

# Differential Space-Time Block-Coded OFDMA for Frequency-Selective Fading Channels

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**Abstract**—Combining differential Alamouti space-time block code (DASTBC) with orthogonal frequency-division multiple access (OFDMA), this paper introduces a multiuser/multirate transmission scheme, which allows full-rate and full-diversity noncoherent communications using two transmit antennas over frequency-selective fading channels. Compared with the existing differential space-time coded OFDM designs, our scheme imposes no restrictions on signal constellations, and thus can improve the spectral efficiency by exploiting efficient modulation techniques such as QAM, APSK etc. The main principles of our design are as follows: OFDMA eliminates multiuser interference, and converts multiuser environments to single-user ones; Space-time coding achieves performance improvement by exploiting space diversity available with multiple antennas, no matter whether channel state information is known to the receiver. System performance is evaluated both analytically and with simulations.

## I. INTRODUCTION

Space-time (ST) coding has gained much attention since it provides significant capacity gain with multiple antennas. ST codes were originally designed for known slow flat fading channels and the single-user environments [1]-[3]. However, it is envisioned that future broadband wireless systems have to support high-data-rate and multiuser communications over frequency-selective fading channels. These requirements make space-time coded (STC) orthogonal frequency-division multiple access (OFDMA) schemes promising techniques because:

- orthogonal frequency-division multiple (OFDM) transforms a broadband frequency-selective fading channel into parallel correlated flat-fading subchannels. Therefore, it is natural to consider STC in the OFDM context [4]-[8].

- Based on OFDMA, multiuser interference (MUI) from all transmit antennas of interfering users can be eliminated determinately regardless of multipath channels [7]-[9].

Most research on STC-OFDM has assumed that perfect estimates of current channel fading conditions are available at the receiver [4]-[8]. However, in some situations, the considerable cost of channel estimation and the degradation of tracking quality in fast time-varying environments make differential STC-OFDM schemes attractive alternatives.

The first differential STC-OFDM scheme was given in [10], which is assumed to utilize the PSK constellations, and only can be used in single-user environments. In this paper, we

propose an *arbitrary-signal-constellation-based* differential ST transmission scheme for wireless finite-impulse response (FIR) channels and *multi-user/multirate* settings by seamlessly integrating differential Alamouti space-time block code (DASTBC) with OFDMA. Two key ideas of our design are as follows: Firstly, OFDMA guarantees deterministic MUI elimination without destroying the orthogonality of ST block code; secondly, ST modulation with only linear decoding complexity provides full diversity gain to improve the system performance, no matter whether channel estimates are available at the receiver.

The rest of the paper is organized as follows. In section II, the model for DASTBC-OFDMA system is introduced. In section III, the design of users' codes in [7] is reviewed. In section IV, we present our new differential ST transmission scheme. In section V, we analyze the performance of our design. Simulation results are provided in section VI. Finally, some conclusions are made in section VII.

*Notation:*  $\otimes$  ( $\circ$ ) denotes Kronecker (Hadamard) product. Subscript  $T$ ,  $*$  and  $H$  stand for transpose, complex conjugate and complex conjugate transpose.  $\text{diag}([d_1, d_2, \dots, d_M]^T)$  is an  $M \times M$  diagonal matrix whose diagonal entries are determined by  $[d_1, d_2, \dots, d_M]^T$ .  $\mathbf{0}_{M \times N}$  ( $\mathbf{I}_{M \times N}$ ) denotes the  $M \times N$  matrix with all zero (one) entries, and  $\mathbf{I}_m$  the  $M \times M$  identity matrix.  $[s(n)](a, b)$  ( $a \leq b$ ) stands for the vector  $[s(a), s(a+1), \dots, s(b)]^T$ .  $[a, b]_I$  is a set of integers, whose element  $x$  satisfies the inequality  $a \leq x \leq b$ .

## II. MODEL FOR DASTBC-OFDMA SYSTEMS

Fig. 1 depicts the baseband model of a chip-sampled quasi-synchronous (QS) multiuser system with two transmit antennas and a single receive antenna for each user. Each of the  $M$  users transmits the modulated data symbols  $\tilde{s}_m(k)$  in block  $\tilde{\mathbf{s}}_m(n) = [\tilde{s}_m(k)](nQ_m, nQ_m + Q_m - 1)$  of size  $Q_m$  ( $m \in [1, M]_I$ ). The *block-based* ST encoder takes two consecutive blocks to output the following  $2Q_m \times 2$  ST coded matrix:

$$\bar{\mathbf{s}}_m(n) = \begin{bmatrix} \bar{s}_{m1}(2n) & \bar{s}_{m1}(2n+1) \\ \bar{s}_{m2}(2n) & \bar{s}_{m2}(2n+1) \end{bmatrix} \quad (1)$$

where  $\bar{s}_{mi}(n) = [\bar{s}_{mi}(l)](nQ_m, nQ_m + Q_m - 1)$  ( $i=1,2$ ) is a  $Q_m \times 1$  block transmitted through the  $i$ th antenna of user  $m$  at the time  $n$ . Before transmission, the user spreading code denoted by the  $P \times Q_m$  matrix  $\mathbf{C}_m$  is applied to  $\bar{\mathbf{s}}_m(n)$  to produce the

