Queuing analysis of simple FEC schemes for Voice over IP

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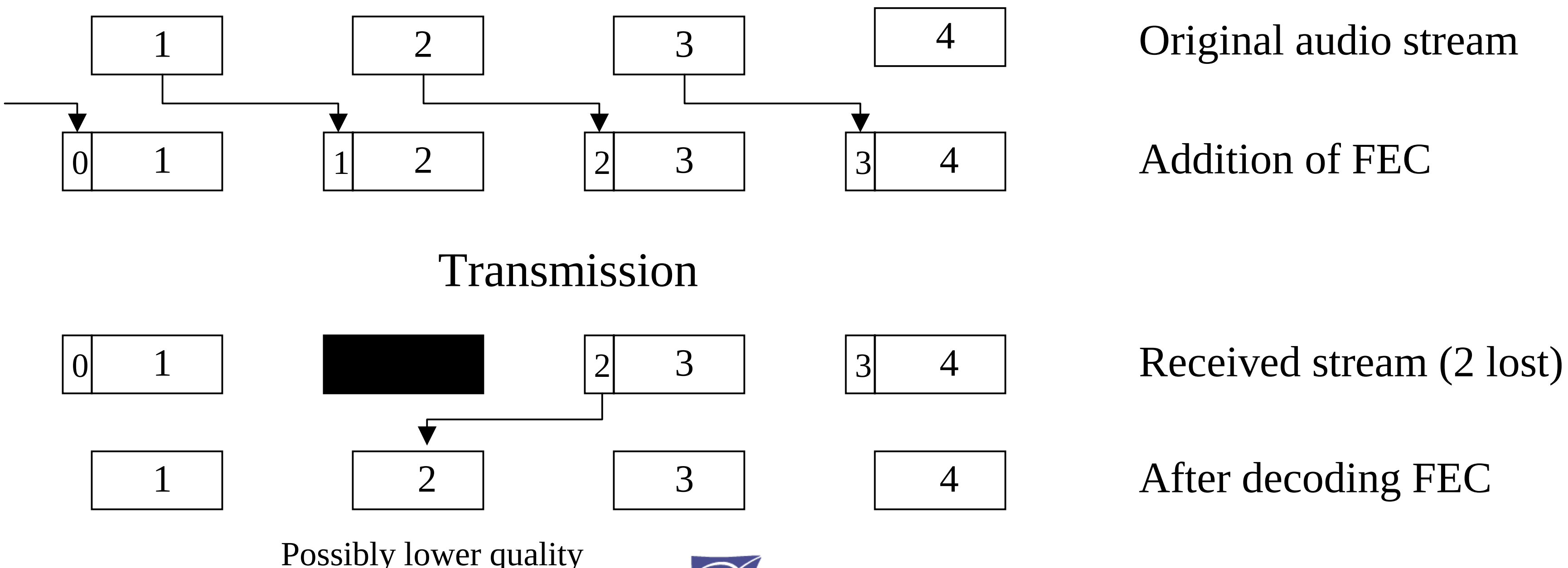
Outline

- ☞ Audio FEC scheme to analyze.
- ☞ Analytical model for audio quality.
- ☞ **Analysis:** Use of a ballot theorem.
- ☞ **Numerical results:** Negative ...
- ☞ Conclusions and perspectives.

FEC for audio application

- **Objective:** Reconstruct packet losses to improve audio quality.
- **Idea:** Add redundant information that can be used when packets are lost.
- Simple FEC scheme (standardized by IETF) (Rat, FreePhone)



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Some questions

- Do we gain in audio quality by adding FEC?
  - *Same original stream:*
    FEC increases the load of the network and hence the packet loss probability (lower quality?).
  - *Reducing the rate of the original stream:*
    Does FEC compensate the original information we did not send?
- When does such a FEC scheme improve the quality, and when it does not?
- Is this simple scheme the optimal one?

