CORTEX INRIA NANCY

A review about what can be computed with spikes . . .

To which extend is the neural code a metric ?

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(Partially supported by the ANR MAPS & the MACCAC ARC)

Global time constraints in spike trains

In computational or biological contexts, spike times are constrained by the neural dynamics and
- [C1] bounded by a refractory period,
- [C2] defined up to some absolute precision,
- [C3] with a minimal delay between one spike and its target
- [C4] there is *often* a maximal inter-spike interval with the next spike (if any)

In *ms*:

r	δt	dt	D
1	0.1	> 0.01	10^{2-3}

* [C4] is not obvious :

X true when leak + conductances (cortical neurons ?)

X false when internal currents (thalamic neurons ?)

Different dynamics. Without [C4]:

A ``vicious'' neuron can remain silent a very long period of time, and then suddenly fire inducing a complete change in the non-linear system.

* [C1-2] yields a spike train information upper-bound

$$N \frac{T}{r} \log_2\left(\frac{T}{\delta\tau}\right)$$

for N during neurons during T, ≃1Kb/neuron/seconds.

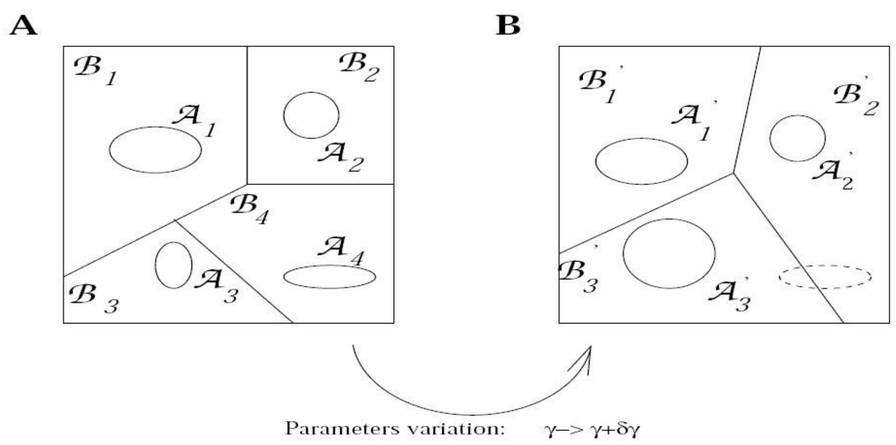
* [C1-3] allows to study time discretized model instances:

[H1] The raster plot is generically periodic, periods are unbounded (⇒ external current or synaptic weights),

[H2] There is a one-to-one correspondence between orbits and raster (raster plots provides a symbolic coding).

True for conductance based leaky integrate and fire neurons.

Likely for any model with reset and contracting dynamics.



[A] The phase space is partitioned into bounded domain and for each initial condition the initial trajectory is attracted to a periodic orbit. [B] If the parameters (input, weights) change, the landscape is modified and several phenomena can occur: change in the basins shape, number of attractors, modification of the attractor.

* [C3-4] allows to optimize event-based simulation

+ Ultra-fast event-time's queue with bounded size
+ Allows to introduce ``lazy´´ event management (next-event time is given after lower-bounds estimations)

A minimal 10Kb C++ kernel with *O(D/dt + N)* buffer size and $10^{1\text{-}1.5}$ operation/spikes → > 10^6 spike/sec on a laptop.

Used as plugin for existing simulators http://enas.gforge.inria.fr

Neural coding and spike train metrics

- Two trains correspond to the ``same neural code´´ → equivalence-relation

E.g.: rank coding ⇆ permutations are equivalence classes

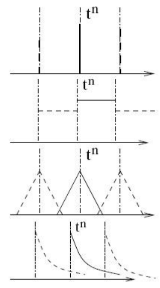
- Two trains correspond ``approximately´´ to the same code → metric-representation

E.g.: - *Binned* metric, spikes grouping in bins (e.g. rate coding),
- *Convolution* metrics, defined on spike train convolution,
- *Spike time metrics,* such as alignment metrics.

* **Convolution metrics**: (linear response)

$$s_i(t) = \sum_{t_i^n \in \mathcal{F}_i} K_i(t - t_i^n) = K_i * \rho_i \in]0,1],$$
$$\rho_i(t) = \sum_{t_i^n \in \mathcal{F}_i} \delta(t - t_i^n),$$

relate the spike-train **ρ** with a continuous signal **s**.



[A] $\delta(t - t_i^n)$

[B] $\chi_{[t_i^{n-1}..t_i^n[}(t)\,\frac{d_{min}}{t_i^n - t_i^{n-1}}$

[C] $\max\left(0, \frac{d_{min} - |t - t_i^n|}{d_{min}}\right)$

[D] $(1 - e^{\frac{-d_{min}}{\tau}})\chi_{[0..\infty[}(t - t_i^n)\, e^{\frac{-(t - t_i^n)}{\tau}}$

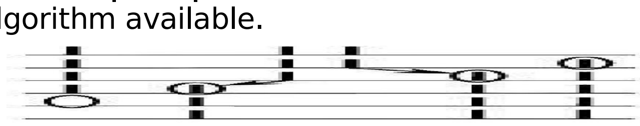
[A] The spike train itself, [B] A causal local frequency measure estimation, [C] A non-causal spike density, [D] A normalized causal exponential profile. Related to: evoked post-synaptic potential, representations using Fourier or Wavelet Transforms,including Mercer scalar-products (``kernel´´ methods). .

- *Kernel identification*: given **s** and **ρ** yields **K**
  → Laplace transform via Parseval theorem
- *Signal deconvolution*: given **s** and **K** yields **ρ**
  → Inverse usual kernels are well-defined
- *Signal reconstruction* (Shanon generalization):
  A [-Ω, Ω] frequency signal **s** is defined by **ρ** iff max [t_i^n, t_i^{n+1}] < Ω / π

* **Alignment metrics**:

Minimum cost **c** of transforming one spike train into the other with:
- spike insertion or spike deletion **c += 1**
- spike shift **c+= |t - t'| / τ**

quadratic algorithm available.



+ Non-linear cost generalization, integrating [C2].
+ Causality integration (``older´´ spikes less matter).
+ Applicable to spike-interval, spike-motifs, ... includes: spike-time differences, rate distance, etc...

- Characterize neuronal variability and coding

- Allow to perform spike train computation/training:

E.g.:considering a SRM model:

$$V_i(t) = \nu(t - t_i^{n-1}) + \sum_{jm} w_{ij}\, \alpha(t - t_j^m),\ t_i^{n-1} < t \leq t_i^n,$$

yields the following formal learning rule:

$$\Delta w_{ij} \equiv \sum_n (t_i^n - \bar{t}_i^n)\, \frac{\partial V_i}{\partial w_{ij}}(t_i^n) \Big/ \frac{\partial V_i}{\partial t_i^n}(t_i^n)$$

Bibliography

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[3] J.D. Victor. Spike train metrics. *Current Opinion in Neurobiology*, 15(5):585–592, 2005.