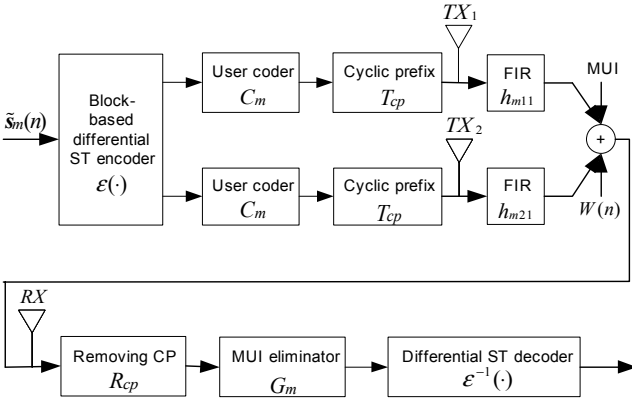


Fig. 1. Block diagram of DASTBC-OFDMA system

block  $\bar{\mathbf{u}}_{mi}(n) = \mathbf{C}_m \bar{\mathbf{s}}_{mi}(n)$  of length  $P$  ( $P > Q_m$ ). In order to eliminate inter-block interference (IBI) caused by the  $L$ th-order FIR channel  $\{h_{m,ij}(l)\}_{l=0}^L$  ( $j$  is used to distinguish the receive antennas), we rely on inserting a cyclic prefix (CP) of length  $L$  at the beginning of  $\bar{\mathbf{u}}_{mi}(n)$ . This CP insertion can be represented by the  $J \times P$  matrix  $\mathbf{T}_{cp} = [\mathbf{I}_{cp}^T \mathbf{I}_p^T]^T$  ( $J = P + L$ ), where  $\mathbf{I}_{cp}$  is formed by the last  $L$  rows of the  $P \times P$  identity matrix  $\mathbf{I}_p$ . After parallel-to-serial converted and pulse-shaped, the  $J$ -long block goes through the channel  $h_{m,ij}(l)$ , whose effect on the transmitted data is denoted by the  $J \times J$  Toeplitz matrix  $\bar{\mathbf{H}}_{m,ij}$  with first row  $[h_{m,ij}(0) \mathbf{0}_{1 \times (J-1)}]$  and first column  $[h_{m,ij}(0) \cdots h_{m,ij}(L) \mathbf{0}_{(P-1)}]^T$ . At the receiver, after sampling the receiver-filter output and discarding the CP, we obtain the receive block  $\tilde{\mathbf{y}}_j(n)$  which still contains MUI and additional noise. Correspondingly, the operation of discarding CP is described by the  $P \times J$  matrix  $\mathbf{R}_{cp} = [\mathbf{0}_{P \times L} \mathbf{I}_P]$ . With  $\mathbf{H}_{m,ij} = \mathbf{R}_{cp} \bar{\mathbf{H}}_{m,ij} \mathbf{T}_{cp}$ , the  $P \times 1$  IBI-free receive block  $\tilde{\mathbf{y}}_j(n)$  at the receive antenna  $j$  is given by [7]:

$$\tilde{\mathbf{y}}_j(n) = \mathbf{x}_{m,j}(n) + \sum_{\mu=1, \mu \neq m}^M \mathbf{x}_{\mu,j}(n) + \mathbf{R}_{cp} \mathbf{w}_j(n) \quad (2)$$

where  $\mathbf{w}_j(n)$  is the white noise vector whose entries are samples of independent complex Gaussian random variables with mean 0, and  $\mathbf{x}_{m,j}(n) = \mathbf{H}_{m,1j} \mathbf{C}_m \bar{\mathbf{s}}_{m1}(n) + \mathbf{H}_{m,2j} \mathbf{C}_m \bar{\mathbf{s}}_{m2}(n)$  denotes the received symbol block from the two transmit antennas of user  $m$ .

Given the block sequences  $\{\tilde{\mathbf{y}}_j(n)\}$ , we will use the following steps to recover  $\bar{\mathbf{s}}_m(n)$ : Firstly, we exploit the user separating code  $\mathbf{G}_m$  to remove MUI; secondly, with only linear processing of MUI-free block  $\mathbf{y}_{m,j}(n) = \mathbf{G}_m \tilde{\mathbf{y}}_j(n)$ , we may utilize ST decoder to collect the embedded antenna diversity built by ST encoder.

### III. THE DESIGN OF USERS' SPREADING CODES

Let's consider a DASTBC-OFDMA system containing  $P$  sub-carriers, which are denoted as  $\exp(j2\pi p/P)$  ( $p \in [0, P-1]$ ), with  $P$  being the sum of all  $Q_m$ s ( $m \in [1, M]$ ). Each user is allocated  $Q_m$  distinct subcarriers, which are represented as indices in the set  $I_m$ , and the user  $m$  is allowed to transmit information only on the subcarriers  $\{z | z = \exp(j2\pi p/P), p \in I_m\}$ .