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. A simple queuing model to understand the problem ...
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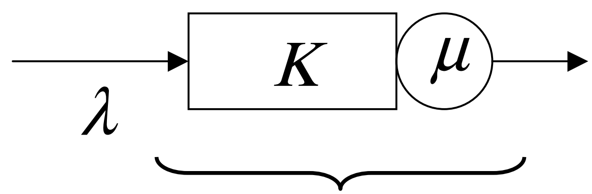
M/M/1/K model

- Before the addition of FEC

Audio flow (s)
Poisson



Network

Exponential service

$$\rho = \lambda / \mu$$

- Holds when:
  - All packets in the network are audio, or when,
  - A per-flow queuing is used in routers.

Audio packet loss probability:

$$\pi = \frac{1-\rho}{1-\rho^{K+1}} \rho^{K}$$

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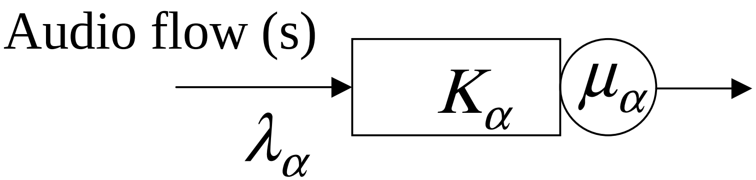
Model in presence of FEC

α : Ratio of FEC and original packet size.

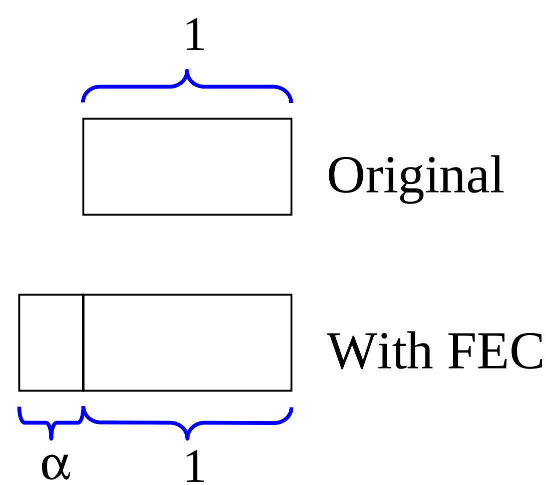


Loss probability $\pi_\alpha = \dfrac{1-\rho_\alpha}{1-\rho_\alpha^{K_\alpha+1}} \rho_\alpha^{K_\alpha}$

Consider first the case when the original audio stream is not changed:

- $\lambda_\alpha = \lambda$
- $K_\alpha = K$ or $K_\alpha = K/(1+\alpha)$
- $\underbrace{\mu_\alpha = \mu/(1+\alpha)}$

All flows are adding FEC, or
Round-robin service in routers.



Audio quality

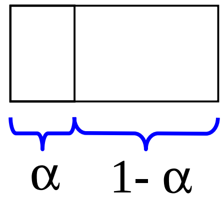
- **Assumptions:**
  - Audio quality increases **linearly** with the volume of data in a packet.
  - "1" is the quality obtained when we correctly receive an original packet.

$$Y_n \in \{0,1\} \text{ : Original packet } n \text{ lost or no.}$$

- Original audio stream is not changed:

$$Q_\alpha = 1.\underbrace{\mathrm{P}\{Y_n = 1\}}_{1-\pi_\alpha} + \alpha.\underbrace{\mathrm{P}\{Y_n = 0\}}_{\pi_\alpha}.\underbrace{\mathrm{P}\{Y_{n+1} = 1 \mid Y_n = 0\}}_{\frac{1}{\rho_\alpha + 1}}$$

With FEC



- The total rate of the audio flow is not changed:

$$Q_\alpha = (1-\alpha).(1-\pi) + \alpha.\pi.\mathrm{P}\{Y_{n+1} = 1 \mid Y_n = 0\}$$

