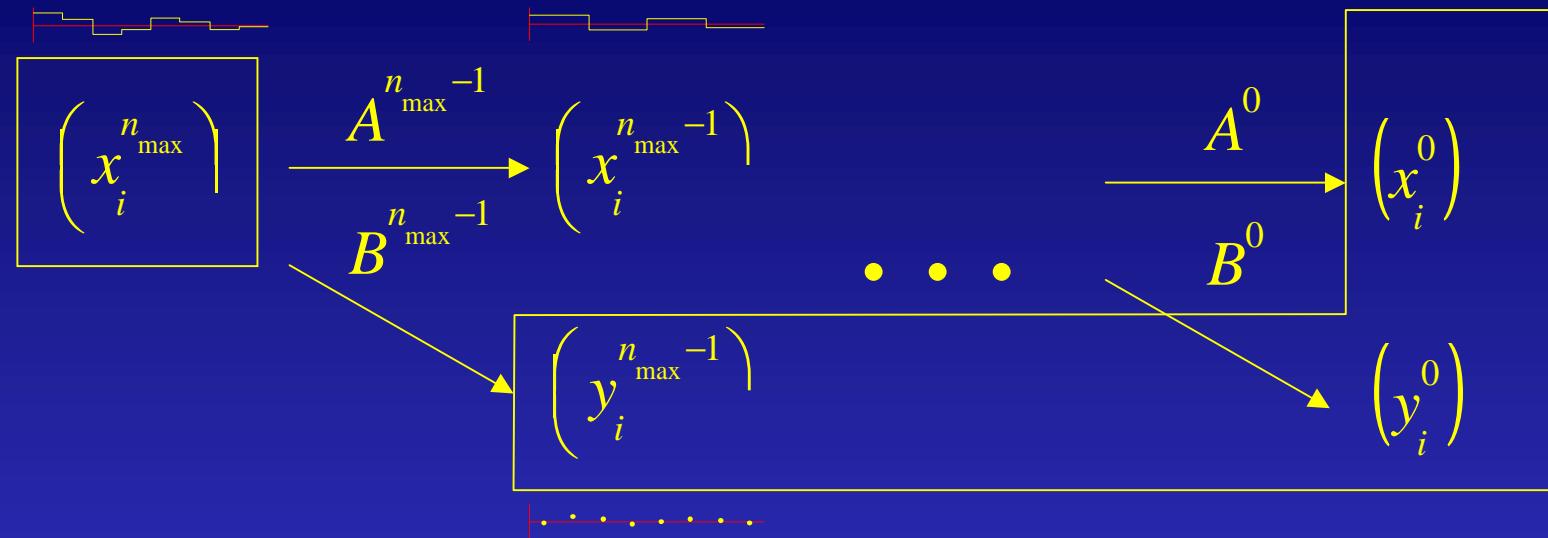
$$\left(\psi_i^n \right) = \left[{}^t Q^n \right] \left(\varphi_i^{n+1} \right)$$

$$\left(\varphi_i^{n+1} \right) = \left[{}^t A^n \right] \left(\varphi_i^n \right) + \left[{}^t B^n \right] \left(\psi_i^n \right)$$