Let  $\mathbf{F}$  denote the  $P \times P$  FFT matrix, whose  $(k, p)$  entry is defined as  $[\mathbf{F}]_{k,p} = P^{-1/2} \exp\{-j2\pi(k-1)(p-1)/P\}$ . Let  $\Phi_m$  be the  $P \times Q_m$  matrix which consists of the  $I_m$  columns of the  $P \times P$  identity matrix, then, the user spreading code  $\mathbf{C}_m$  and the user separating code  $\mathbf{G}_m$  are designed as [7]:

$$\mathbf{C}_m = \mathbf{F}^H \Phi_m, \quad \mathbf{G}_m = \Phi_m^T \mathbf{F}. \quad m \in [1, M] \quad (3)$$

Using (2), (3), we obtain [7]

$$\mathbf{y}_{m,j}(n) = \mathbf{D}_{m,1j} \bar{\mathbf{s}}_{m1}(n) + \mathbf{D}_{m,2j} \bar{\mathbf{s}}_{m2}(n) + \mathbf{n}_{m,j}(n) \quad (4)$$

where  $\mathbf{n}_{m,j}(n) = \mathbf{G}_m \mathbf{R}_{cp} \mathbf{w}_j(n)$  is also Gaussian white noise,  $\mathbf{D}_{m,ij}$  is a  $Q_m \times Q_m$  diagonal matrix holding the frequency response of the channel  $h_{m,ij}(l)$  at the  $I_m$  subcarriers:  $\mathbf{D}_{m,ij} = \text{diag}([H_{m,ij}(\rho_{m1}), \dots, H_{m,ij}(\rho_{mQ_m})]^T)$  ( $\rho_{mk} = \exp(j2\pi p_k/P)$ ,  $p_k \in I_m$ ,  $k \in [1, Q_m]$ ), and  $H_{m,ij}(\rho)$  is defined as

$$H_{m,ij}(\rho) = \sum_{l=0}^L h_{m,ij}(l) \rho^{-l}.$$

From (4), it can be seen that MUI elimination does not require the channel state information at the receiver.

### IV. DIFFERENTIAL ST ENCODING AND DECODING

*Definition:* Let  $\mathbf{s}_i$  ( $i = 1, 2$ ) denote  $Q \times 1$  complex column vector, whose  $k$ th entry is expressed as  $s_i(k)$ . If  $\alpha_k^2 = |s_1(k)|^2 + |s_2(k)|^2 \neq 0$  holds for any given  $k$  ( $k \in [1, Q]$ ), then, the  $Q \times Q$  diagonal matrix  $\mathbf{A}(\mathbf{s}_1, \mathbf{s}_2) = \text{diag}([\alpha_1^{-1}, \alpha_2^{-1}, \dots, \alpha_Q^{-1}]^T)$  is called Alamouti normalized matrix (ANM) determined by  $\mathbf{s}_1, \mathbf{s}_2$ , the  $Q \times 1$  vector  $\mathbf{e}(\mathbf{s}_1, \mathbf{s}_2) = [\alpha_1^2, \alpha_2^2, \dots, \alpha_Q^2]^T$  is called energy vector given by  $\mathbf{s}_1, \mathbf{s}_2$ , and Alamouti quasi-diagonal matrix (AQDM) is defined as:

$$\mathbf{AQD}(\mathbf{s}_1, \mathbf{s}_2) = \begin{bmatrix} \text{diag}(\mathbf{s}_1) \cdot \mathbf{A}(\mathbf{s}_1, \mathbf{s}_2) & -[\text{diag}(\mathbf{s}_2) \cdot \mathbf{A}(\mathbf{s}_1, \mathbf{s}_2)]^* \\ \text{diag}(\mathbf{s}_2) \cdot \mathbf{A}(\mathbf{s}_1, \mathbf{s}_2) & [\text{diag}(\mathbf{s}_1) \cdot \mathbf{A}(\mathbf{s}_1, \mathbf{s}_2)]^* \end{bmatrix}. \quad (5)$$

#### A. Encoding

According to our design, the coded matrix at the  $n$ th time block is determined by

$$\bar{\mathbf{S}}_m(n) = \mathbf{AQD}(\bar{\mathbf{s}}_{m1}(2n-2), \bar{\mathbf{s}}_{m2}(2n-2)) \cdot \begin{bmatrix} \bar{\mathbf{s}}_m(2n) & -\bar{\mathbf{s}}_m^*(2n+1) \\ \bar{\mathbf{s}}_m(2n+1) & \bar{\mathbf{s}}_m^*(2n) \end{bmatrix} \quad (6)$$

where  $\bar{\mathbf{s}}_{m1}(0) = \mathbf{I}_{Q_m \times 1}$ ,  $\bar{\mathbf{s}}_{m2}(0) = \mathbf{0}_{Q_m \times 1}$ ,  $\bar{\mathbf{s}}_{m1}(1) = \mathbf{0}_{Q_m \times 1}$ ,  $\bar{\mathbf{s}}_{m2}(1) = \mathbf{I}_{Q_m \times 1}$ . On the basis of (6), we reorganize (4) as follows