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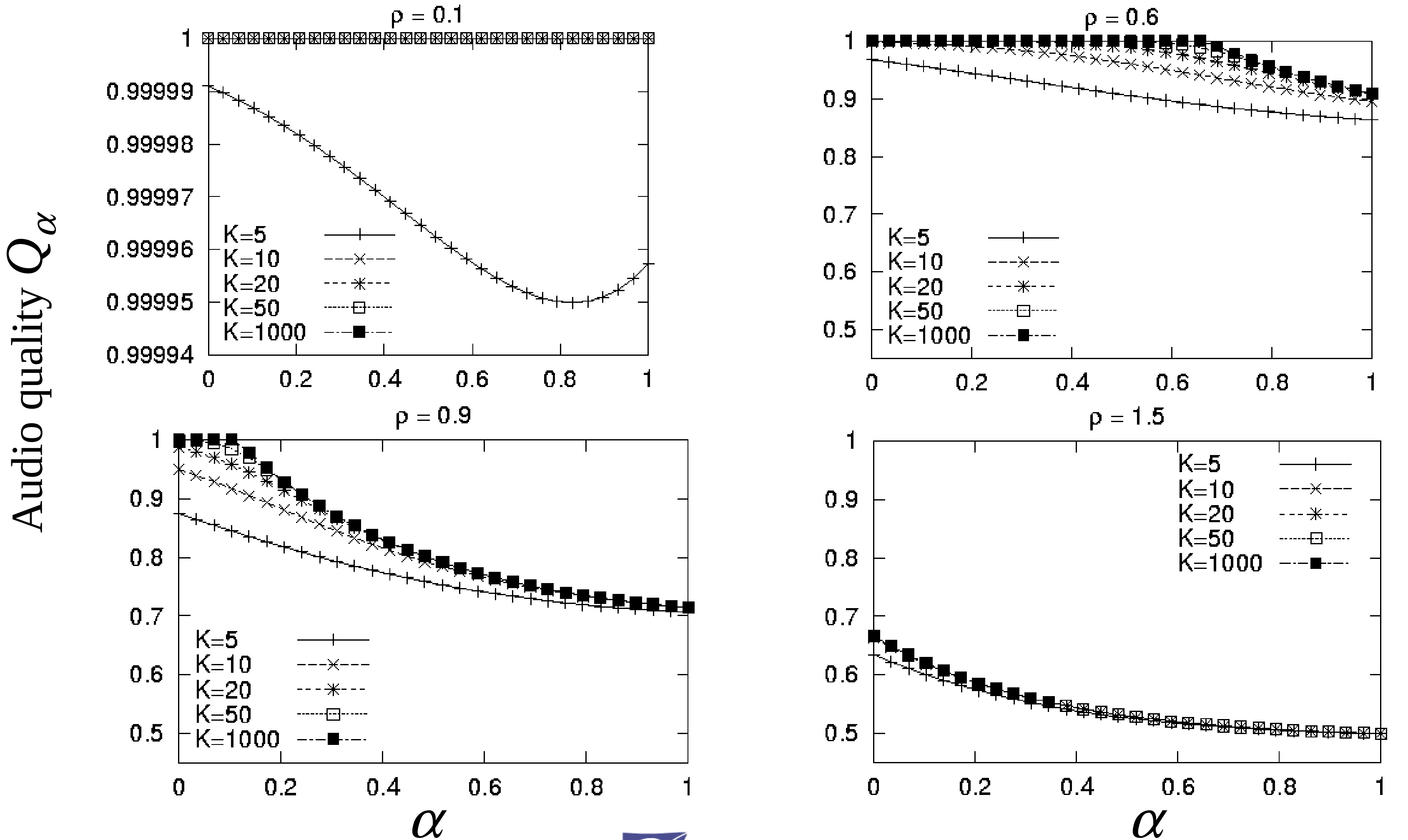
Numerical results: Negative ...