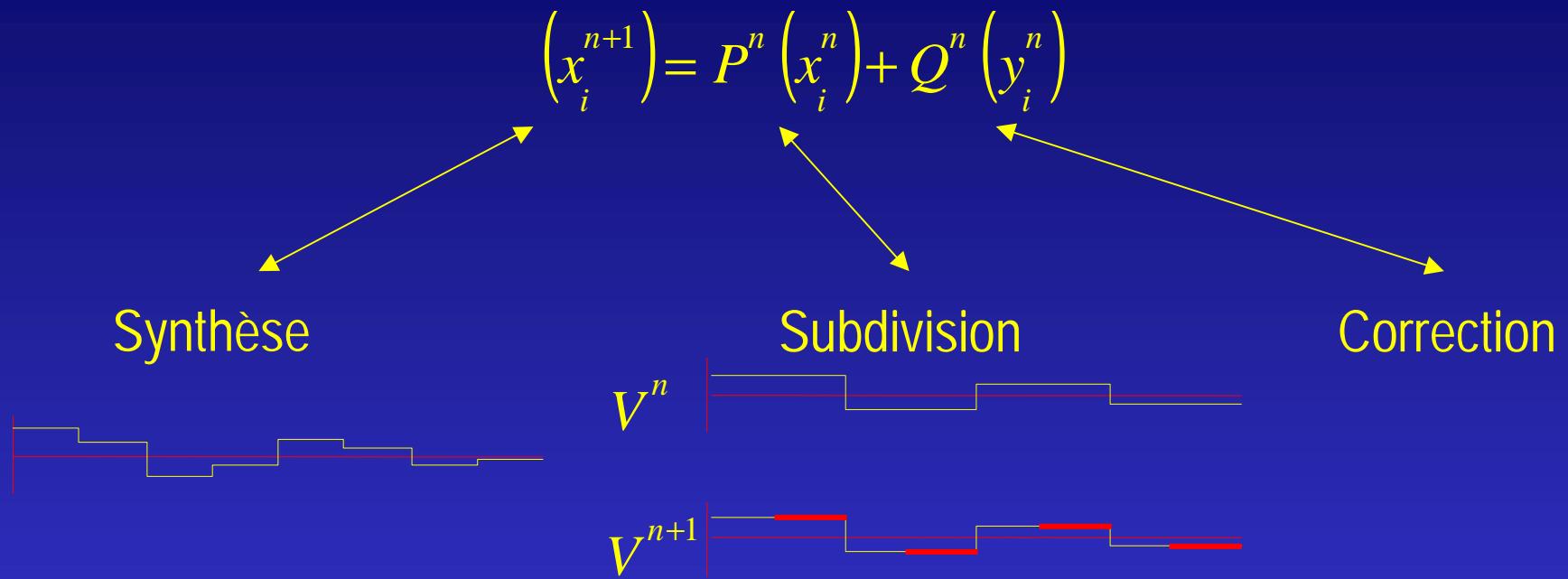
- Condition de reconstruction

$$\left[P^n Q^n \right] \begin{bmatrix} A^n \\ B^n \end{bmatrix} = I$$

Analyse/Synthèse



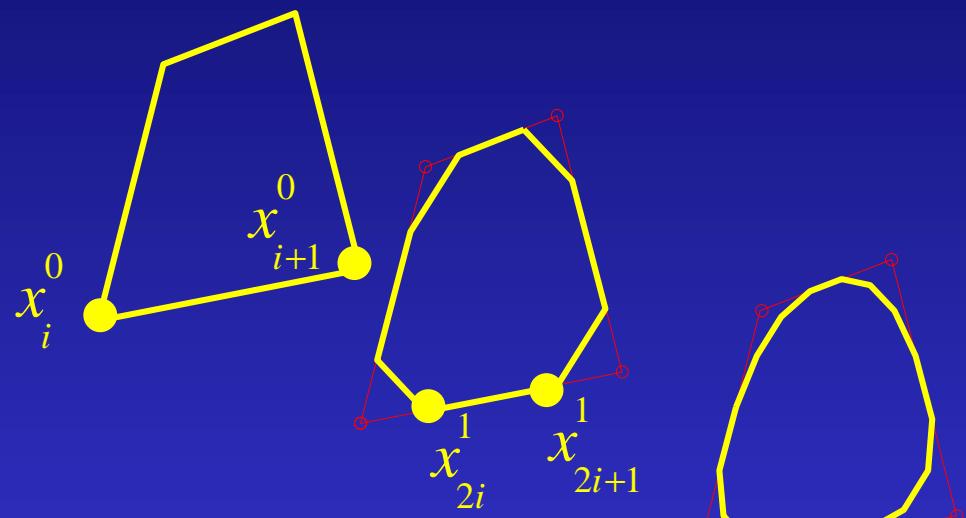
Synthèse = Subdivision + Correction



- Applications en géométrie: la subdivision doit converger vers des formes lisses

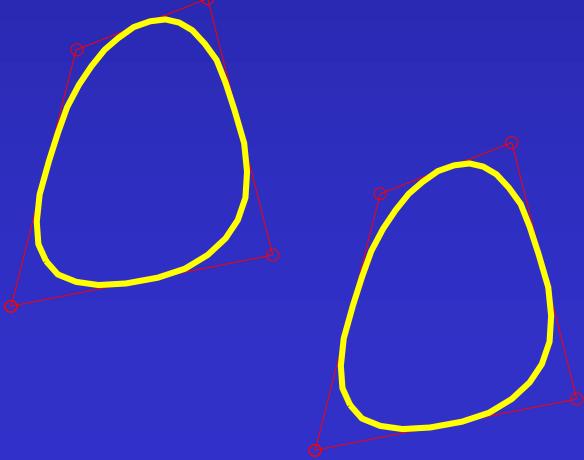
Courbes de subdivision uniforme

Ex I: Chaikin (74)



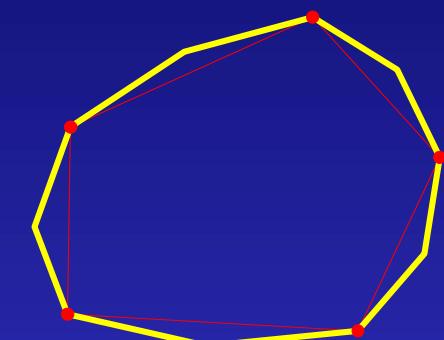
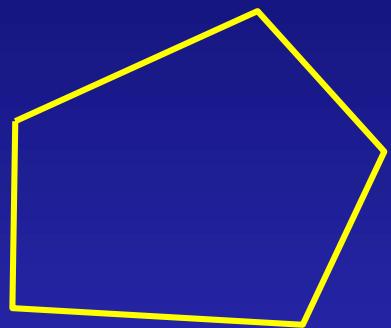
$$x_{2i}^{n+1} = \frac{3}{4} x_i^n + \frac{1}{4} x_{i+1}^n$$

$$x_{2i+1}^{n+1} = \frac{1}{4} x_i^n + \frac{3}{4} x_{i+1}^n$$



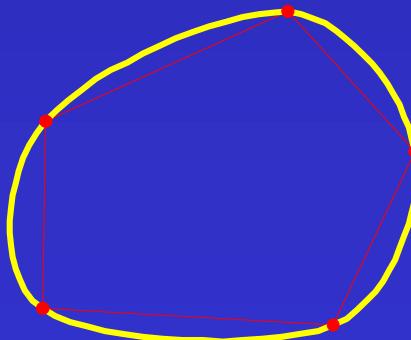
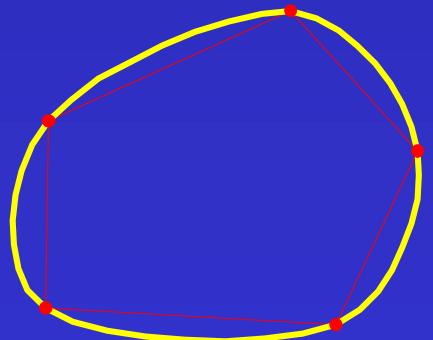
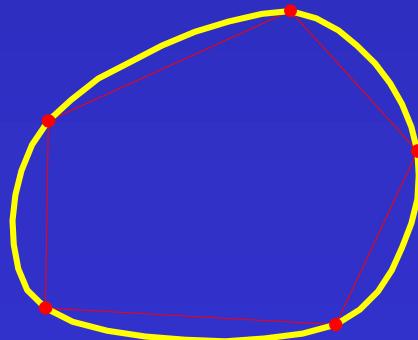
Courbes de subdivision uniforme