$$\begin{bmatrix} \mathbf{y}_{m,j}^T(2n) \\ \mathbf{y}_{m,j}^T(2n+1) \end{bmatrix} = [\bar{\mathbf{D}}_{m,1j}(n) \quad \bar{\mathbf{D}}_{m,2j}(n)] \begin{bmatrix} \bar{\mathbf{s}}_m(2n) & -\bar{\mathbf{s}}_m^*(2n+1) \\ \bar{\mathbf{s}}_m(2n+1) & \bar{\mathbf{s}}_m^*(2n) \end{bmatrix} + \mathbf{n}_{m,j}(n) \quad (7a)$$

$$[\bar{\mathbf{D}}_{m,1j}(n) \quad \bar{\mathbf{D}}_{m,2j}(n)] = [\mathbf{D}_{m,1j} \quad \mathbf{D}_{m,2j}] \cdot \mathbf{AQD}(\bar{\mathbf{s}}_{m1}(2n-2), \bar{\mathbf{s}}_{m2}(2n-2)) \quad (7b)$$

where  $\bar{\mathbf{D}}_{m,ij}$  ( $i=1,2$ ) is the  $Q_m \times Q_m$  diagonal matrix.

#### B. Decoding

In the differential case, we assume that the channel keeps unchanged within two consecutive time blocks, and the receiver doesn't know the channel. For simplicity, we denote the ANM given by  $\bar{\mathbf{s}}_{m1}(2n-2), \bar{\mathbf{s}}_{m2}(2n-2)$  as  $\mathbf{A}(n)$ . Let  $\mathbf{Y}_{ij}(n) = \text{diag}(\mathbf{y}_{m,j}(2n-3+i))$ ,  $\mathbf{N}_{ij}(n) = \text{diag}(\mathbf{n}_{m,j}(2n-3+i))$  ( $i=1,2$ ). Using (4), (7b) and the equations  $\bar{\mathbf{s}}_{m1}(2n+1) = -\bar{\mathbf{s}}_{m2}^*(2n)$ ,  $\bar{\mathbf{s}}_{m2}(2n+1) = \bar{\mathbf{s}}_{m1}^*(2n)$  derived from (6), we obtain

$$\bar{\mathbf{D}}_{m,ij}(n) = [\mathbf{Y}_{ij}(n) - \mathbf{N}_{ij}(n)] \cdot \mathbf{A}(n), \quad i=1,2. \quad (8)$$

Plugging (8) into (7a), we arrive at

$$\begin{bmatrix} \mathbf{y}_{m,j}^T(2n) \\ \mathbf{y}_{m,j}^T(2n+1) \end{bmatrix} = \begin{bmatrix} [\mathbf{Y}_j(n)\mathbf{A}(n)]^T \\ [\mathbf{Y}_j(n)\mathbf{A}(n)]^T \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_m(2n) & -\tilde{\mathbf{s}}_m^*(2n+1) \\ \tilde{\mathbf{s}}_m(2n+1) & \tilde{\mathbf{s}}_m^*(2n) \end{bmatrix} + \hat{\boldsymbol{\eta}}_{m,j}(n) \quad (9a)$$

$$\hat{\boldsymbol{\eta}}_{m,j}(n) = - \begin{bmatrix} [\mathbf{N}_1(n)\mathbf{A}(n)]^T \\ [\mathbf{N}_2(n)\mathbf{A}(n)]^T \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_m(2n) & -\tilde{\mathbf{s}}_m^*(2n+1) \\ \tilde{\mathbf{s}}_m(2n+1) & \tilde{\mathbf{s}}_m^*(2n) \end{bmatrix} + \boldsymbol{\eta}_{m,j}(n). \quad (9b)$$

In order to exploit space diversity, ST decoder forms two  $Q_m \times 1$  decision vectors  $\tilde{\mathbf{z}}_{m,j}(2n)$ ,  $\tilde{\mathbf{z}}_{m,j}(2n+1)$

$$\begin{bmatrix} \tilde{\mathbf{z}}_{m,j}(2n) \\ \tilde{\mathbf{z}}_{m,j}(2n+1) \end{bmatrix} = \begin{bmatrix} [\mathbf{Y}_j(n)\mathbf{A}(n)]^* & \mathbf{Y}_j(n)\mathbf{A}(n) \\ [\mathbf{Y}_j(n)\mathbf{A}(n)]^* & -\mathbf{Y}_j(n)\mathbf{A}(n) \end{bmatrix} \begin{bmatrix} \mathbf{y}_{m,j}(2n) \\ \mathbf{y}_{m,j}^*(2n+1) \end{bmatrix}. \quad (10)$$

