CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

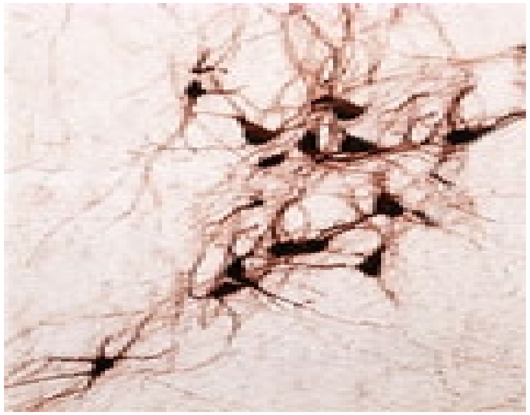
INRIA

Université Nice SOPHIA ANTIPOLIS

Statistics of spikes trains, synaptic plasticity and Gibbs distributions.

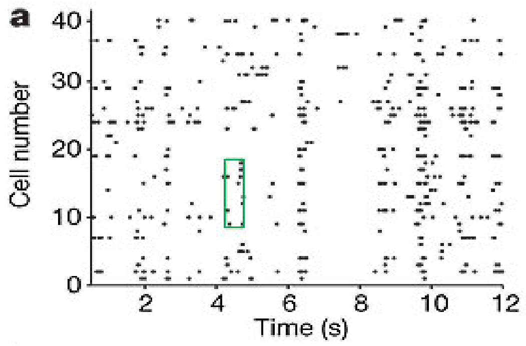
Bruno Cessac, Horacio Rostro,

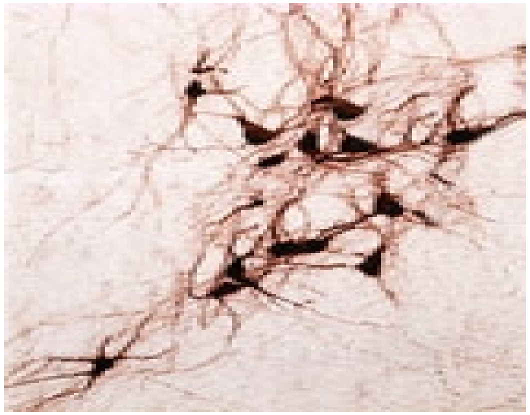
Juan-Carlos Vasquez, Thierry Viéville



•Multiples scales.
•Non linear and collective dynamics.
• Adaptation.
• Interwoven evolution.

Neural network activity.





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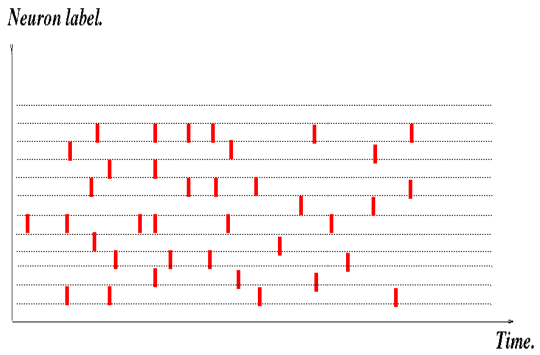

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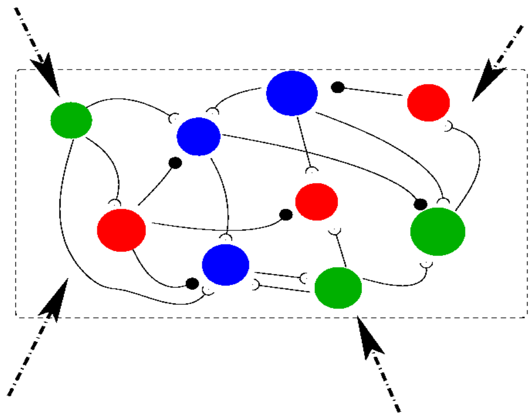

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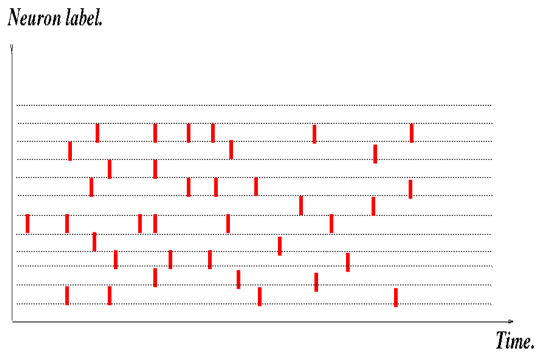

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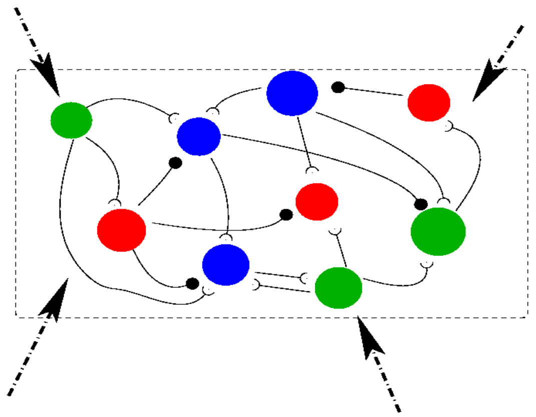

