Biologically Plausible Trajectory Generator

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(Presentation thanks to Pierre Kornprobst)

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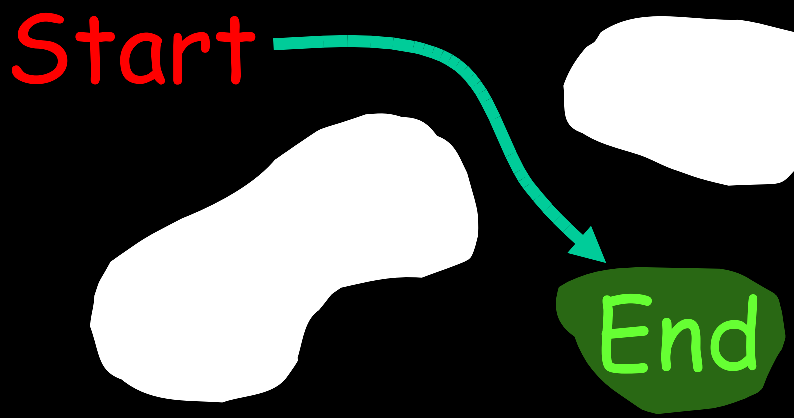
Trajectory Generation Problem



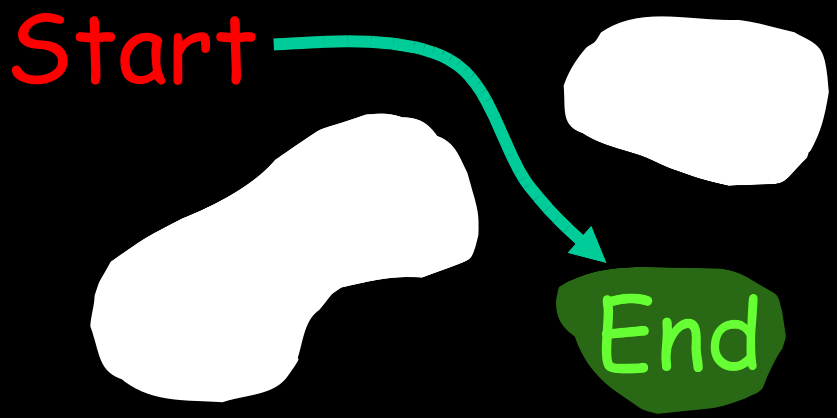
- A Very General Problem

  Finding your ``way´´ in geographic map

  Making a gesture in a complex environment

  Driving without skid or with a power bound

Trajectory Generation Problem



- But also a High-dimensional abstract problem…

State of the Art

- In robotics (two keys)

  Latombe (1991) : still a hard problem !

  Connolly-Grupen (199*) : harmonic potential. Elegant solution of the problem but subject to the curse of dimensionality

- In biology (see e.g. Poucet et al. 2004)

  Seems easy !

Four Key Aspects

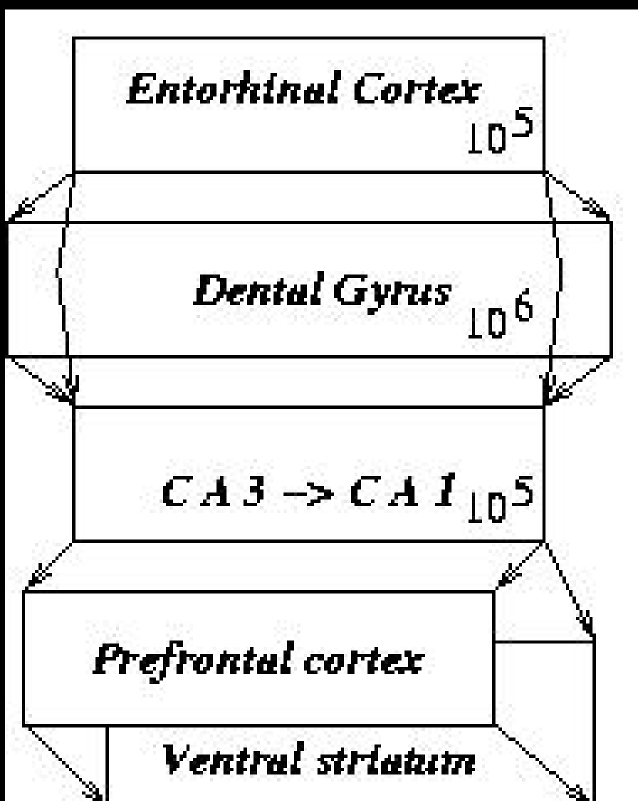
- The problem is to be solved:
  - **Action**: Exploration decouvre obstacle + generation trajectoire pour aller but
  - **Global level**: e.g. labyrinth
  - **Dynamics**: Look and move paradigm (here)
  - **Degrees of freedom**: A large number of

Very difficult as a Mathematical problem !

Very easily solved by a mice brain !