Let  $\bar{n}$  be the integral part of  $n/2$ . Now Substituting (9a) into (10), we obtain

$$\tilde{\mathbf{z}}_{m,j}(n) = [\mathbf{A}(\bar{n})]^2 \left[ (\mathbf{Y}_j(\bar{n}))^* \mathbf{Y}_j(\bar{n}) + (\mathbf{Y}_j(\bar{n}))^* \mathbf{Y}_j(\bar{n}) \right] \tilde{\mathbf{s}}_m(n) + \hat{\boldsymbol{\eta}}_{m,j}(n). \quad (11)$$

Using (6) and the definition of AQDM, we may easily show  $\mathbf{e}(\tilde{\mathbf{s}}_m(2n-2), \tilde{\mathbf{s}}_m(2n-2)) = \mathbf{e}(\tilde{\mathbf{s}}_m(2n-2), \tilde{\mathbf{s}}_m(2n-1))$ . Hence, exploiting the data blocks already detected at the previous time block, we can obtain the estimation of  $\mathbf{A}(n)$ , which will be used in (11) to complete the current decoding. Let  $\hat{\boldsymbol{\eta}}_{m,j}(n) = [\hat{\boldsymbol{\eta}}_{m,j}^T(2n) \hat{\boldsymbol{\eta}}_{m,j}^T(2n+1)]^T$ ,  $\bar{\mathbf{A}}(n) = \mathbf{I}_2 \otimes \mathbf{A}(n)$ , then, we have

$$\hat{\boldsymbol{\eta}}_{m,j}(n) = [\bar{\mathbf{A}}(n)]^2 \begin{bmatrix} \mathbf{Y}_j^*(n) & \mathbf{Y}_j(n) \\ \mathbf{Y}_j^*(n) & -\mathbf{Y}_j(n) \end{bmatrix} \begin{bmatrix} -\mathbf{N}_1(n)\tilde{\mathbf{s}}_m(2n) - \mathbf{N}_2(n)\tilde{\mathbf{s}}_m(2n+1) \\ \mathbf{N}_1^*(n)\tilde{\mathbf{s}}_m(2n+1) - \mathbf{N}_2^*(n)\tilde{\mathbf{s}}_m(2n) \end{bmatrix} + \bar{\mathbf{A}}(n) \begin{bmatrix} \mathbf{Y}_j^*(n) & \mathbf{Y}_j(n) \\ \mathbf{Y}_j^*(n) & -\mathbf{Y}_j(n) \end{bmatrix} \begin{bmatrix} \mathbf{n}_{m,j}(2n) \\ \mathbf{n}_{m,j}^*(2n+1) \end{bmatrix}. \quad (12)$$

This means that  $\hat{\boldsymbol{\eta}}_{m,j}(n)$  is colored in general, and thus the maximum-likelihood (ML) detection based on (11) leads to the exponential complexity of the ML decoders. However, while signal-to-noise ratio (SNR) is high, we can neglect the second-order noise terms in  $\hat{\boldsymbol{\eta}}_{m,j}(n)$  and approximate  $\hat{\boldsymbol{\eta}}_{m,j}(n)$  by a complex Gaussian random vector  $\tilde{\boldsymbol{\eta}}_{m,j}(n)$

$$\tilde{\boldsymbol{\eta}}_{m,j}(n) = \bar{\mathbf{A}}(n) \begin{bmatrix} \bar{\mathbf{D}}_{m,1j}^*(n) & \bar{\mathbf{D}}_{m,2j}(n) \\ \bar{\mathbf{D}}_{m,2j}^*(n) & -\bar{\mathbf{D}}_{m,1j}(n) \end{bmatrix} \begin{bmatrix} -\mathbf{N}_1(n)\tilde{\mathbf{s}}_m(2n) - \mathbf{N}_2(n)\tilde{\mathbf{s}}_m(2n+1) \\ \mathbf{N}_1^*(n)\tilde{\mathbf{s}}_m(2n+1) - \mathbf{N}_2^*(n)\tilde{\mathbf{s}}_m(2n) \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{D}}_{m,1j}^*(n) & \bar{\mathbf{D}}_{m,2j}(n) \\ \bar{\mathbf{D}}_{m,2j}^*(n) & -\bar{\mathbf{D}}_{m,1j}(n) \end{bmatrix} \begin{bmatrix} \mathbf{n}_{m,j}(2n) \\ \mathbf{n}_{m,j}^*(2n+1) \end{bmatrix}. \quad (13)$$

Using  $\bar{\mathbf{D}}_{m,1j}^*(\bar{n})\bar{\mathbf{D}}_{m,1j}(\bar{n}) + \bar{\mathbf{D}}_{m,2j}^*(\bar{n})\bar{\mathbf{D}}_{m,2j}(\bar{n}) = \bar{\mathbf{D}}_{m,1j}^* \mathbf{D}_{m,1j} + \bar{\mathbf{D}}_{m,2j}^* \mathbf{D}_{m,2j}$ , we can obtain the covariance matrix of  $\tilde{\boldsymbol{\eta}}_{m,j}(n)$  from (13)