Ex II: Dyn, Levin, Gregory (83)



$$x_{2i}^{n+1} = x_i^n$$

$$x_{2i+1}^{n+1} = -\frac{1}{16}x_{i-1}^n + \frac{9}{16}x_i^n + \frac{9}{16}x_{i+1}^n - \frac{1}{16}x_{i+2}^n$$



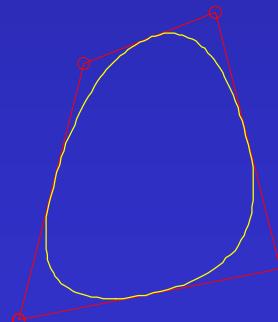
Courbes de subdivision uniforme

Cas général:

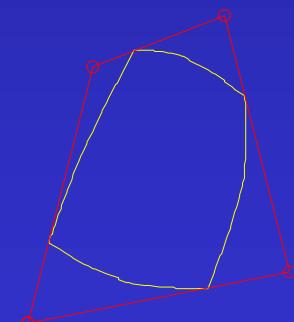
$$x_{2i}^{n+1} = a_j x_{i+j}^n$$

$$x_{2i+1}^{n+1} = b_j x_{i+j}^n$$

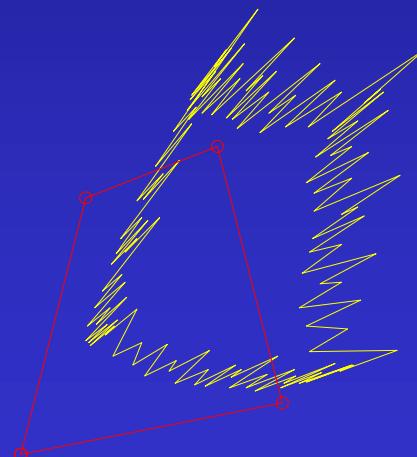
Pbm: Étude de la convergence



Convergence C1



Convergence C0

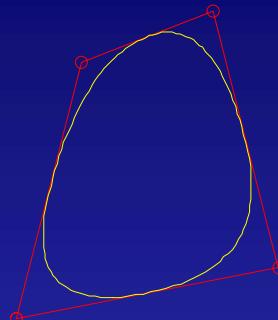


Divergence

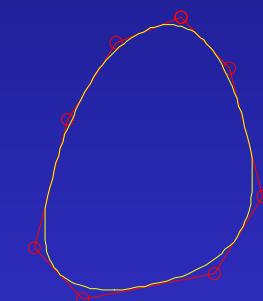
Chaikin: espaces d'approximation

 V^0

Ensemble des courbes obtenues
en partant de 4 points

 V^1

Ensemble des courbes obtenues
en partant de 8 points

 V^n

Ensemble des courbes obtenues
en partant de 2^{n+2} points

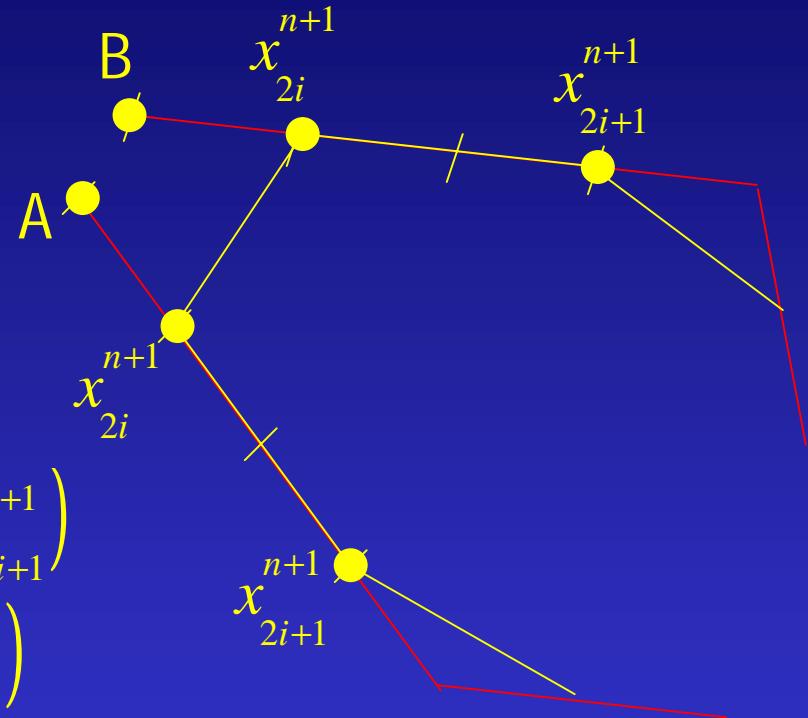
Chaikin: Analyse

$$x_i^n = \frac{A+B}{2}$$

$$y_i^n = \frac{B-A}{2}$$

$$x_i^n = \frac{1}{4} \left(-x_{2i-2}^{n+1} + 3x_{2i-1}^{n+1} + 3x_{2i}^{n+1} - x_{2i+1}^{n+1} \right)$$

$$y_i^n = \frac{1}{4} \left(x_{2i-2}^{n+1} - 3x_{2i-1}^{n+1} + 3x_{2i}^{n+1} - x_{2i+1}^{n+1} \right)$$

