Phase Space Structure of Spiking Neural Network

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We have investigated the structure of the so called "edge of chaos" in an Integrate-and-Fire Neural Network with discrete time dynamics. The existence of a "complexity" order parameter allows us to study the network dynamics and its response to an external input current. This could be applied to identify regions of enhanced dynamical sensitivity and response variability, which are important for many tasks such as discrimination in vision.

1. General Framework

- Aims

Neural Network — Dynamical Complexity → CODING ← Stimuli Response; CODING → For Computation; CODING ↔ Plasticity, Learning

- Neuron Model

Integrate-and-Fire

$$V(t) = \begin{cases} \tau_m \frac{dV}{dt} = -V(t) + R_m I(t) \\ V(t+\epsilon) = V_{reset}, \quad \text{if } V(t) \geq \theta \end{cases}$$

⇒ **Beslon-Mazet-Soula (2006)**

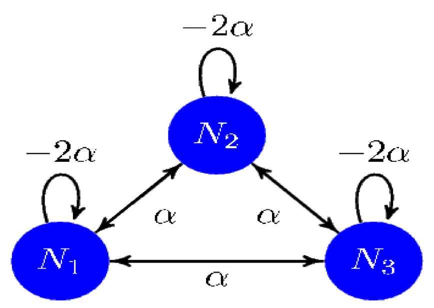
$$V(t+1) \equiv F[V(t)] = \begin{cases} \gamma V(t) + I(t) \\ V_{reset} + I(t), \quad \text{if } V(t) \geq \theta \end{cases}$$

- Network Structure : **Laplacian** 1-D

$$V_i(t+1) = F_i(\mathbf{V}(t)) = \gamma V_i(t)(1 - Z[V_i(t)]) + \sum_{j=1}^{N} W_{ij} Z[V_j(t)] + I_i^{ext}(t)$$

where $Z[x] = 1$ if $x \geq \theta$ or 0 otherwise

↔ $W_{i\pm1,i} = \alpha, \quad W_{ii} = -2\alpha,$ and $W_{ij} = 0$ otherwise



- Which is the response to Stimuli?

Asymptotic Dynamics: Configuration Space $\mathbb{M} \subseteq \mathbb{R}^N$, *ω-limit set*

$$\omega(\mathbb{M}) = \left\{ \mathbf{V}' \mid \exists \{t_k\}_{k=0}^{+\infty}, \mathbf{V} \in \mathbb{M}, \mathbf{F}^{t_k}(\mathbf{V}) \to \mathbf{V}' \text{ as } t_k \to \infty \right\}$$

Firing rate:

$$PD_i = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T} Z[V_i(t)] \qquad \forall i = 1, \ldots N$$

⇒

$$dIP = \inf_{\mathbf{V} \in \omega(\mathbb{M})} \sum_{i=1}^{N} (I_i - PD_i)^2 \qquad (*)$$

Dayan et Abbot(01), Soula et al.(06)

2. Theoretical Results

- **Symbolic Coding for the dynamics :**

Bounded Configuration Space $\mathbb{M} = [V_{min}, V_{max}]^N$

Defining $B_0 = [V_{min}, \theta)$ and $B_1 = [\theta, V_{max}]$

⇒ **Natural partition:** $\mathbb{M} = \bigcup_{\eta}^{2^N} \mathbb{M}_\eta$, where

$\eta = \{\eta_1, ..., \eta_N\} \in \mathbf{\Lambda} = \{0,1\}^N$ and $\mathbb{M}_\eta = \left\{ \times_{i=1}^{N} B_{\eta_i} \right\}$

- **Edge of Chaos** : Non generic region where the system presents sensitivity to initial conditions.

$$S = \{\mathbf{V} \in \mathbb{M} \mid \exists i, V_i = \theta\}$$

$$d(\mathbf{V}^+, S) = \inf_{t \geq 0} \min_{i=1,\ldots N} |V_i(t) - \theta|$$

$$dAS = d(\omega(\mathbb{M}), S) = \inf_{\mathbf{V} \in \omega(\mathbb{M})} d(\mathbf{V}^+, S)$$