About Biological Trajectory Generator

- Hyppocampal and related structures are involved in trajectory generation



Sensoru inputs

Environment loci

Present location

Sparse locus map(s)

Shifts of the present location

Goal location

Motor command

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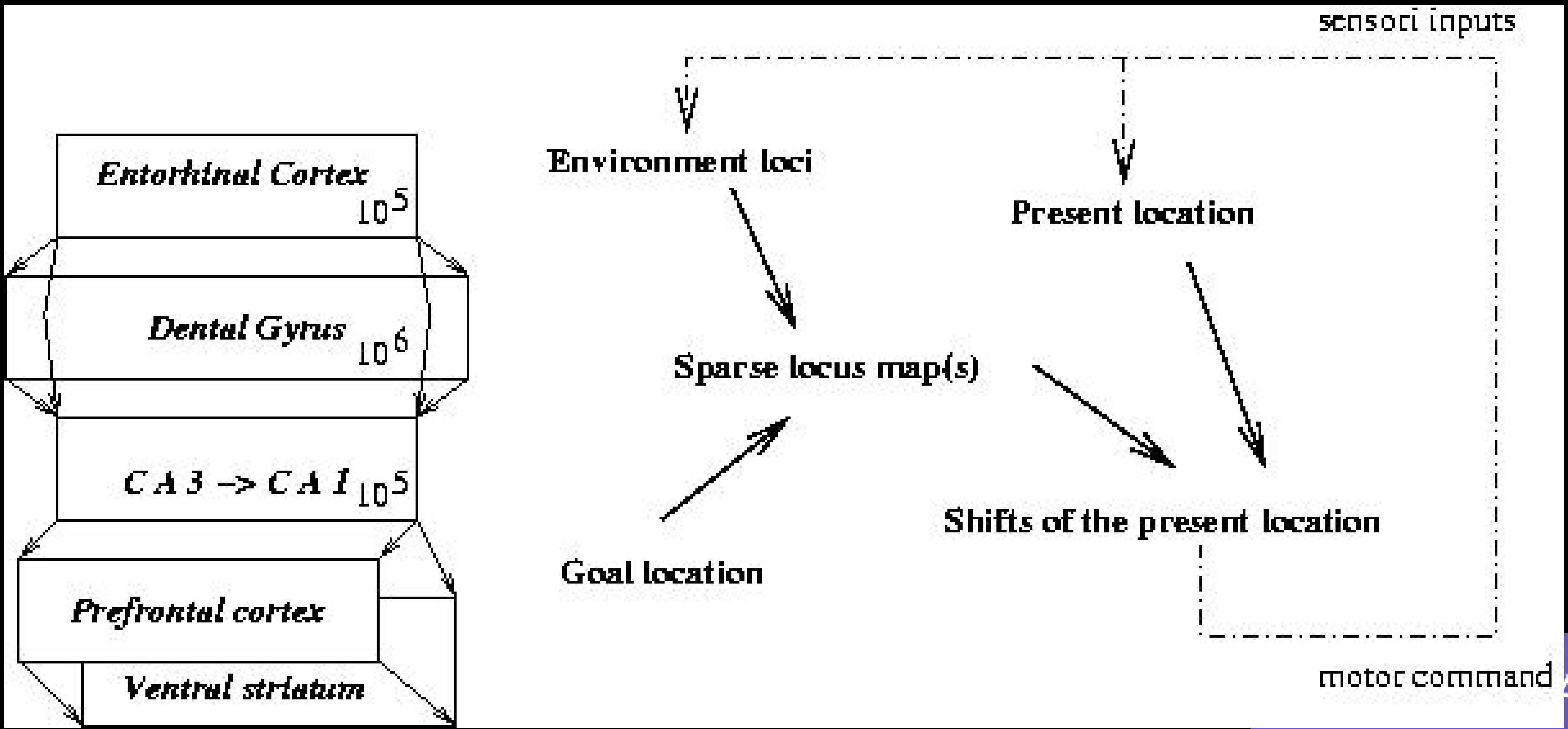

About Biological Trajectory Generator

- Internal representation:
  place fields (PF) within locus maps (LM)
- No topography but recruitment on request
- Reactivation of LM when spatial-area changes
- Obstacles PF -> elongated shapes
  Intermediate goals -> isotropic shapes

- Used for [goal oriented + wandering] behavior
- How is this use in/as a sensori-motor loop ?

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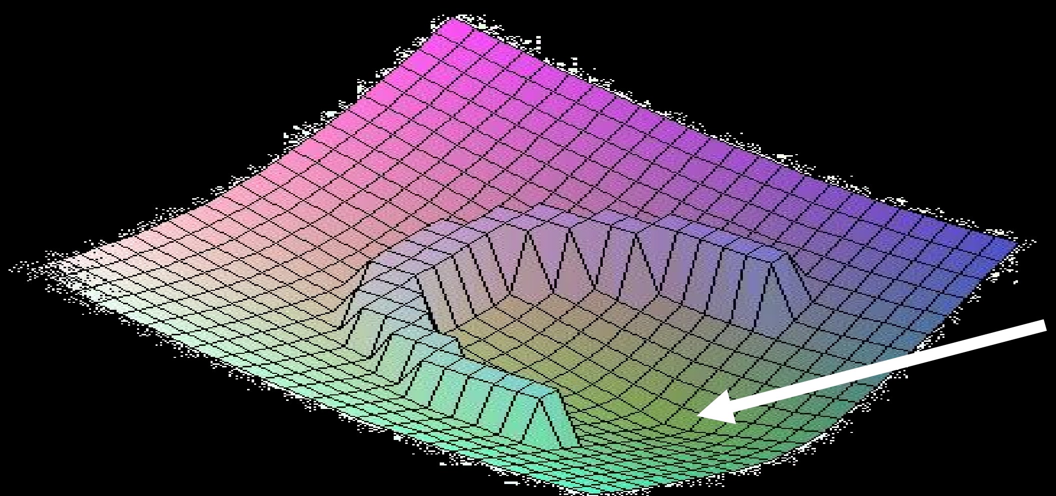
Potential and Trajectory