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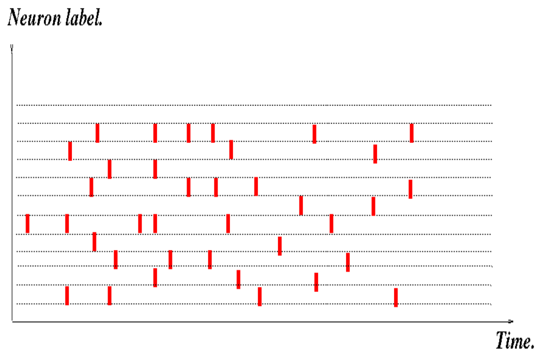

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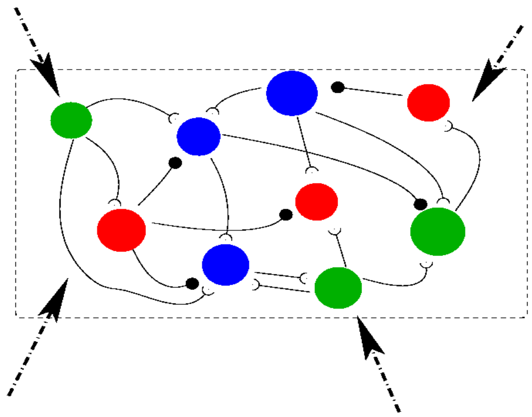

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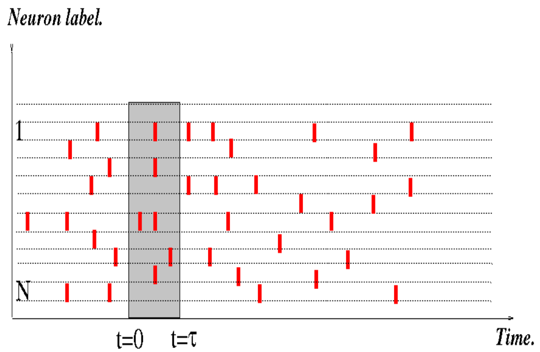

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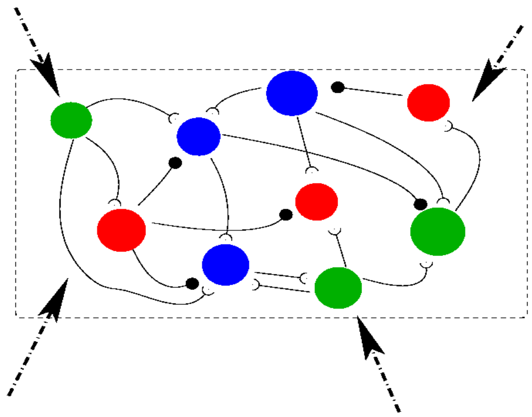

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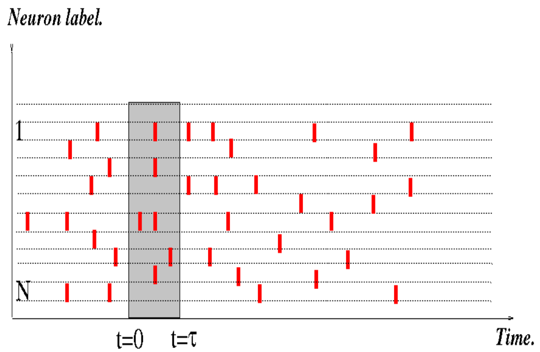

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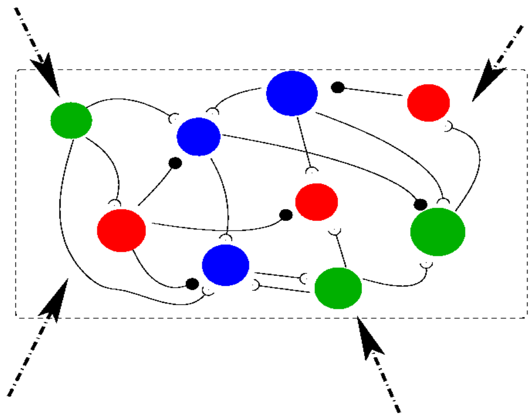

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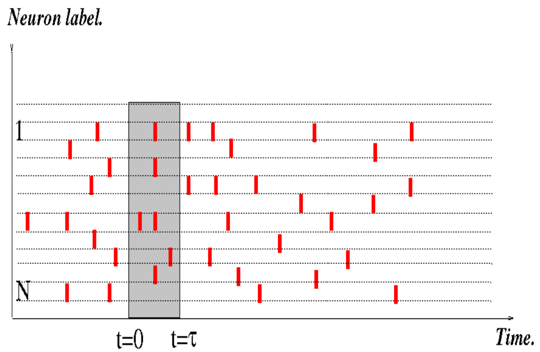

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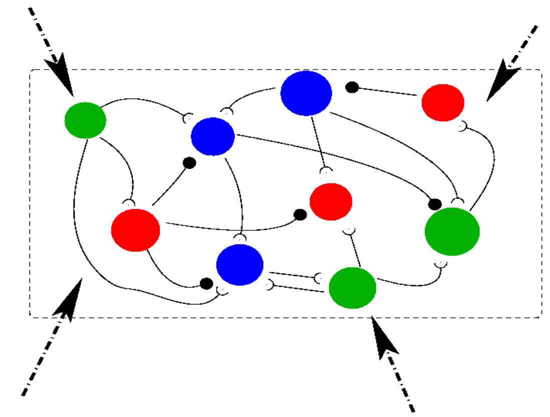