→ **Theorem**

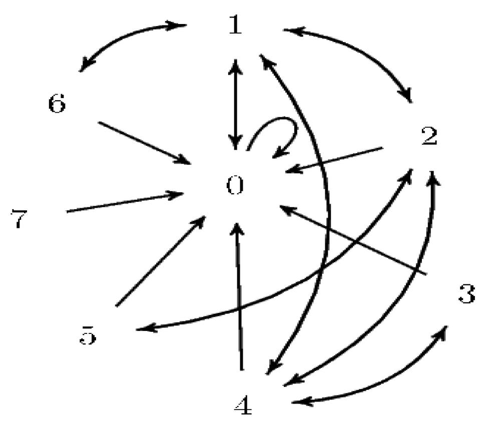
If $dAS = d(\omega(\mathbb{M}), S) > \epsilon > 0$ then:

i. *Call $\mathbf{F}^t$ the t-th iterate of $\mathbf{F}$. There is a finite T, depending on $s(\omega(\mathbb{M}), S)$, such that $T \to +\infty$ when $d(\omega(\mathbb{M}), S) \to 0$ and such that there exists a finite Markov partition for $\mathbf{F}^T$.*

ii. *$\omega(\mathbb{M})$ is a finite union of stable periodic orbits with a finite period.*

- **Transition Graphs**

example N=3

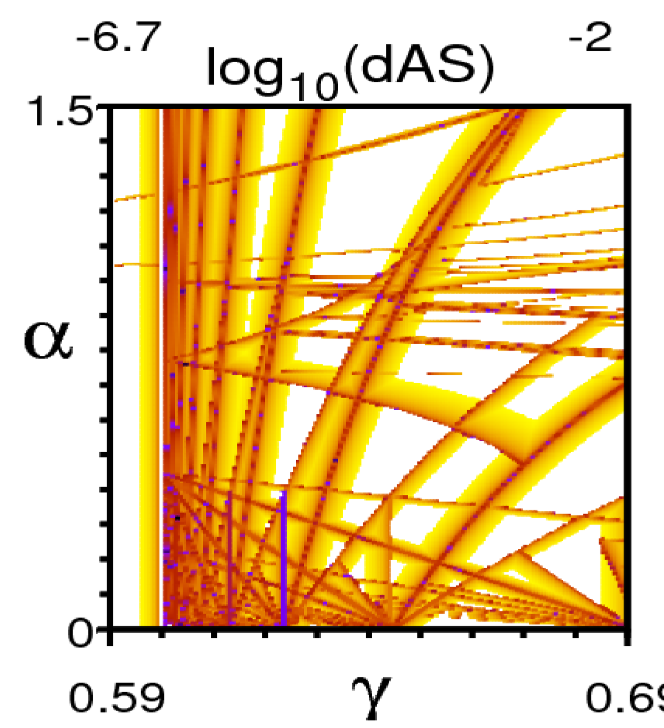


If $I^{ext} > \theta(1-\gamma)$ (**) for at least one neuron, then transitions from the inactive state **0** to some other states are allowed, which increases the number and length of orbits.

Cessac(08)