- Obstacles to avoid are the potential max
- The goal(s) corresponds to its min
- Let's throw a *regular sheet* on this space
- And let it ``roll-down´´ along this relief



Potential and Trajectory

- Two caveats with potentials

  Local minima !

  Solutions: Convex profiles, VonBaumgarten curve, ..

  Here we use harmonic potential

  minimize collision probability

  very general (e.g. non holonomic constraints)

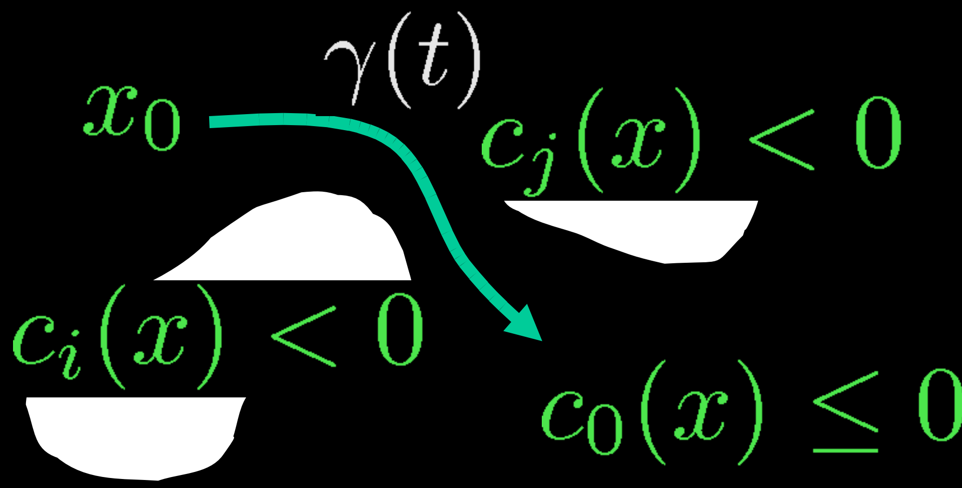
  Connolly-Grupen (1994)

  Curse of dimensionality !

  (exponential complexity with dimension)

  Solution: Parametric potential

Potential and Trajectory



Let us consider (here functions are at least twice differentiable) :
(a) a system, defined by a *state vector* $\mathbf{x} \in \mathcal{R}^n$, $n \geq 2$
(b) an *initial state*, written $\mathbf{x}_0 \in \mathcal{R}^n$,
(c) r *constraints* defined by scalar inequalities $c_i(\mathbf{x}) > 0, i \in \{1..r\}$,
(d) a *goal* defined by an constraint of the form $c_0(\mathbf{x}) \leq 0$,
We consider a connected domain:

$$\mathcal{U} = \cup_{i \in \{0..r\}} \mathcal{U}_i \text{ with } \mathcal{U}_i = \{\mathbf{x}, c_i(\mathbf{x}) > 0\} \text{ with } \mathbf{x}_0 \in \mathcal{U}$$

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Potential and Trajectory

Here we define harmonic potentials $V : \mathcal{U} \rightarrow \mathcal{R}$ thus

$$\forall \mathbf{x} \in \mathcal{U}, \Delta V(\mathbf{x}) = 0 \Rightarrow ||\nabla V(\mathbf{x})|| > 0$$

with

$$\lim_{\mathbf{x} \rightarrow \partial \mathcal{U}_0} V(\mathbf{x}) = -\infty \text{ and } \forall i > 0, \lim_{\mathbf{x} \rightarrow \partial \mathcal{U}_i} V(\mathbf{x}) = +\infty$$

and consider trajectory $\gamma : \mathcal{R}^+ \rightarrow \mathcal{R}^n$ such that:

- starting at the initial point $\gamma(0) = \mathbf{x}_0$, with $\gamma'(t) = -\nabla V(\gamma(t))$
- verifying the problem constraints $\forall t \in \mathcal{R}^+, \gamma(t) \in \mathcal{U}$,
- and reaching the goal $\lim_{t \rightarrow \infty} c_0(\gamma(t)) = 0$ (asymptotically)

Potential and Trajectory

- Global goal influence (adaptive gain)
- Obstacle local influence

Suitable harmonic potential are *adelicians*

$$V(\mathbf{x}) = \sum_{i=1}^{r} \mathcal{A}_i(\mathbf{x}) - \mathcal{A}_0(\mathbf{x})$$