$$\text{Cov}[\tilde{\boldsymbol{\eta}}_{m,j}(n)] = \delta_w^2 \begin{bmatrix} \mathbf{D}_{m,j} & \mathbf{0}_{Q_m \times Q_m} \\ \mathbf{0}_{Q_m \times Q_m} & \mathbf{D}_{m,j} \end{bmatrix} (\mathbf{I}_{2Q_m} + [\bar{\mathbf{A}}(n)\bar{\mathbf{C}}(n)]^2) \quad (14)$$

where  $\bar{\mathbf{C}}(n) = [\bar{\mathbf{A}}(n+1)]^{-1}$ . The above covariance matrix shows that the entries of  $\tilde{\boldsymbol{\eta}}_{m,j}(n)$  are independent. Noting that  $\mathbf{Y}_j(\bar{n})$  ( $i=1,2$ ) are diagonal, we can reasonably conclude that the independent detection for each entry of  $\tilde{\mathbf{s}}_m(n)$  based on (11) will incur little performance loss compared with the ML detection of  $\tilde{\mathbf{s}}_m(n)$ .

## V. PERFORMANCE ANALYSIS

In this section, we'll consider the case where each user is equipped with  $N_R$  receive antennas. To improve the system performance, we form a combined decision vector  $\tilde{\mathbf{z}}_m(n)$

$$\tilde{\mathbf{z}}_m(n) = \sum_{j=1}^{N_R} \tilde{\mathbf{z}}_{m,j}(n) = [\mathbf{A}(\bar{n})]^2 \left[ \sum_{j=1}^{N_R} \mathbf{Y}_j(\bar{n}) \right] \cdot \tilde{\mathbf{s}}_m(n) + \sum_{j=1}^{N_R} \hat{\boldsymbol{\eta}}_{m,j}(n), \quad (15)$$

where  $\mathbf{Y}_j(\bar{n}) = (\mathbf{Y}_j(\bar{n}))^* \mathbf{Y}_j(\bar{n}) + (\mathbf{Y}_j(\bar{n}))^* \mathbf{Y}_j(\bar{n})$ . While SNR is large enough, (15) can be rewritten as (c.f. (8))

$$\tilde{\mathbf{z}}_m(n) \approx \left[ \sum_{j=1}^{N_R} \mathbf{D}_{m,j} \right] \cdot \tilde{\mathbf{s}}_m(n) + \sum_{j=1}^{N_R} \hat{\boldsymbol{\eta}}_{m,j}(n) \quad (16)$$

If we use the column vector  $\mathbf{snr}_{m, \text{DI}}(n)$  to denote the average SNRs of the decision metrics in  $\tilde{\mathbf{z}}_m(n)$ , then we have (c.f. (14) and (16))

$$\mathbf{snr}_{m, \text{DI}}(n) \approx \delta_w^2 \left[ \sum_{j=1}^{N_R} \mathbf{D}_{m,j} \right] \cdot (\mathbf{I}_{Q_m} + [\mathbf{A}(n)\mathbf{C}(n)]^2)^{-1} \cdot [\tilde{\mathbf{s}}_m(n) \circ \tilde{\mathbf{s}}_m^*(n)] \quad (17)$$

where  $\mathbf{C}(n) = [\mathbf{A}(n+1)]^{-1}$ . For the purpose of performance comparison, let us firstly derive the SNRs of the decision metrics used in coherent STC-OFDMA systems [7]. Assuming that we do not consider the linear precoding in [7], then, the combined decision vector  $\mathbf{z}_m(n)$  used to detect  $\tilde{\mathbf{s}}_m(n)$  can be expressed as (c.f. (8) of [7])

$$\mathbf{z}_m(n) = \left[ \sum_{j=1}^{N_R} \mathbf{D}_{m,j} \right] \cdot \tilde{\mathbf{s}}_m(n) + \sum_{j=1}^{N_R} \hat{\boldsymbol{\eta}}_{m,j}(n). \quad (18)$$

Let  $\hat{\boldsymbol{\eta}}_{m,j}(n) = [[\hat{\boldsymbol{\eta}}_{m,j}^T(2n) \hat{\boldsymbol{\eta}}_{m,j}^T(2n+1)]^T]^T$ . From (9) of [7], we may deduce that

$$\hat{\boldsymbol{\eta}}_{m,j}(n) = \begin{bmatrix} \mathbf{D}_{m,1j}^* & \mathbf{D}_{m,2j} \\ \mathbf{D}_{m,2j}^* & -\mathbf{D}_{m,1j} \end{bmatrix} \begin{bmatrix} \mathbf{n}_{m,j}(2n) \\ \mathbf{n}_{m,j}^*(2n+1) \end{bmatrix}, \quad (19)$$

where the covariance matrix of  $\hat{\boldsymbol{\eta}}_{m,j}(n)$  is given by

$$\text{Cov}[\hat{\boldsymbol{\eta}}_{m,j}(n)] = \delta_w^2 \begin{bmatrix} \mathbf{D}_{m,j} & \mathbf{0}_{Q_m \times Q_m} \\ \mathbf{0}_{Q_m \times Q_m} & \mathbf{D}_{m,j} \end{bmatrix}. \quad (20)$$