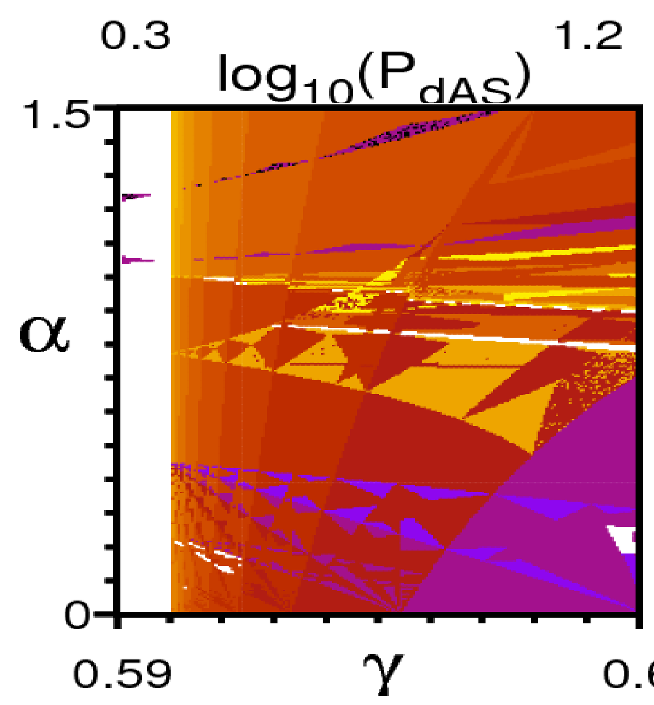
3. Tracking the Edge of Chaos : Simulation Results N=5 , I^{ext}=(0 , 0 , 0.4 , 0.4, 0.4)

PHASE SPACE STRUCTURE



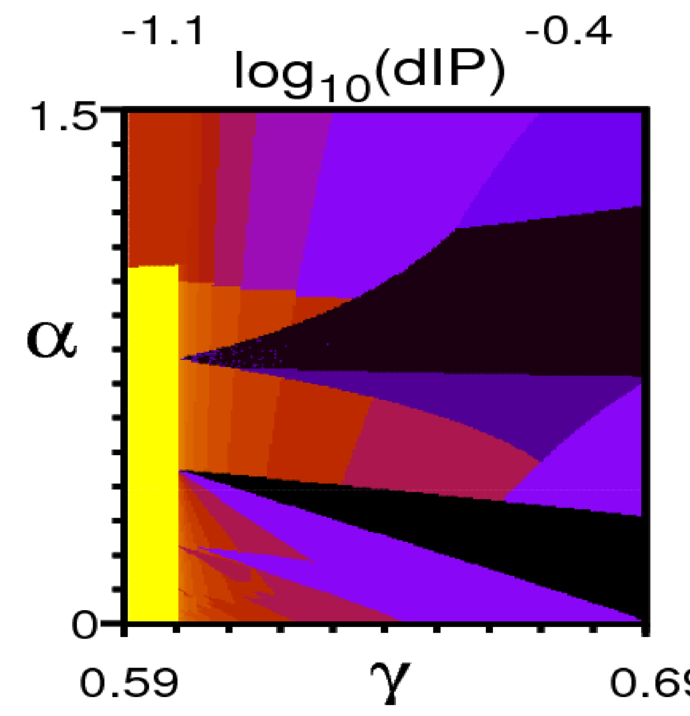
$\log(dAS)$, the edge of chaos is at $dAS = 0$; white denotes: $dAS < 10^{-7}$ at $\gamma = 0.6$ and $dAS \geq 10^{-2}$ everywhere else.

DYNAMICAL BEHAVIOR



$\log(P)$, where P is the period of the orbit that minimizes dAS; white denotes: $P = 1$ at $\gamma < 0.6$ and $P > 20$ everywhere else.

RESPONSE TO STIMULI



$\log(dIP)$, where dIP is the minimal distance between the input vector and the firing rate vector (eq.$(*)$).

4. Conclusions

- Depending on the external Input value, the structure of **the Edge of Chaos** and the response variability to the stimuli can be both drastically modified.
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- More Precisely, the condition $I^{ext} > \theta(1-\gamma)$ (**) defines the region that gets strongly modified, since the network is able to come back to an activity state from the totally inactive one.
- If the equality holds in the relation (**), the input takes infinity time to produce a firing after reaching the inactive state (Ghost Orbit).

5. Perspectives

- Is it possible to predict the phase space structure of the edge of chaos and under which conditions?
- How this results might appear on more general kind of networks?
- Analysis of networks under the action of Spike-Timing Dependent Plasticity (STDP) rules, namely in/ near the edge of chaos.
- Tuning Networks for robust pattern recognition of filtering tasks.

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