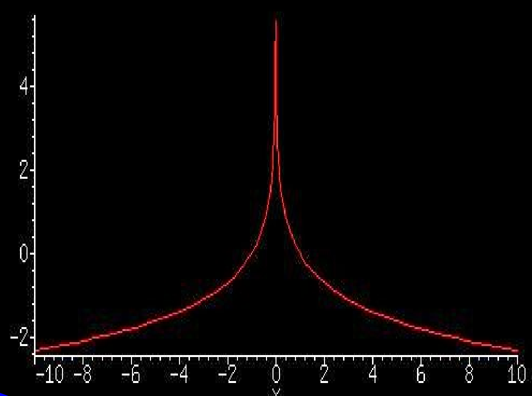
with

$$\sum_{\in \partial \mathcal{U}_i} \mu_{y_{ij}} \Phi(\mathbf{x} - \mathbf{y}_{ij})$$

Only closest points

al radial symmetric harmonic

$$= \frac{\lambda}{||\mathbf{x}||^{n-2}})$$



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Potential and Trajectory

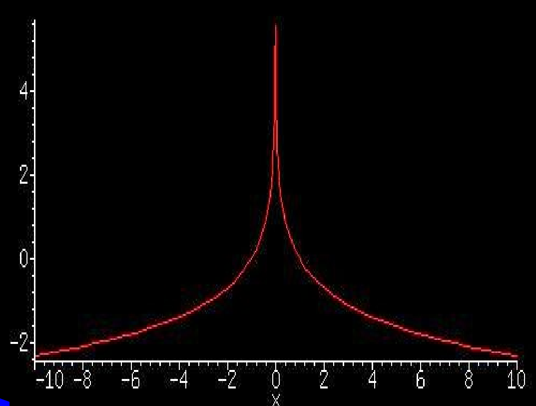
- Global goal influence (adaptive gain)
- Obstacle local influence

Suitable harmonic potential are *adelicians* :

$$V(\mathbf{x}) = \sum_{i=1}^{r} \mathcal{A}_i(\mathbf{x}) - \mathcal{A}_0(\mathbf{x}) \text{ with } \mathcal{A}_i(\mathbf{x}) = \sum_{\mathbf{y}_{ij} \in \partial \mathcal{U}_i} \mu_{\mathbf{y}_{ij}} \Phi(\mathbf{x} - \mathbf{y}_{ij})$$

where $\Phi()$ is the fundamental radial symmetric harmonic function

$$(\text{for } n > 2,\ \Phi(\mathbf{x}) = \frac{\lambda}{||\mathbf{x}||^{n-2}})$$