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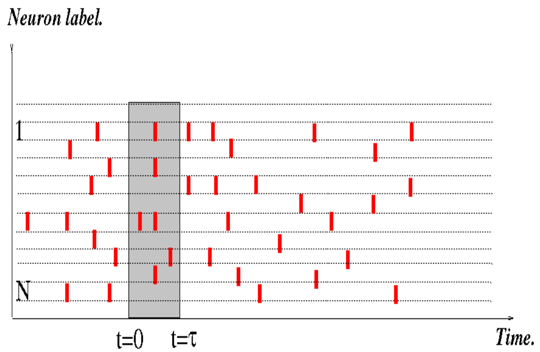

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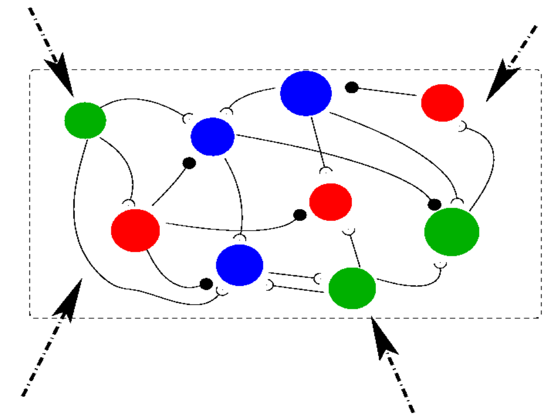

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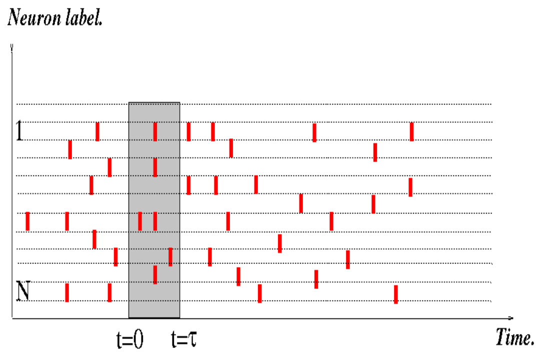

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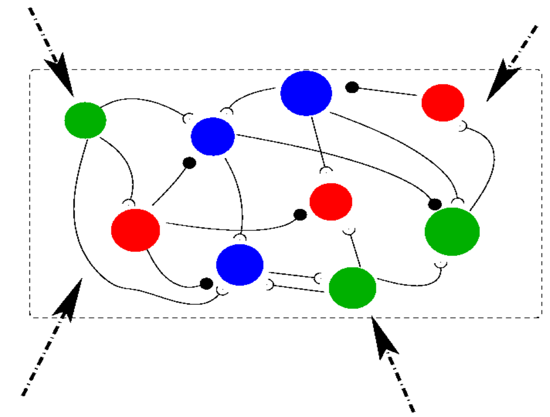

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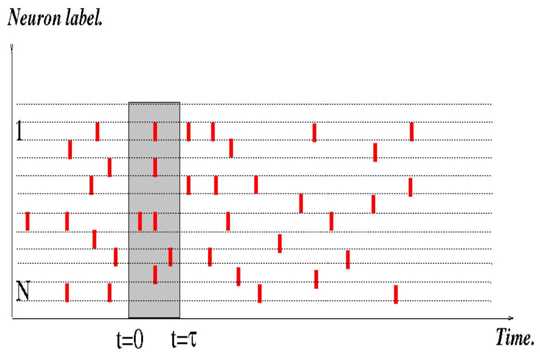

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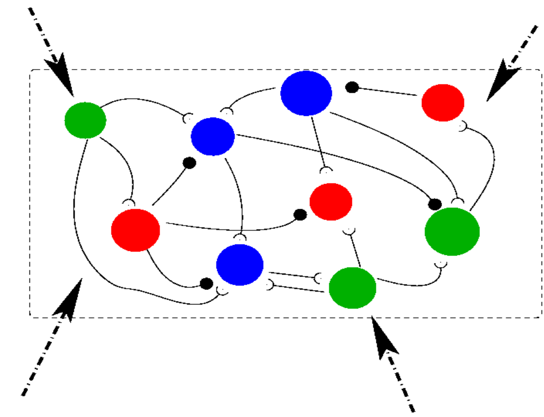

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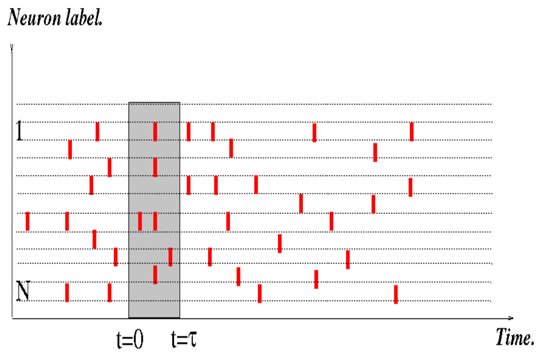