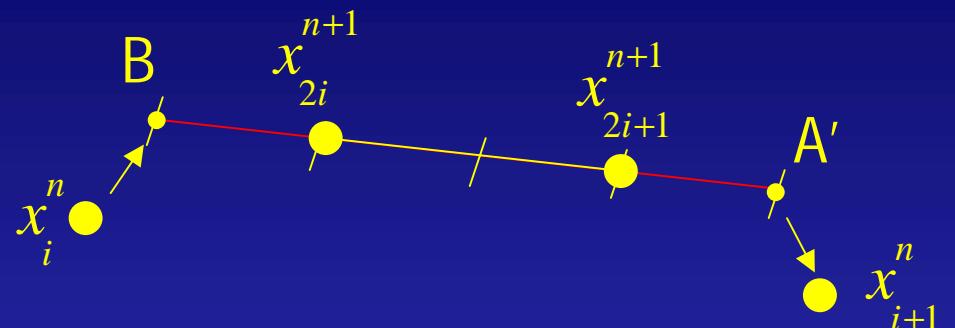


- Subdivision exacte => A=B
=> détail = 0

Chaikin: Synthèse

$$B = x_i^n + y_i^n$$

$$A' = x_{i+1}^n - y_{i+1}^n$$

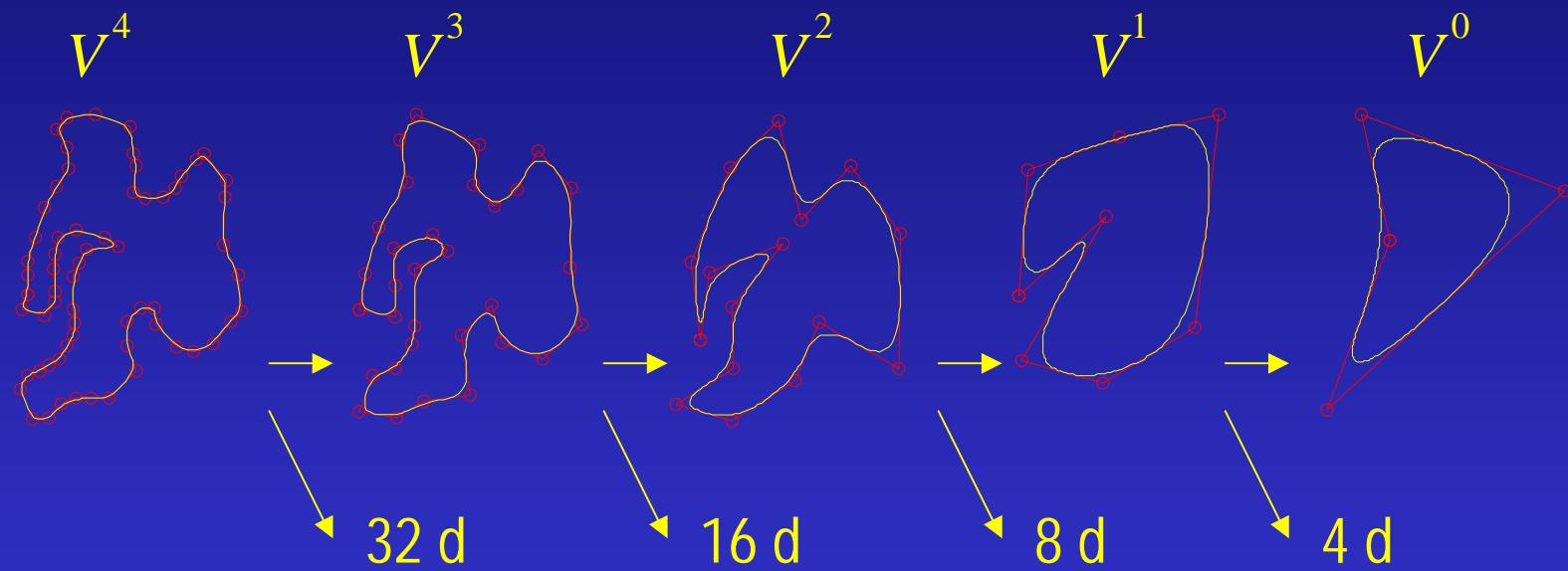


$$x_{2i}^{n+1} = \frac{3}{4} \left(x_i^n + y_i^n \right) + \frac{1}{4} \left(x_{i+1}^n - y_{i+1}^n \right)$$

$$x_{2i+1}^{n+1} = \frac{1}{4} \left(x_i^n + y_i^n \right) + \frac{3}{4} \left(x_{i+1}^n - y_{i+1}^n \right)$$