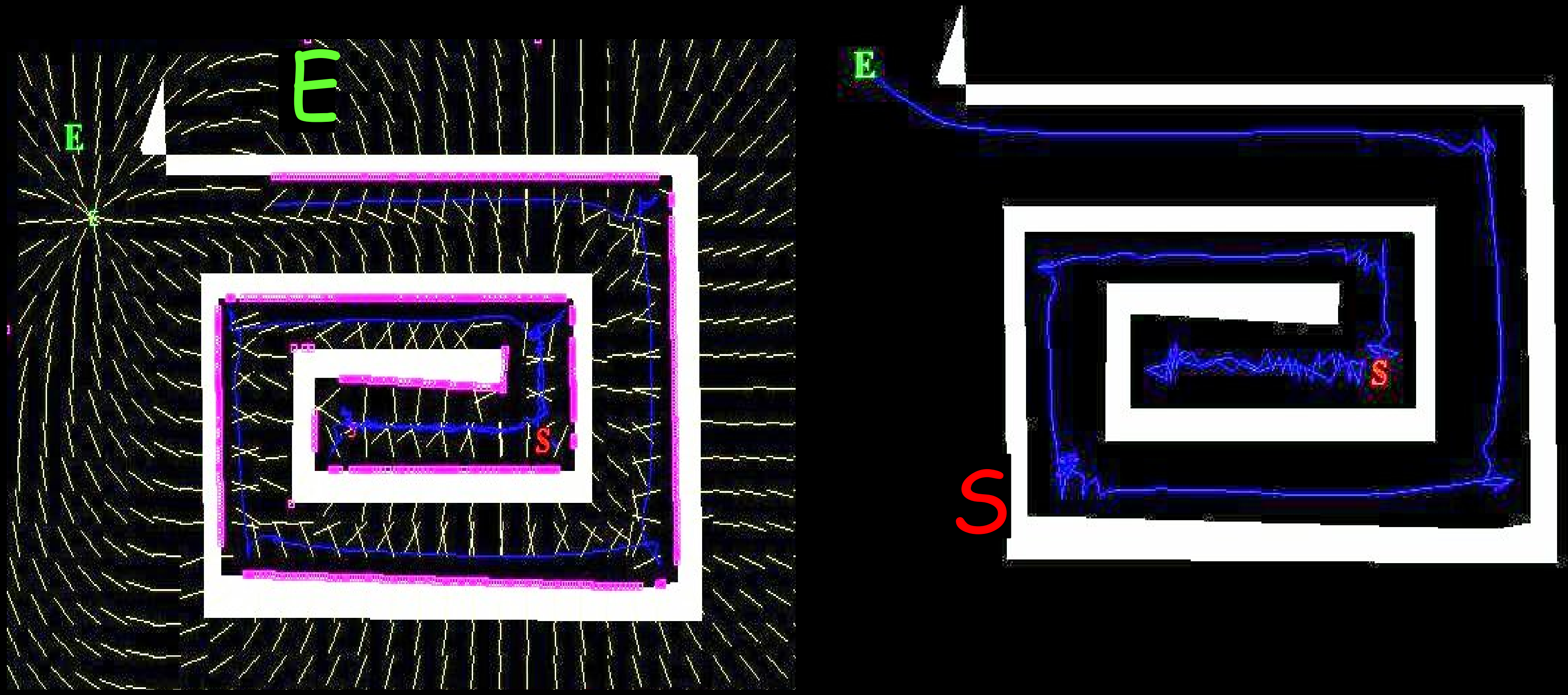
Only closest points

Potential and Trajectory

- Yields a sparse map representation
  O(trajectory-length) ! no direct dim. curse
- Adaptive representation
  exploration within navigation
  sensori-motor loop embending
  (including gain control)

- Biological plausibility is well-founded

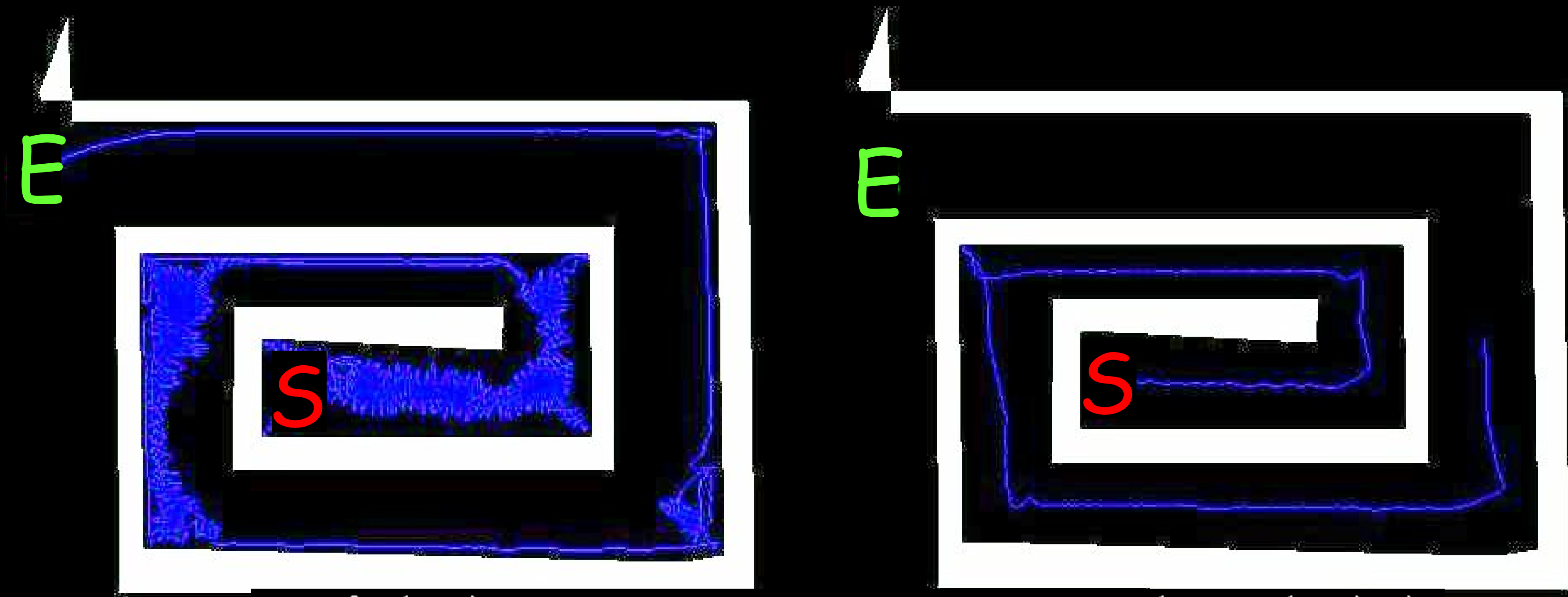
Labyrinth Experiment



Start End Map's loci Potential field Trajectory

Various Behaviors

Exploration . . versus . . navigation
(simple gain adjustment)