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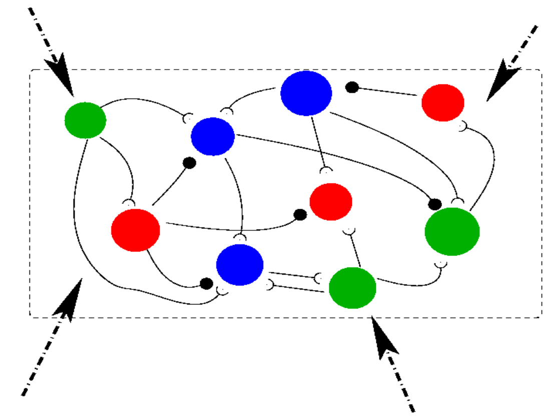

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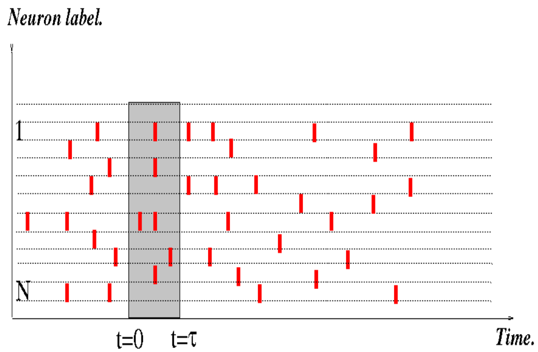

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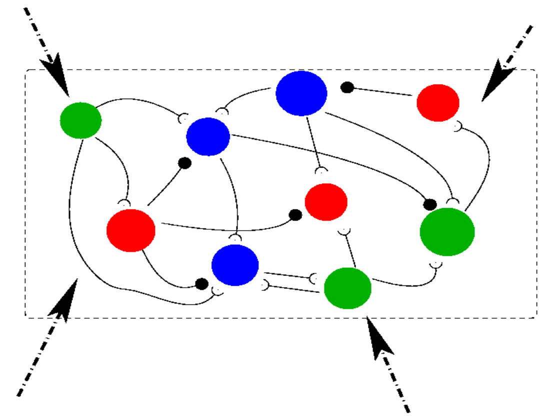

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$$P[R|S] = \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} n_S(R)$$

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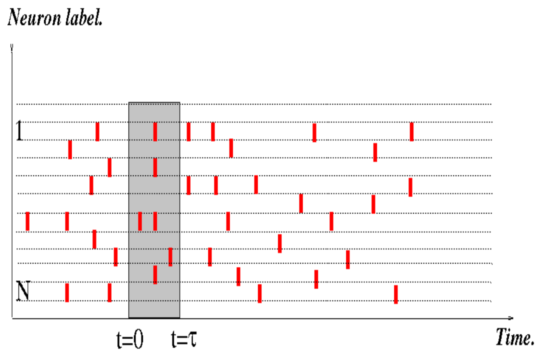

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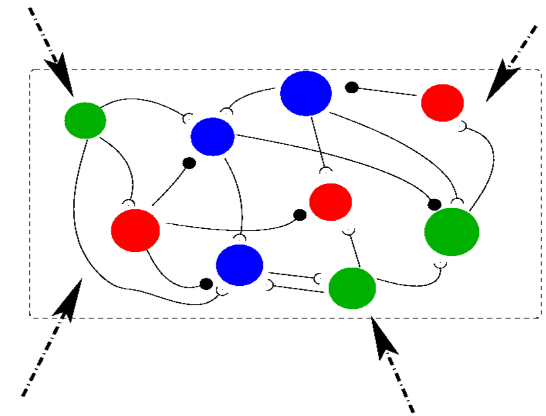

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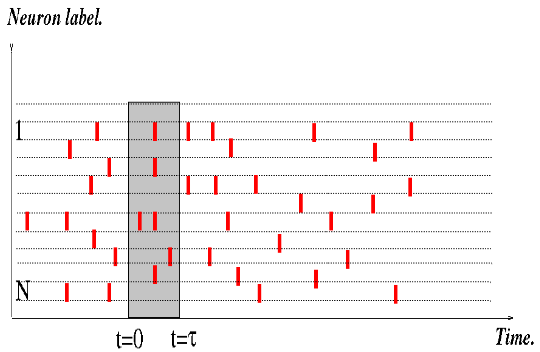

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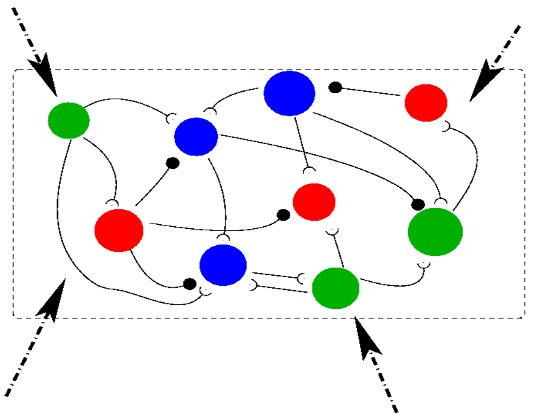