Since  $\mathbf{D}_{m,ij}$  ( $i=1,2$ ) are diagonal, we may easily derive from (18) and (20) a linear-complexity coherent decoder, which can independently detect each entry of  $\tilde{\mathbf{s}}_m(n)$  without loss in performance. Similar to (17), we utilize  $Q_m \times 1$  vector  $\mathbf{snr}_{m, \text{coherent}}(n)$  to denote the average SNRs of the decision metrics in  $\mathbf{z}_m(n)$ . Based on (18) and (20), we may obtain the average SNR under the coherent condition

$$\mathbf{snr}_{m, \text{coherent}}(n) = \delta_w^{-2} \left[ \sum_{j=1}^{N_R} \mathbf{D}_{m,j} \right] \cdot [\tilde{\mathbf{s}}_m(n) \circ \tilde{\mathbf{s}}_m^*(n)] \quad (21)$$

From (17) and (21), we can see that our STC-OFDMA system will recover modulated data symbols with full antenna diversity, no matter whether channel state information is available at the receiver.

To investigate the performance degradation in noncoherent detection, Let us consider a special case where PSK constellation is employed. We assume that all the modulated data symbols have the same amplitudes  $(2)^{-0.5} A_P$ . In such situation, the encoder (6) is simplified to

$$\bar{\mathbf{s}}_m(n) = \frac{1}{A_P} \begin{bmatrix} \text{diag}(\bar{\mathbf{s}}_1(n)) & -[\text{diag}(\bar{\mathbf{s}}_2(n))]^* \\ \text{diag}(\bar{\mathbf{s}}_2(n)) & [\text{diag}(\bar{\mathbf{s}}_1(n))]^* \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_m(2n) & -\tilde{\mathbf{s}}_m^*(2n+1) \\ \tilde{\mathbf{s}}_m(2n+1) & \tilde{\mathbf{s}}_m^*(2n) \end{bmatrix}, \quad (22)$$

where  $\bar{\mathbf{s}}_i(n) = \bar{\mathbf{s}}_{mi}(2n-2)$  ( $i=1,2$ ), and (10) reduces to

$$\begin{bmatrix} \tilde{\mathbf{z}}_{m,j}(2n) \\ \tilde{\mathbf{z}}_{m,j}(2n+1) \end{bmatrix} = \frac{1}{A_P} \begin{bmatrix} [\mathbf{Y}_j(n)]^* & \mathbf{Y}_j(n) \\ [\mathbf{Y}_j(n)]^* & -\mathbf{Y}_j(n) \end{bmatrix} \begin{bmatrix} \mathbf{y}_{m,j}(2n) \\ \mathbf{y}_{m,j}^*(2n+1) \end{bmatrix}. \quad (23)$$

That is, the differential decoding algorithm is independent of the previous detection results. Since  $\mathbf{A}(n) = [\mathbf{C}(n)]^{-1} = (A_P)^{-1} \mathbf{I}_{Q_m}$

holds for PSK modulation, while SNR is high, we have

$$\text{snr}_{m, \text{DI}}(n) \approx 0.5 \cdot \delta_w^{-2} \left[ \sum_{j=1}^{N_R} \mathbf{D}_{m,j} \right] \cdot [\tilde{s}_m(n) \circ \tilde{s}_m^*(n)]. \quad (24)$$

Comparing this result with the SNR for the coherent detection case (see (21)), we know that there is about a 3dB loss in the differential decoding.

To proceed further with our performance analysis, we assume that QPSK signal constellations are employed. Our figure of merit is the average bit error rate (BER), defined as

$$\bar{P}_{em}(n) = (Q_m)^{-1} \sum_{k=0}^{Q_m-1} P_{m,k}(n) \quad (25)$$

where  $P_{m,k}(n)$  denotes the BER for the modulated data symbol  $\tilde{s}_m(nQ_m + k)$  whose energy we denote by  $E_{m,k}(n)$ . It follows from (17) and (21) that [11]

$$\bar{P}_{em}(n) = (Q_m)^{-1} \sum_{k=0}^{Q_m-1} Q \left( \sqrt{\alpha_{m,k}(n) \cdot \delta_w^{-2} \cdot E_{m,k}(n) \cdot H_{m,k}} \right) \quad (26a)$$

$$\alpha_{m,k}(n) = \begin{cases} 1 & \text{coherent.} \\ \left[ 1 + \frac{E_{m,k}(n) + E_{m,k}(n+Q_m)}{E_{m,k}(n-2Q_m) + E_{m,k}(n-Q_m)} \right]^{-1} & \text{DI, } n \text{ is even.} \\ \left[ 1 + \frac{E_{m,k}(n-Q_m) + E_{m,k}(n)}{E_{m,k}(n-3Q_m) + E_{m,k}(n-2Q_m)} \right]^{-1} & \text{DI, } n \text{ is odd.} \end{cases} \quad (26b)$$

$$H_{m,k} = \sum_{i=1}^2 \sum_{j=1}^{N_R} |H_{m,ij}(\exp(j2\pi p_k / P))|^2 \quad (26c)$$

where  $Q(x) = (2\pi)^{-0.5} \int_x^\infty e^{-t^2/2} dt$  ( $x \geq 0$ ), DI denotes the differential case, and  $p_k$  is the value of the  $k$ th element in the set  $I_m$ .