Audio quality for a general offset

☞ **Idea:** Move away the redundancy from the original packet.

☞ **Motivation:** Audio packets are quite often lost in bursts.

ϕ : Offset between redundancy and original packet

$$Q_\alpha = (1-\pi_\alpha) + \alpha.\pi_\alpha.\mathrm{P}\{Y_{n+\phi} = 1 \mid Y_n = 0\}$$

☞ **Analysis:** We proved that the quality is indeed an increasing function of ϕ.

Maximum quality for infinite ϕ :

$$Q_\alpha = (1-\pi_\alpha) + \alpha.\pi_\alpha.(1-\pi_\alpha)$$

Not feasible for reasons of end-to-end delay, but still an upper bound ...

Audio quality for finite offset

Problem: Calculation of $P\{Y_{n+\phi} = 1 \mid Y_n = 0\}$

Let Z_j = Nb of packets served between the arrival of packets ($n+j-1$) and ($n+j$)

Theorem: For $\phi \le K_\alpha$ (which is quite acceptable)

$$P\{Y_{n+\phi} = 0 \mid Y_n = 0\} = P\{Z_\phi + \cdots + Z_r < (\phi - r + 1) \quad \text{for} \quad r = 1, \ldots, \phi\}$$

And the Ballot Theorem [Takacs,1967] says that if $\sum_{r=1}^{\phi} Z_r = k$ we have,

$$P\{Z_\phi + \cdots + Z_r < (\phi - r + 1) \quad \text{for} \quad r = 1, \ldots, \phi\} = 1 - \frac{k}{\phi}$$

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Audio quality for finite offset

Thus,

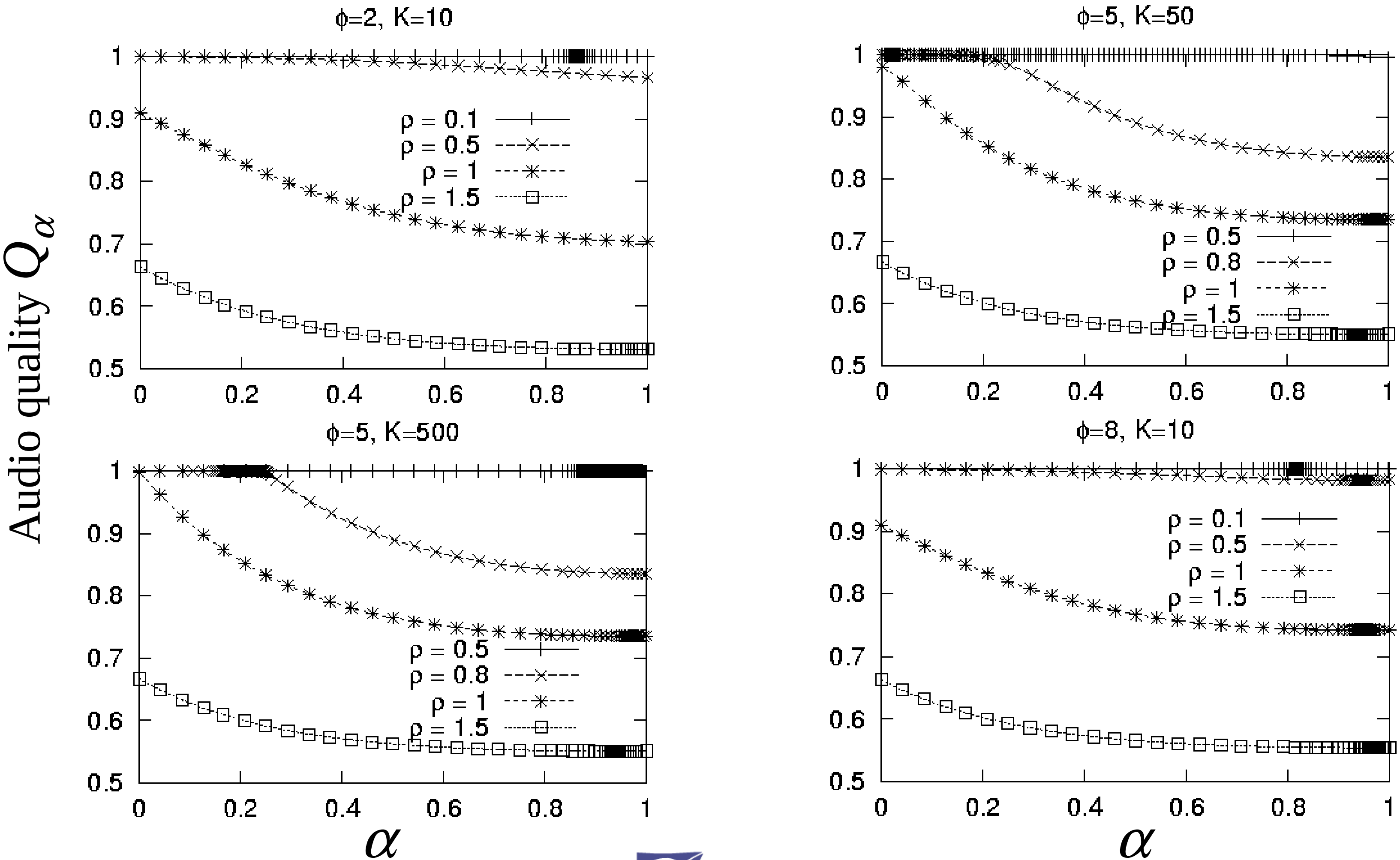
$$P\{Y_{n+\phi} = 0 \mid Y_n = 0\} = \sum_{k=0}^{\phi-1} \left(1 - \frac{k}{\phi}\right) . P\left\{\sum_{r=1}^{\phi} Z_r = k\right\}$$