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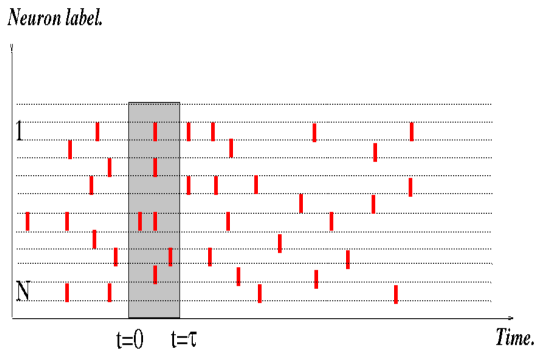

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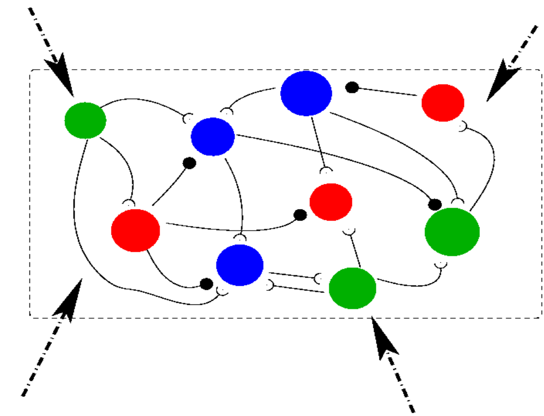

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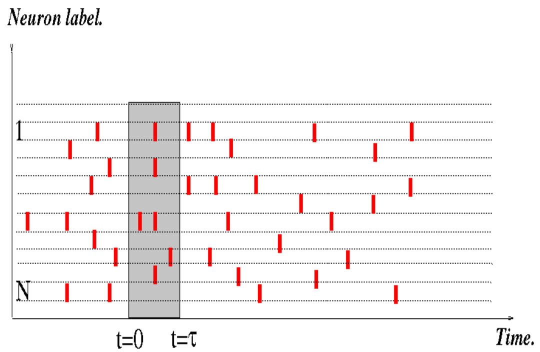

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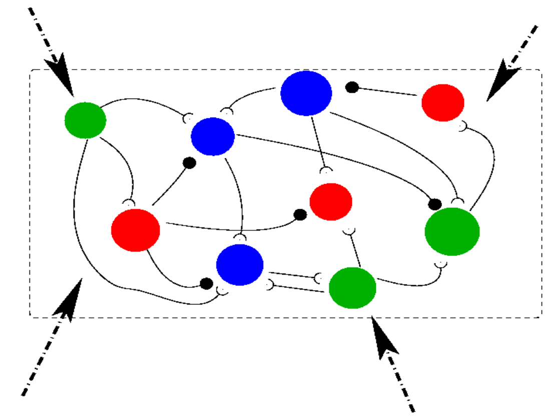

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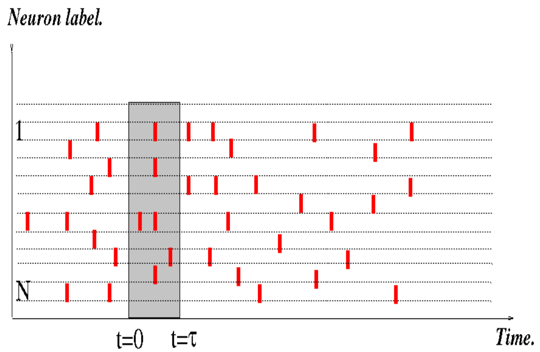

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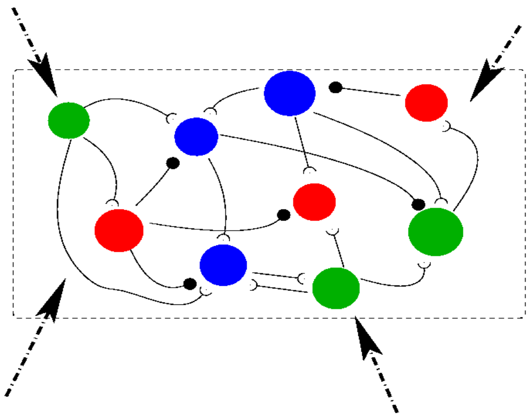
I-F models are (maybe) good enough.

Approximating real raster plots from orbits of IF models with suitable parameters.

R. Jolivet, T. J. Lewis, W. Gerstner (2004)J. Neurophysiology 92: 959-976



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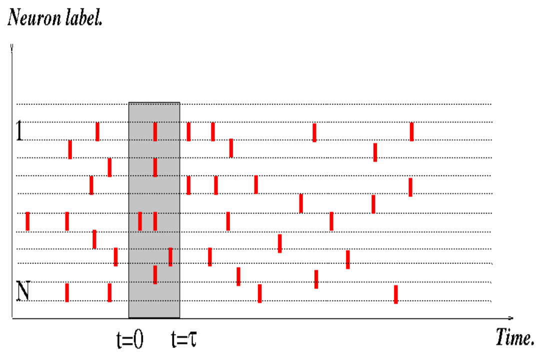