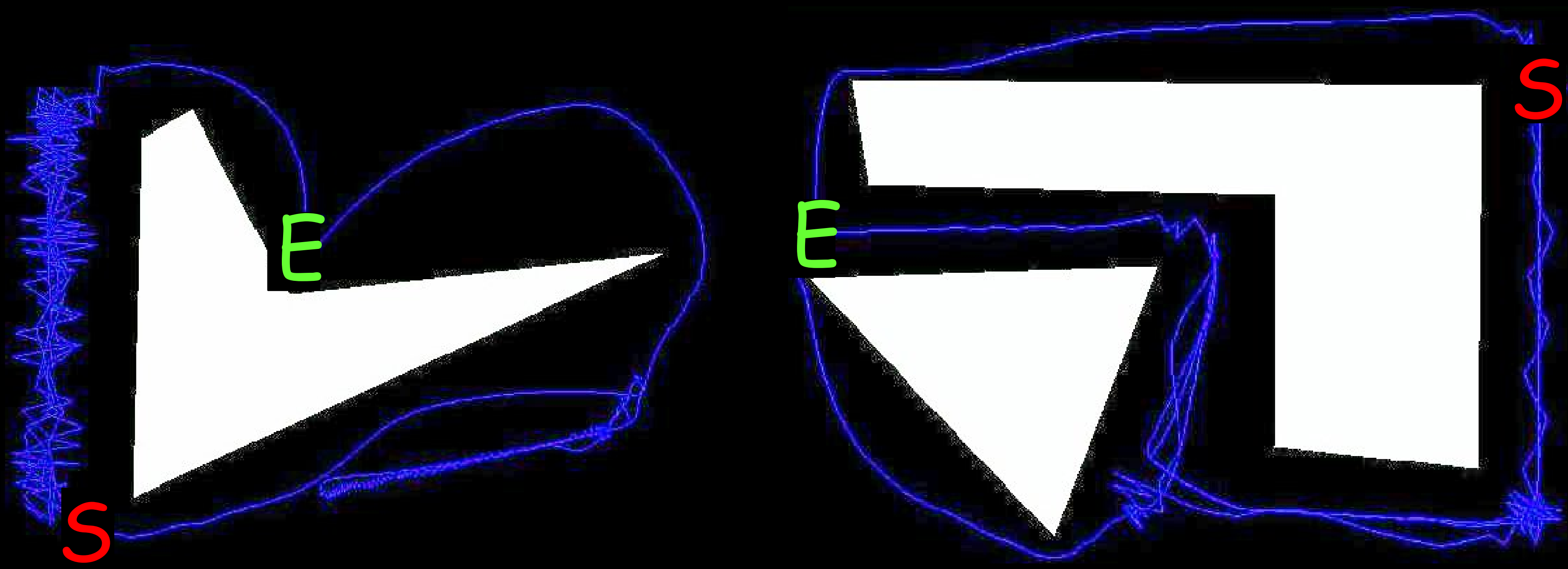


$$\gamma'(t) = -G\nabla V(\gamma(t))$$

Learning When Re-run

Improved trajectory

. . and . . further exploration



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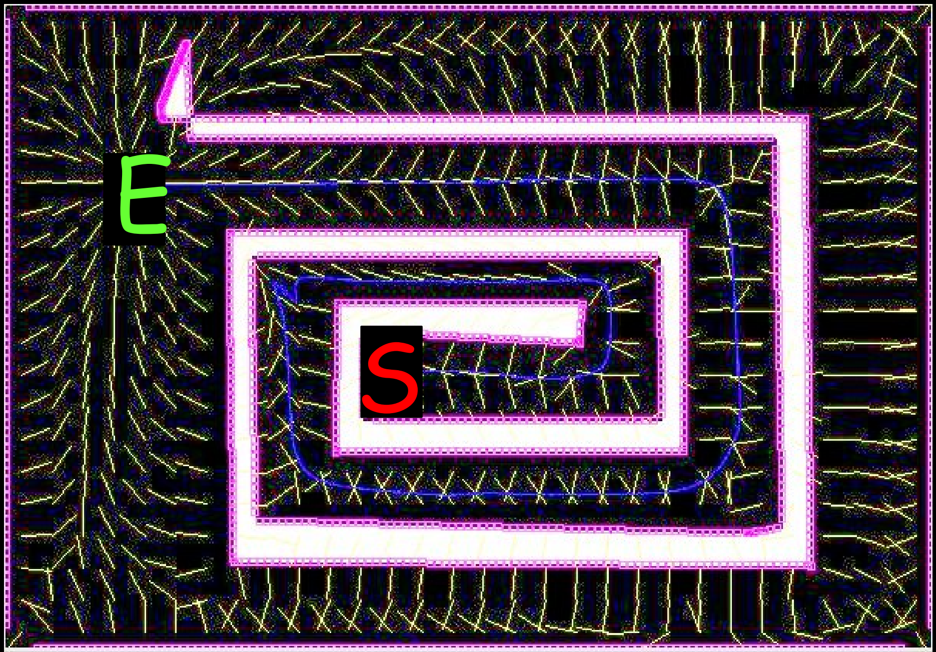
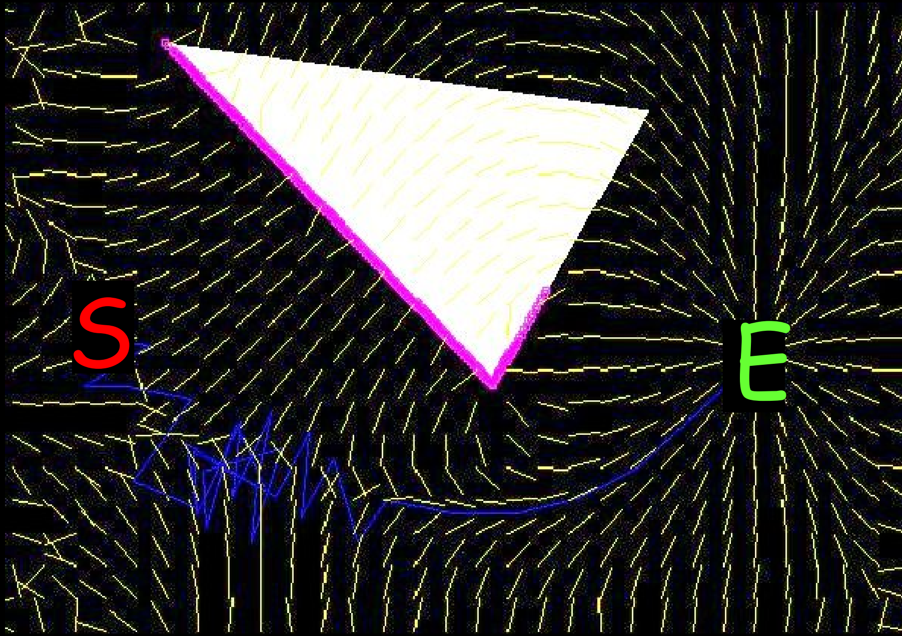
Using Various Data Input