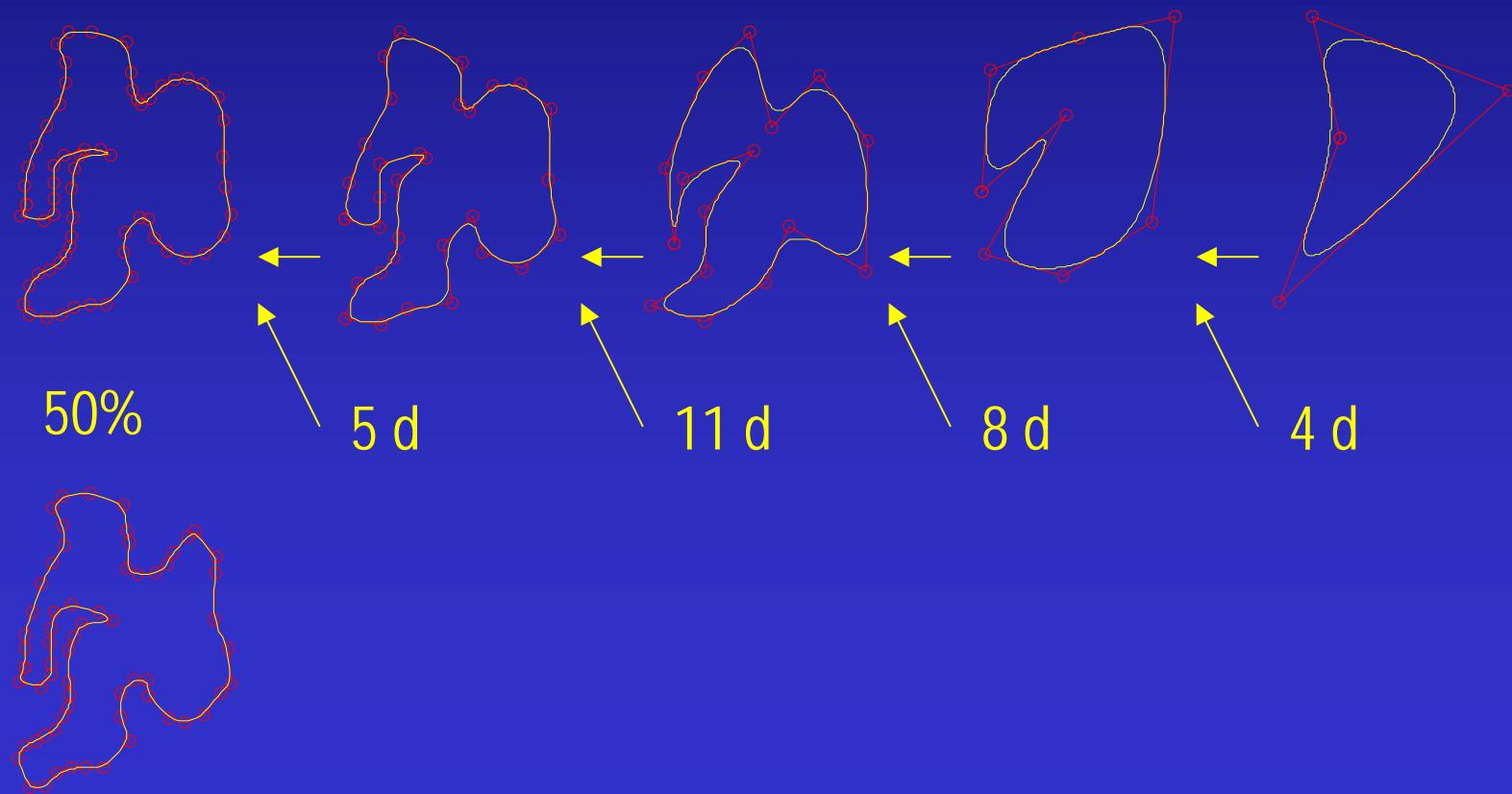
- Subdivision+Correction

Chaikin: exemple d'analyse



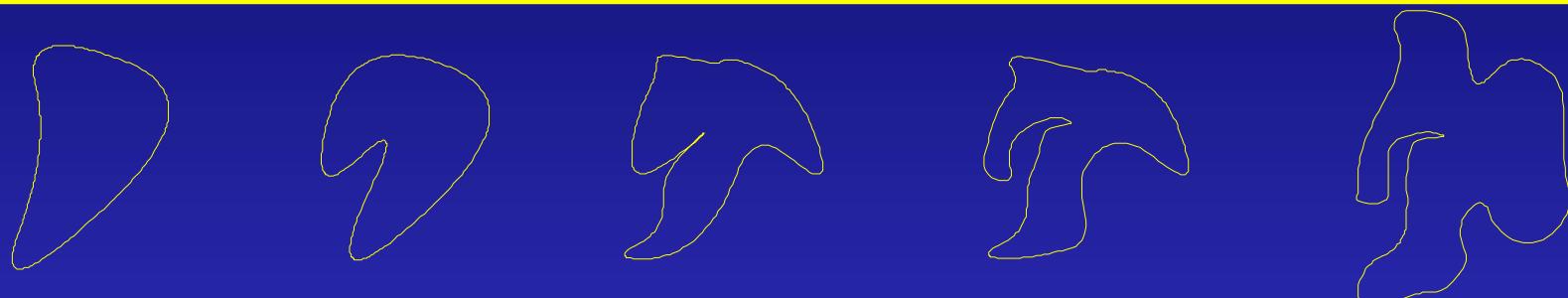
Chaikin: exemple d'approximation

Reconstruction en utilisant les coefficients de détail de norme > 1.6

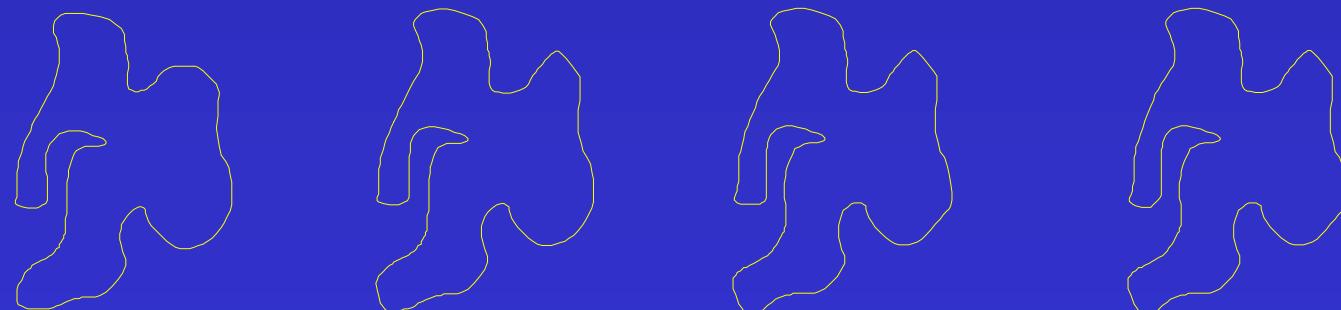


Reconstruction progressive

Seuil	15	10	5	3	1.6
#détail	4	5	10	16	32
%coeff	6.25%	7.81%	15.62%	25%	50%

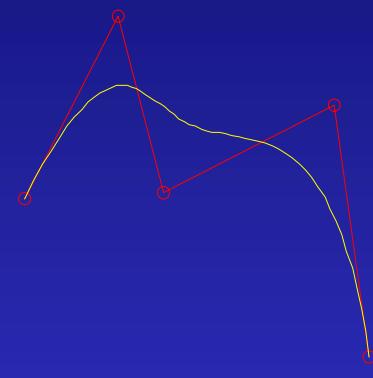
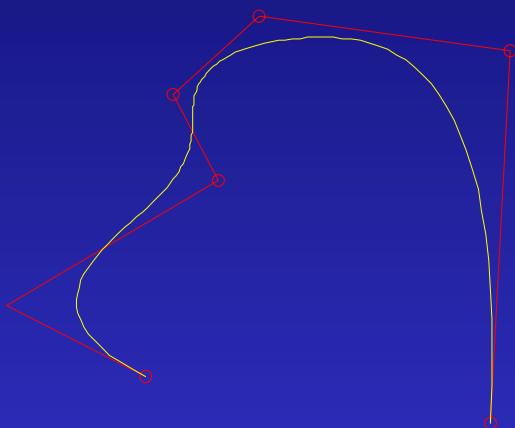


1	0.5	0.1	0
40	48	62	64
62.5%	75%	96.88%	100%