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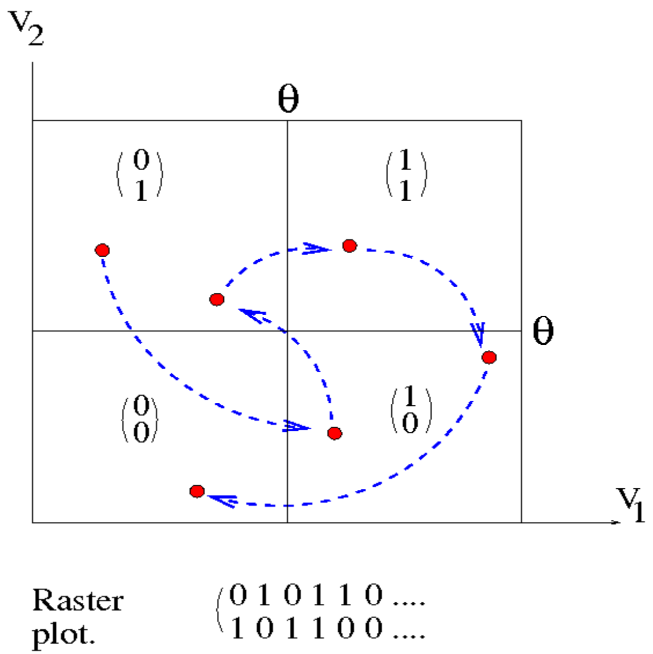

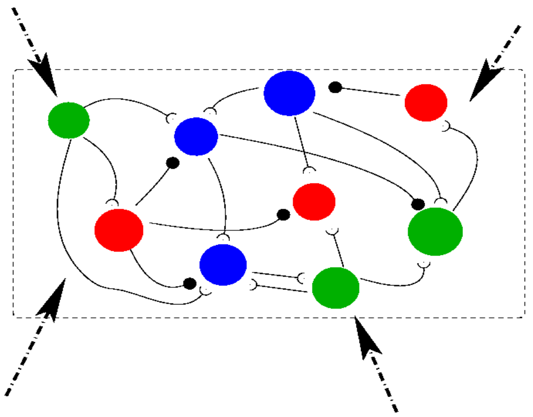

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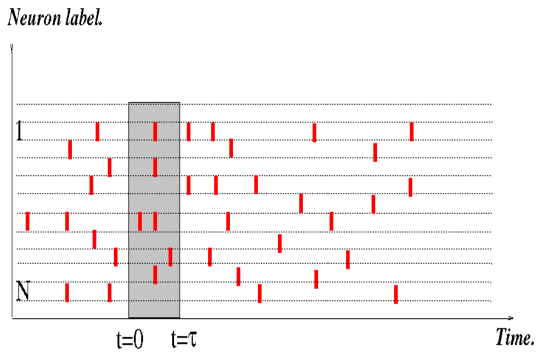

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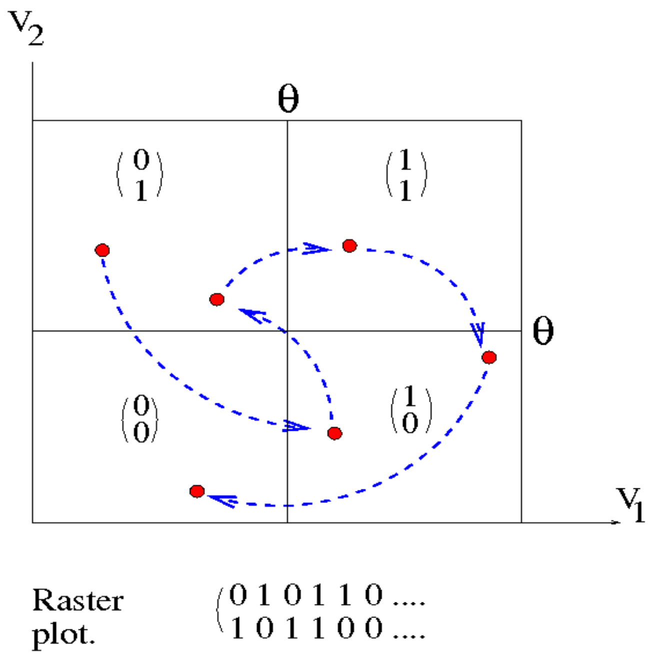

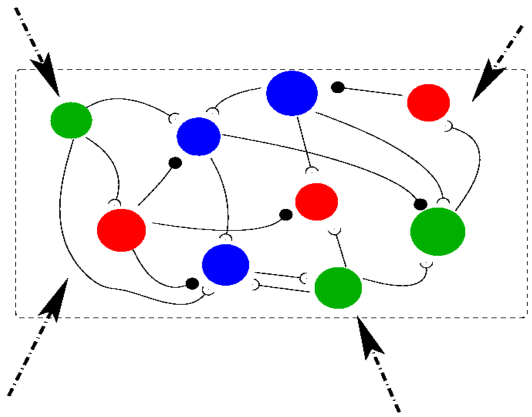

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- **Non linear and collective dynamics.**
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gIF Models M. Rudolph, A. Destexhe, Neural Comput., 18, 2146–2210 (2006).

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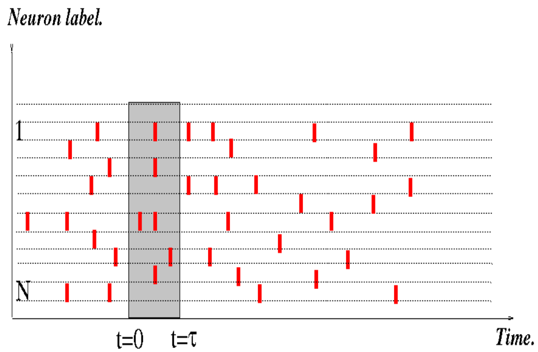
Neural network activity.