## VI. SIMULATION RESULTS

In this section, we present simulation results for  $N_R = 1$  and  $N_R = 2$  receive antennas. The OFDMA system in our simulation has 2MHz bandwidth and use the parameter  $P = 256$ . Two high-rate users, two middle-rate users and four low-rate users are tested. With each user being equipped with two transmit antennas, the high-rate user is allocated 64 subcarriers, the middle-rate user has 32 subcarriers, whereas the low-rate user is assigned 16 subcarriers. The simulations are carried out for various  $E/N_0$  (SNR) points, where  $N_0$  is the noise power spectral density level, and  $E$  is the average signal power at each receive antenna. To evaluate the system performance in actual application environment, we consider a Rayleigh fading channel model recommended by GSM Recommendation 05.05 [12], with the maximum delay spread equal to  $5 \mu s$  ( $L = 10$ ). For simplicity, we assume that the random channels  $h_{m,ij}(l)$  with 20Hz Doppler frequency are independent for different  $m, i, j$ . Since the performance curves are almost the same for the users with different data rates, we only discuss the simulation results for the middle-rate users.

Fig. 2 presents the system performance while  $N_R = 1$  receive antenna is used at the receiver. Observing that the BER gain achieved by 16QAM-DASTBC-OFDMA is about

1.5dB at  $\text{BER} = 2 \times 10^{-3}$  compared with 16PSK-DASTBC-OFDMA, it is deduced that our multi-amplitude-and-multi-phase design for differential ST coding indeed results in BER improvement. In fact, it is well known that the performance of 16QAM modulation is superior to that of 16PSK modulation [11]. Utilizing the simple expression for the decoder described by (11), we may easily understand above simulation results. Another notable fact in Fig. 2 is that the noncoherent decoding of 16QAM-DASTBC-OFDMA achieves the same space-time diversity, but with a SNR degradation of 3~4dB while compared with the results of coherent decoding.

Fig. 3 provides the performance curves for the case of  $N_R = 2$ . At a BER of  $10^{-3}$ , the simulation shows that the performance of 16QAM-DASTBC-OFDMA is about 2.5dB better than that of 16PSK-DASTBC-OFDMA. In addition, it can be seen from the simulation results that the curves of Fig. 3 decline more rapidly than that of Fig. 2 when the same constellations are employed. This is due to the fact that the gain of antenna diversity will become greater when the number of receive antennas increases.

## VII. CONCLUSION

Relying on symbol blocking and OFDMA, we present a differential ST coded multiuser/multirate system suitable for frequency-selective fading channels. Since our schemes impose no restrictions on underlying signal constellation, we can expect the spectral efficiency to be increased by carrying information not only on phases but also on amplitudes. The goal of our system design is twofold: Firstly, OFDMA is exploited to eliminate MUI determinately; secondly, ST coding is designed to recover transmitted symbols with full diversity gain and linear complexity, no matter whether channel estimates are available at the receiver. With a little modification, our scheme can be integrated with linear precoding to further improve the system performance. Future research will be focused on designing differential multiuser/multirate transmission system that can exploit space, time and frequency diversity for fast-fading multipath channels.

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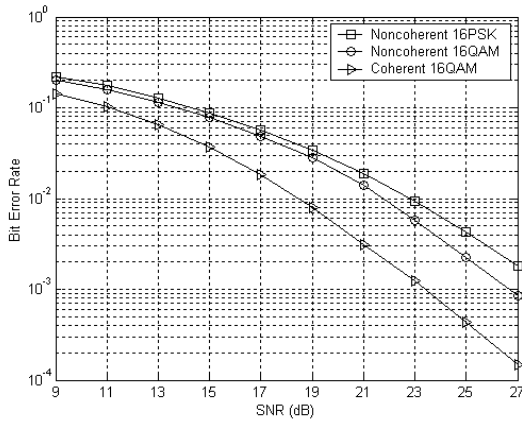


Fig. 2 Performance curves for one receive antenna.

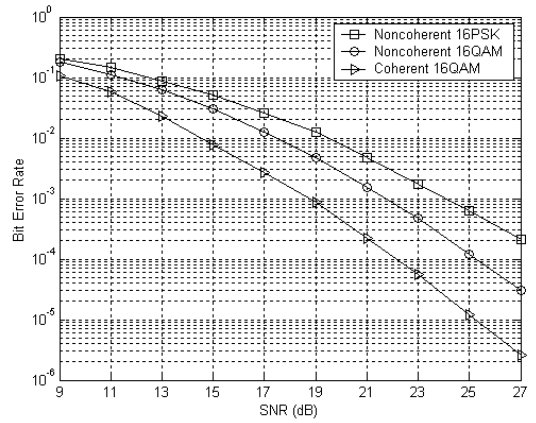


Fig. 3 Performance curves for two receive antennas.

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