Courbes de subdivision non uniforme

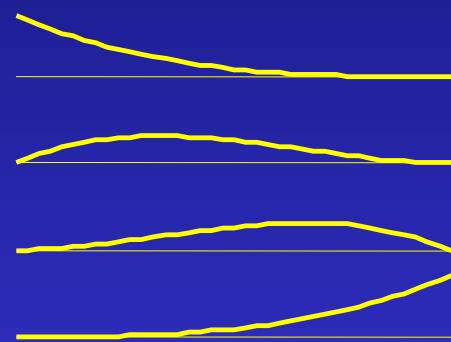
- Insertion récursive de nœuds dans l'intervalle de définition

 V^0  V^1  V^2

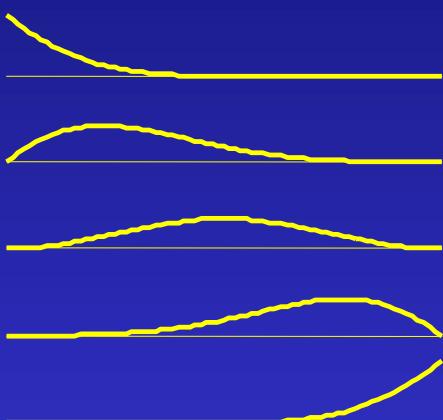
• degré d => $V^n : 2^n + d$ points de contrôle

Fonctions échelles

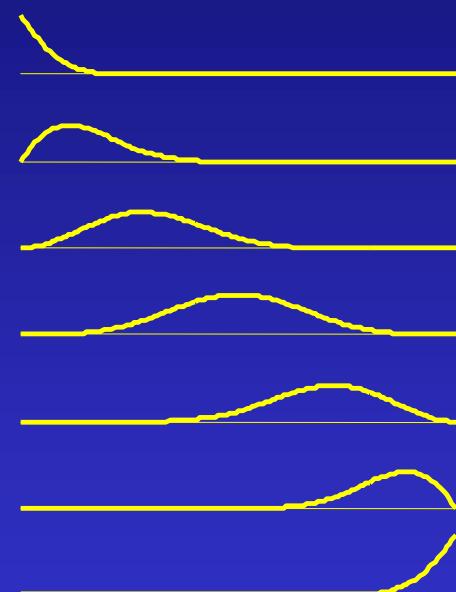
- Fonctions échelles cubiques



$$V^0$$



$$V^1$$

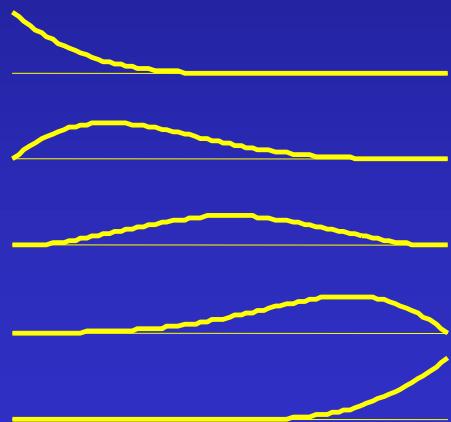


$$V^2$$

Fonctions ondelettes

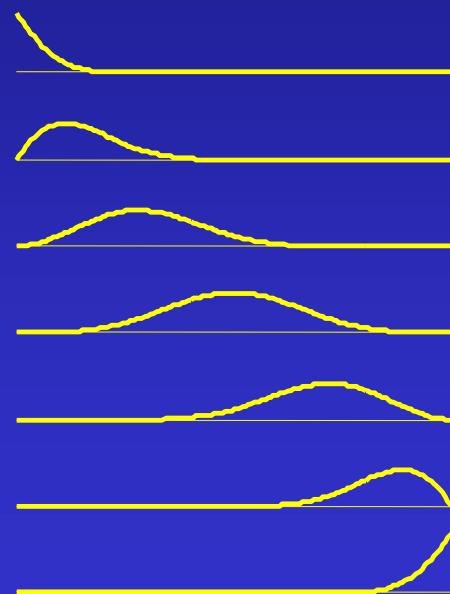
• Semi-orthogonalité: $V^{n+1} = V^n \overset{\perp}{\oplus} W^n$