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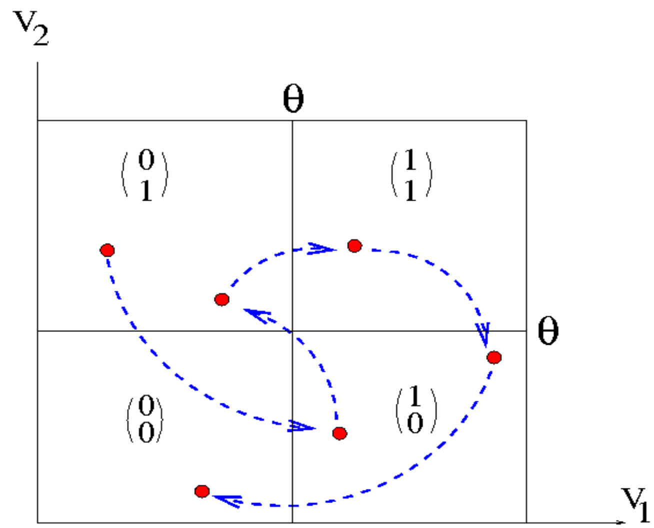
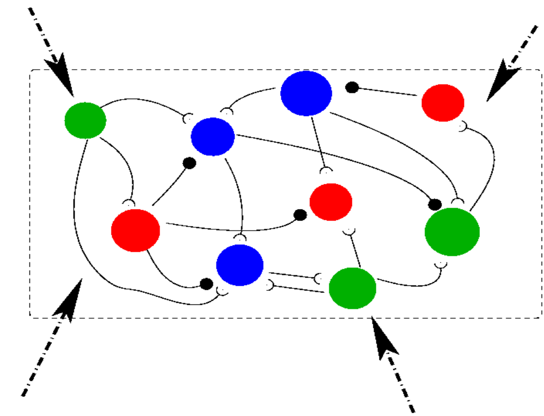


Raster plot. $\begin{pmatrix}0 & 1 & 0 & 1 & 1 & 0 & \ldots\\ 1 & 0 & 1 & 1 & 0 & 0 & \ldots\end{pmatrix}$



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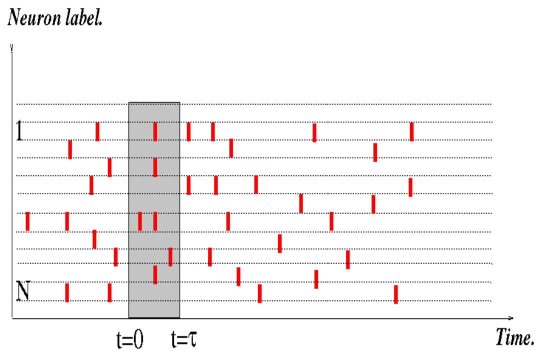

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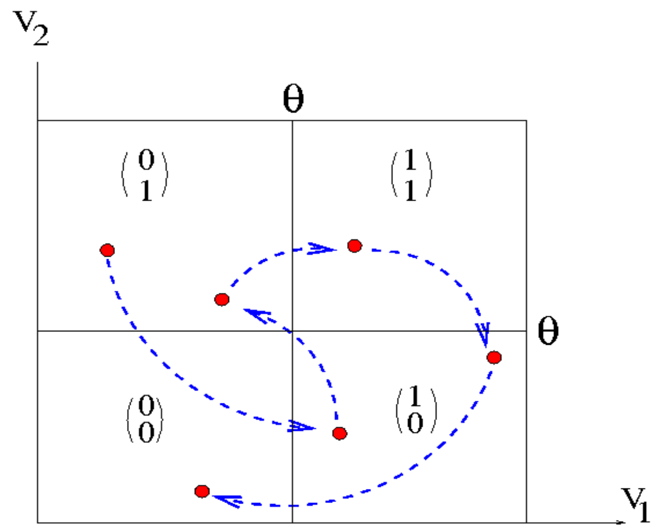

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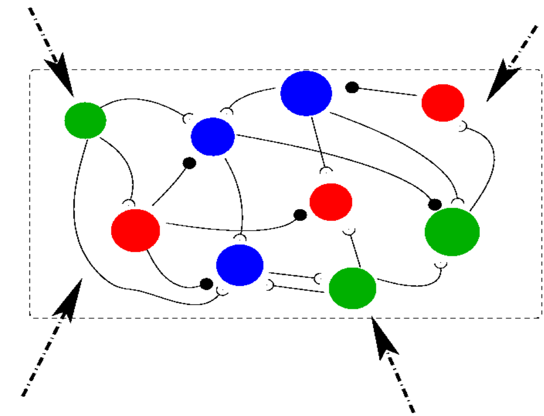

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Spikes trains provide a **symbolic coding**.

To a given “input” one can associate a **finite number** of **periodic orbits** (depending on the initial condition).