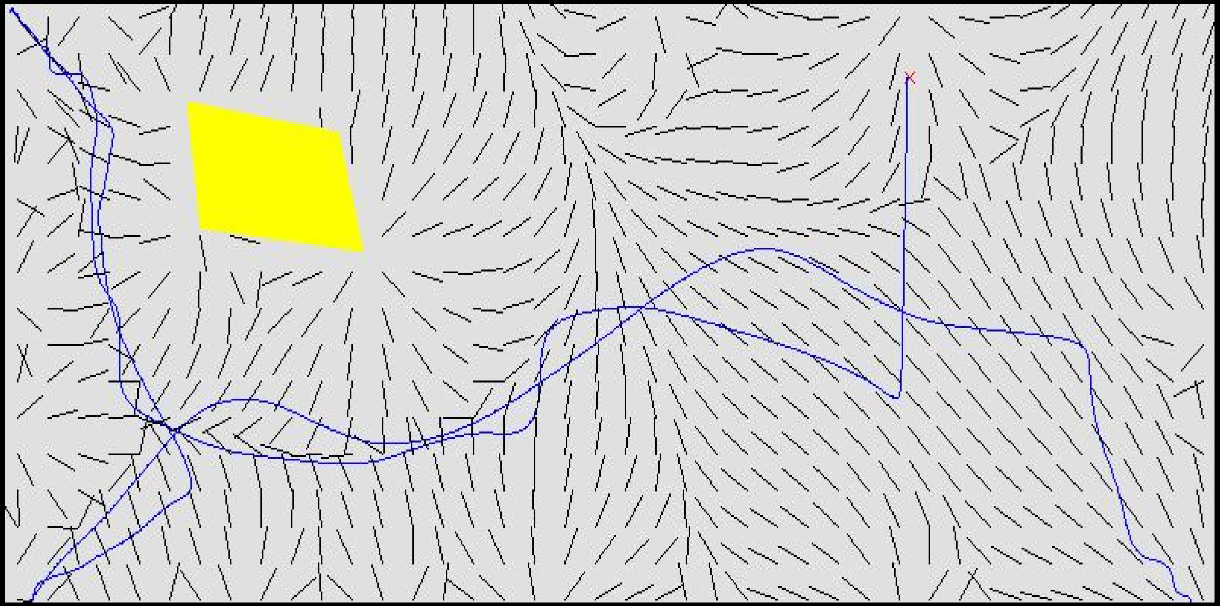


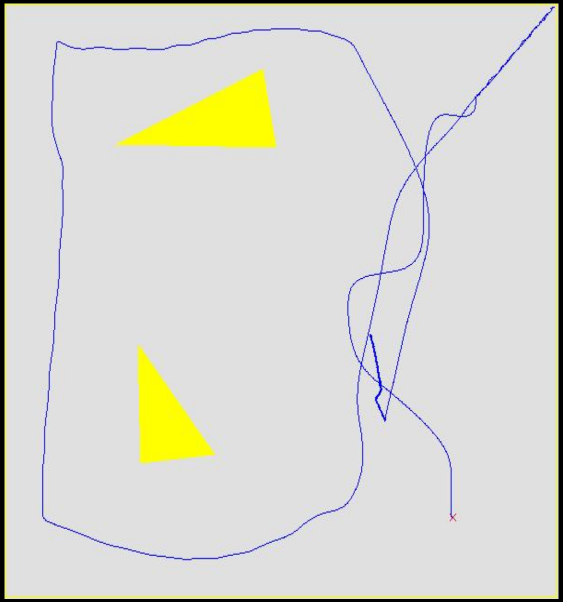
When the map is known



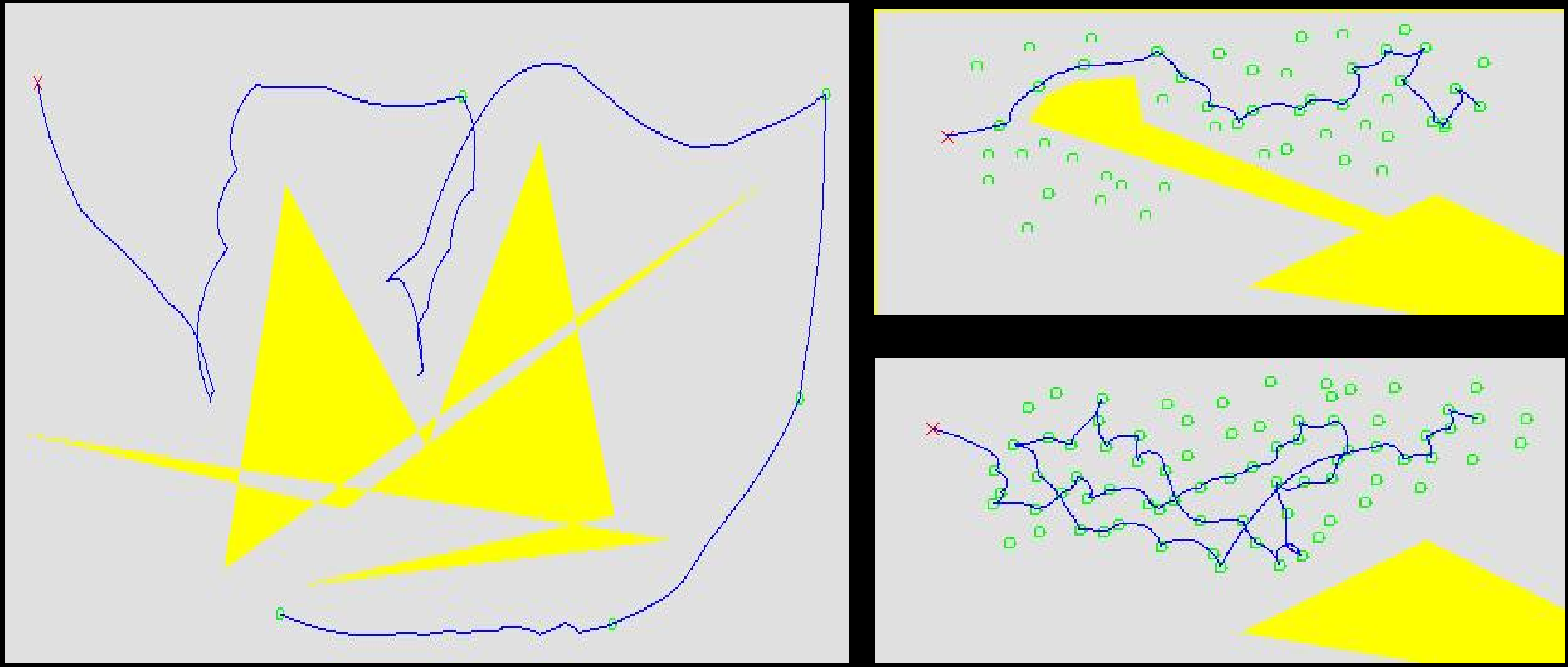
With only the line of sight as input

Wandering Behavior (no goal)





With Intermediate goals



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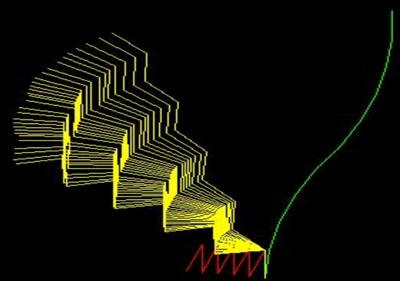
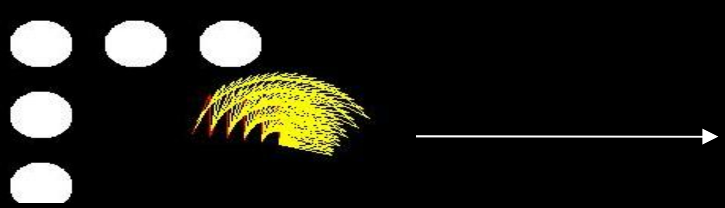
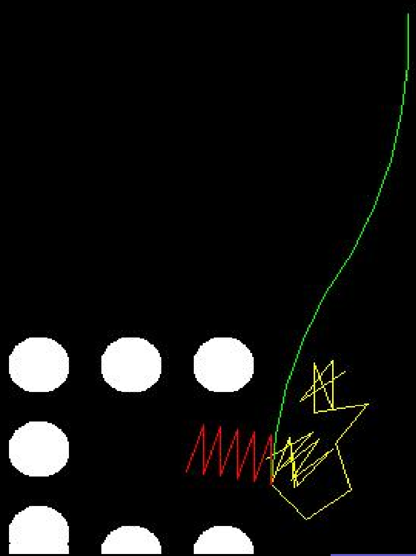
Extension to a 10 d.of. arm

- Considering a high dimensional problem



from a trap,
begins . . and escape :





Conclusion

- Biological sparse locus maps have inspired a solution against the harmonic potential curse of dim.
- Improved harmonic potential methods show that biological locus maps are sufficient mechanisms to explain exploration / navigation in partially known environments.

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Perspectives

- Better link with biological models
- More general behavior generation (gestures, manoeuvres, plans, ..)

Questions ?

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Software avalaible + on-line demos

http://www-sop.inria.fr/odyssee/imp/trajectory

Detailled report

http://www-sop.inria.fr/rapports/sophia/RR-4539.html

Large-public presentation

http://interstices.info/display.jsp?id=c_14155