=> meilleure approximation entre V^{n+1} et V^n



V^n, ϕ_i^n

$$q_{ki} \phi_k^{n+1}$$



W^n, ψ_i^n

V^{n+1}, ϕ_i^{n+1}

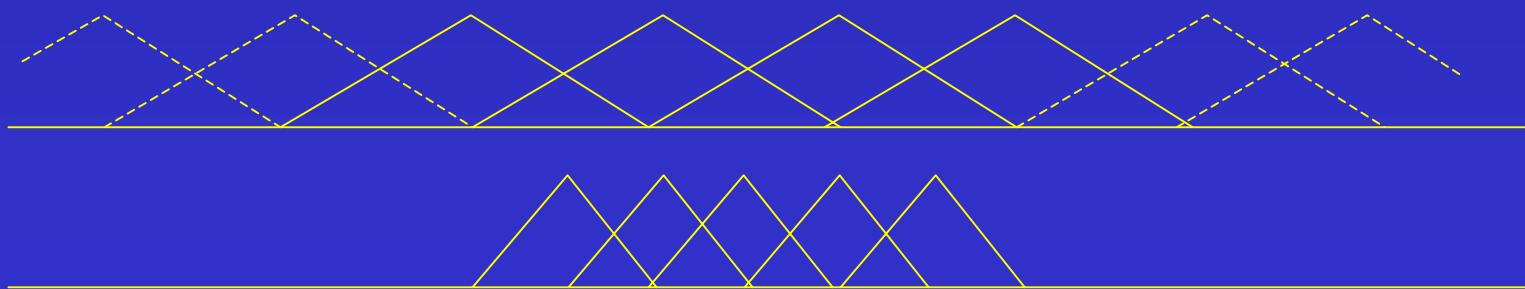
Fonctions ondelettes

$$\psi_i^n = \sum_k q_{ki} \varphi_k^{n+1}$$

Inconnues: q_{ki}

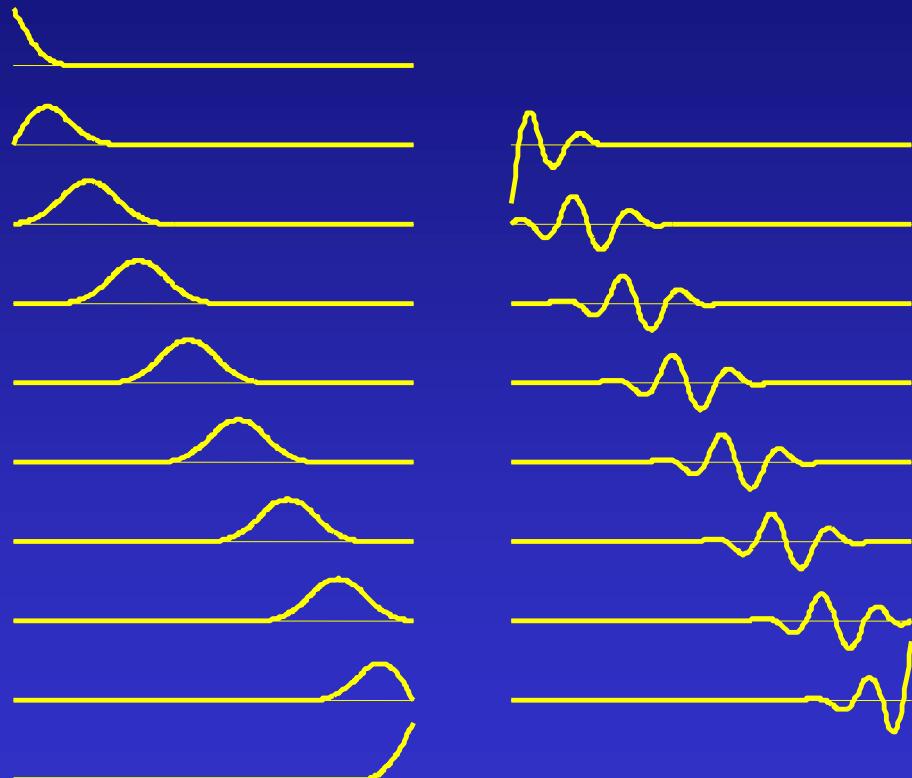
$$\langle \psi_i^n | \varphi_j^n \rangle = 0 \text{ : conditions linéaires en les inconnues}$$

- Quand on augmente le nombre d'inconnues, le nombre d'équations augmente moins vite => existence de solutions
- Recherche de la solution de support minimal