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E. Schneidman, M.J. Berry, R. Segev, W. Bialek, Nature,440, (2006)

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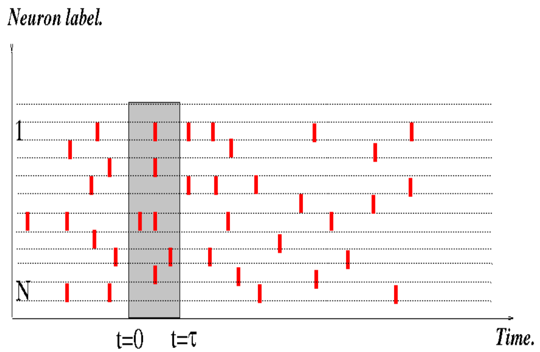

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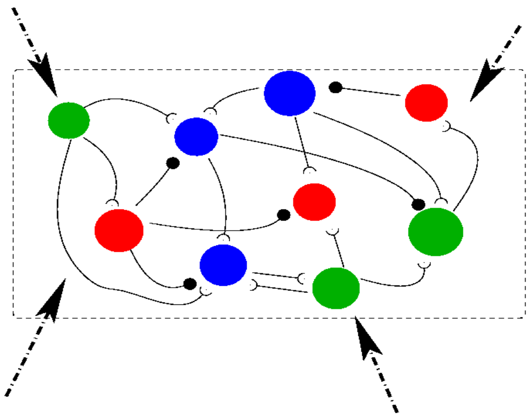

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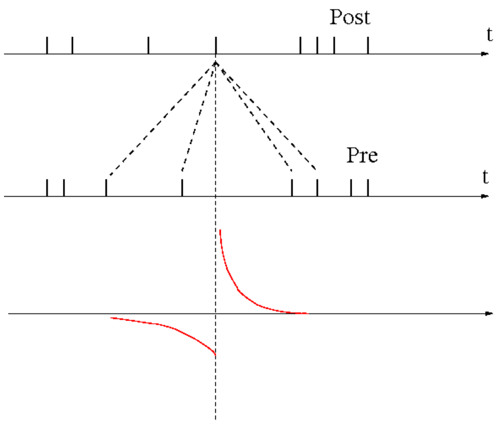

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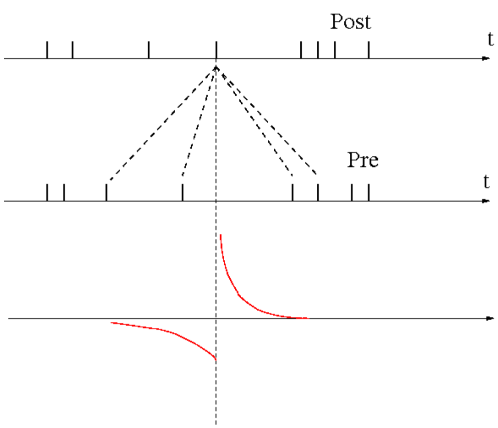

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$$\delta W_{ij}(t) = g(W_{ij}(t), [\omega_i]_{t,t-T_s}, [\omega_j]_{t,t-T_s})$$

$$\delta W_{ij}^{(\tau)} = W_{ij}^{(\tau+1)} - W_{ij}^{(\tau)}$$

$$= \epsilon \pi^{(T)} \left[\phi_{ij}(W_{ij}^{(\tau)}, [\omega_i^{(\tau)}], [\omega_j^{(\tau)}]) \right]$$

Convergence

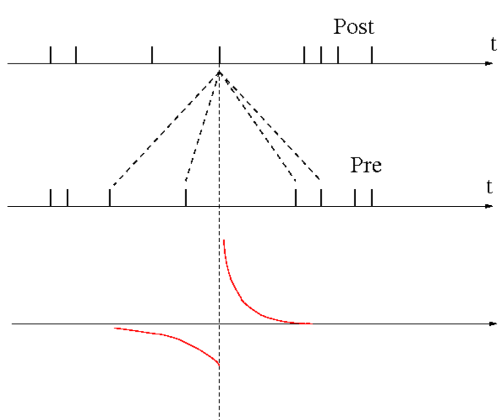
$$\delta W_{ij} = 0 \Rightarrow \pi^{(T)} \left[\phi_{ij}(W_{ij}^{(\tau)}, [\omega_i^{(\tau)}], [\omega_j^{(\tau)}]) \right] = 0$$

Example

$$\delta W_{ij}^{(\tau)} = \epsilon \left[(\lambda - 1) W_{ij}^{(\tau)} + \frac{1}{T - 2C} \sum_{t=C}^{T-C} \sum_{u=-C}^{C} f(u) \omega_i^{(\tau)}(t+u) \omega_j^{(\tau)}(t) \right]$$

$$f(x) = \begin{cases} A_- e^{\frac{x}{\tau_-}}, & x < 0; \\ A_+ e^{-\frac{x}{\tau_+}}, & x > 0; \\ 0, & x = 0, \end{cases}$$

$$C = \max(\tau_+, \tau_-)$$



Dynamics and statistics evolution

Changing synaptic weights

changing membrane potential dynamics

changing raster plots dynamics
and statistics

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Dynamics and statistics evolution

Changing synaptic weights

$\Downarrow$

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Dynamics and statistics evolution

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- Regular periods are separated by sharp variations of synaptic weights (phase transitions).
- If the synaptic adaptation rule “converges” then the corresponding statistical model is a Gibbs measure with potential

$$\sum_{i,j} \lambda_{ij} \phi_{ij}$$

The synaptic adaptation "rule" fixes the statistical model.

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- **Check on numerical examples ?**

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- **Check on real data ?**