Theorem: Given that the $\{Z_r\}$ are i.i.d., it is easy to show that

$$P\left\{\sum_{r=1}^{\phi} Z_r = k\right\} = \binom{\phi + k - 1}{\phi - 1} . \left(\frac{\rho_\alpha}{1+\rho_\alpha}\right)^{\phi} . \left(\frac{1}{1+\rho_\alpha}\right)^{k}$$

This concludes the calculation of $P\{Y_{n+\phi} = 1 \mid Y_n = 0\}$, and hence of the audio quality for a finite offset.

Numerical results: Negative ...



Audio quality for infinite offset

If we exclude the negative impact of the delay, the best audio quality that we could obtain is given as follows ...

☞ When the total audio rate is not changed:

$$Q_\alpha = (1-\alpha).(1-\pi) + \alpha.\pi.(1-\pi)$$

Clearly, always decreasing with α!

☞ When the size of original packets is kept the same:

$$Q_\alpha = (1-\pi_\alpha) + \alpha.\pi_\alpha.(1-\pi_\alpha)$$

Again, numerical results show that the quality is always decreasing with α ...

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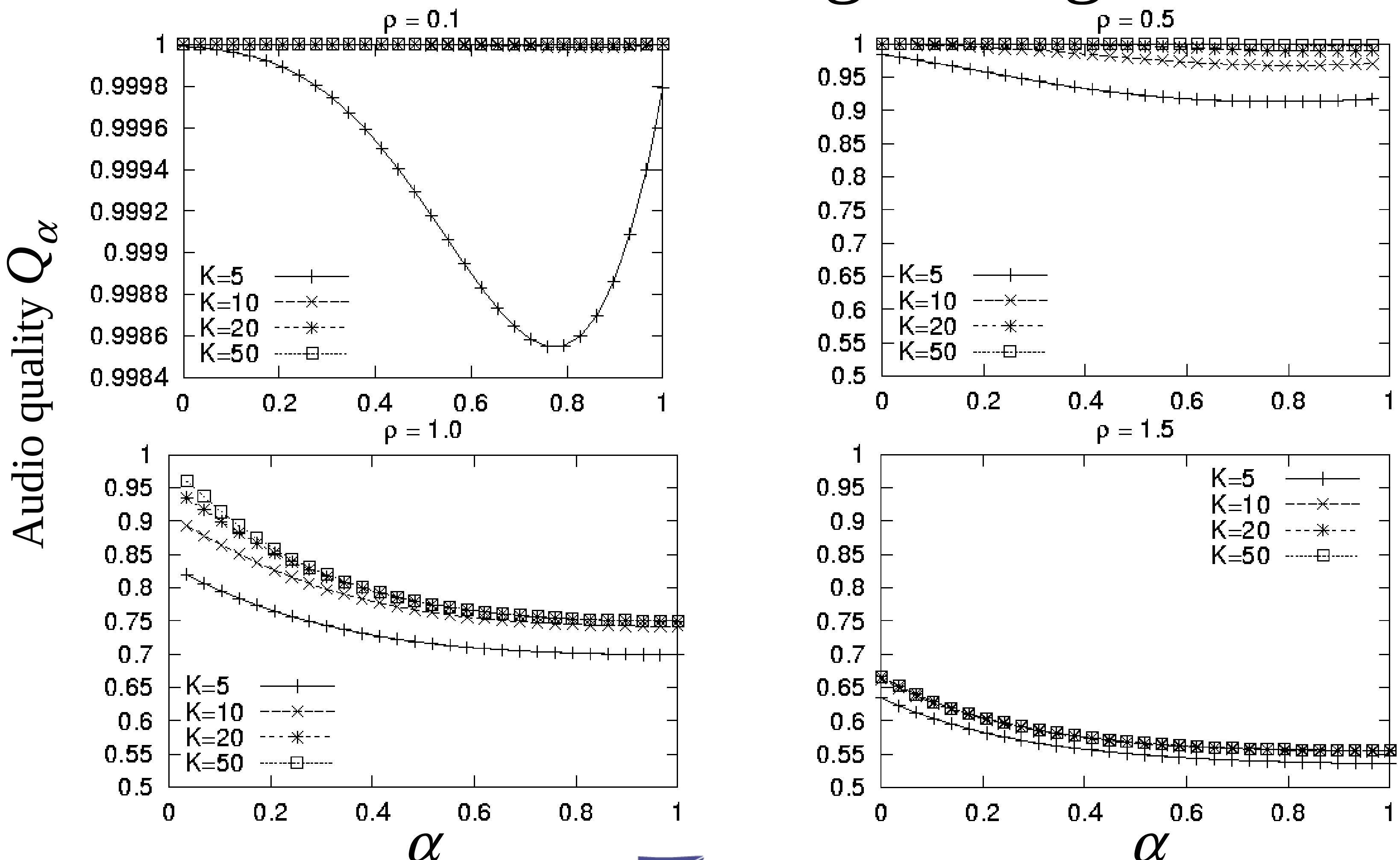
Numerical results: Again negative ...



Discussion of the results

- **Interpretation:** We lose in the quality of the original stream more that we gain from the addition of FEC.
- **Reasons:**
  - Big impact of FEC on network load (loss rate).
  - Low quality of FEC compared to original audio packets.
  - The redundancy only protects one packet (inefficient utilization).
- **Cases when we may gain:**
  - High quality of a small amount of FEC (higher coding rate, e.g., GSM).
  - Compete with other flows that do not use FEC (low influence on loss rate).

Perspectives

☞Include the impact of exogenous traffic not implementing FEC

(our analysis here shows the negative performance of FEC when all flows use it, hence it shows that this simple FEC scheme is not viable).

☞Account for cases when redundancy is coded with a higher-rate codec.

☞Consider the fact that the audio quality is not really linear with the packet size.

☞Define and evaluate more intelligent FEC schemes (e.g., code the redundancy using multiple audio packets).