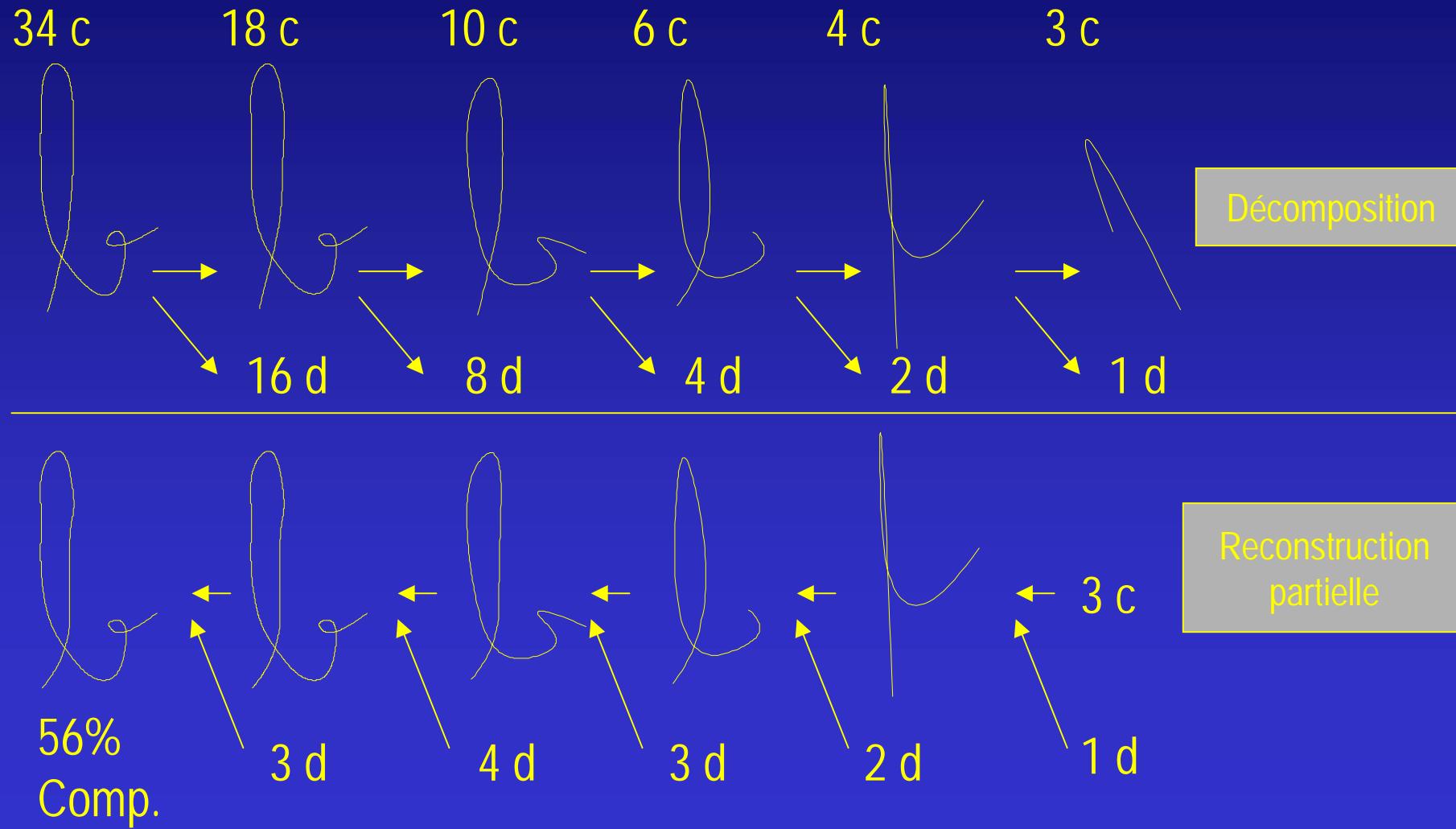


Fonctions ondelettes

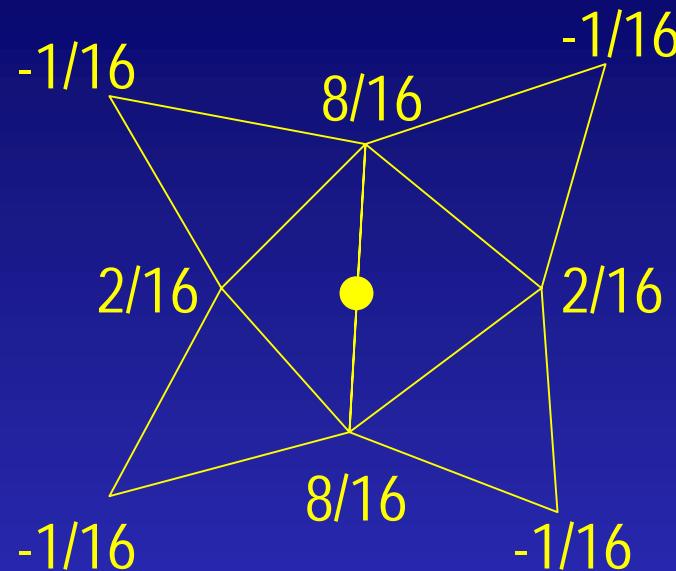
- Degré 2, niveau 3 de subdivision

 V^3 W^3

Exemple de reconstruction partielle

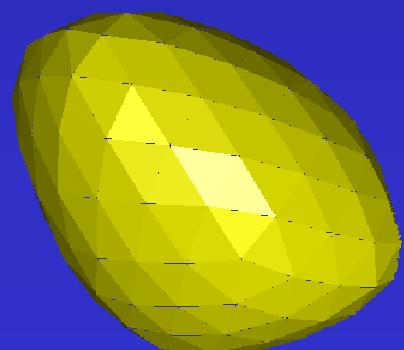


Surfaces de subdivision uniforme

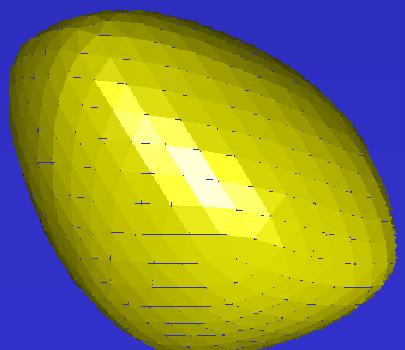


Butterfly Scheme (Dyn, Levin, Gregory)

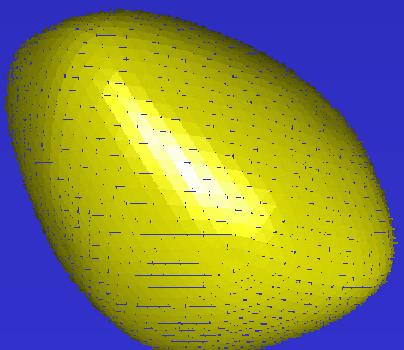
320 t



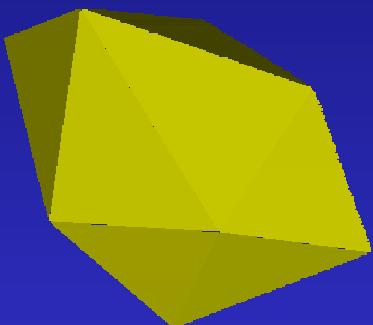
1280 t



5120 t



20 t



80 t

