PHASE SPACE STRUCTURE OF SPIKING NEURAL NETWORK WITH LAPLACIAN COUPLING

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Let us consider a Leaky Integrate-and-Fire neural network with discrete time dynamics given by

$$V_i(t+1) = \gamma V_i(t)[1 - Z(V_i(t))] + \sum_j W_{ij} Z(V_j(t)) + I_i \quad (1)$$

with V_i the membrane potential of neuron i, $\gamma \in [0,1]$ the leak, $Z(x) = 0$ if $x < \theta$ and 1 otherwise, where θ is the threshold. It has been shown in [1] that the attractors of eq.1 are generically periodic orbits, although the period can be arbitrarily long. Moreover, the dynamics is chaotic, namely sensitive to initial conditions and perturbations, in a non generic region of the parameter's space, traditionally called the "edge of chaos". Interpreted in terms of neural outputs (raster plots) as the response to an input current (I_i), one can show that the system either exhibits stable input-output responses with low variability, or a high variability with low stability. The largest variability occurs at the edge of chaos, but what is the structure of this set? In order to investigate its structure we consider here a $1D$ circular network with (discrete) Laplacian coupling, i.e. with a diffusive interaction given by $W_{i\,i\pm1} = \alpha$, $W_{ii} = -2\alpha$ and 0 otherwise with $\alpha > 0$. In Figure 1(Left), dAS is the maximal amplitude of a perturbation that leaves the orbit stable. The edge of chaos corresponds to $dAS = 0$. Figure 1(center) shows the period of the most sensitive orbit. Figure 1(right) depicts the minimal distance between the input vector and the firing rate vector corresponding to allowed orbits. These results show that even in this simple network structure an input has drastic and non trivial effects on the global dynamics, in the structure of the edge of chaos and also on the response variability to the stimuli.

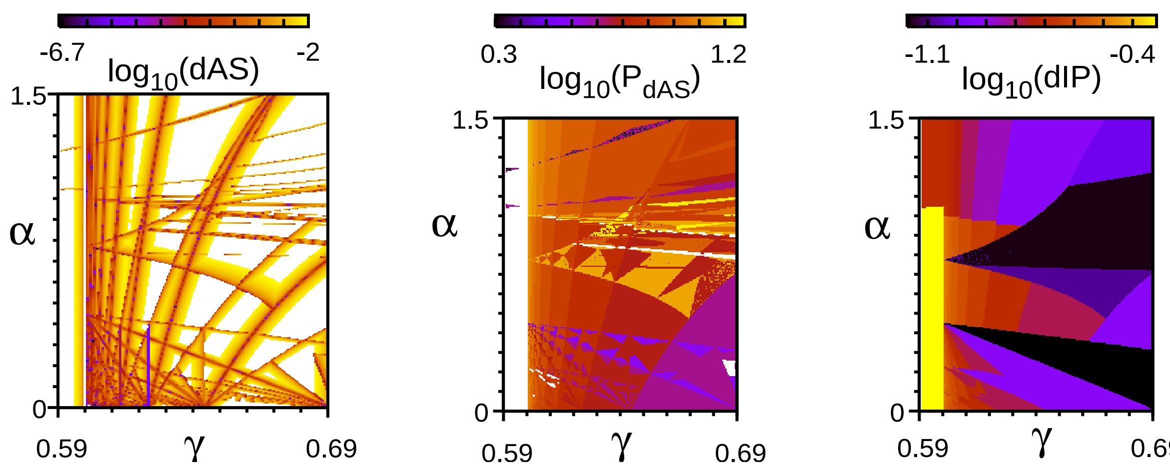

Figure 1. Simulation of 5 Neurons with threshold $\theta = 1.0$ and input intensity $I = 0.4$: **(Left)** $\log(dAS)$, white denotes $dAS < 10^{-7}$ at $\gamma = 0.6$ and $dAS \geq 10^{-2}$ everywhere else in the graphic. **(Center)** $\log(P)$, where P is the period of the orbit that minimizes dAS, white denotes $P = 1$ at $\gamma < 0.6$ and $P > 20$ everywhere else. **(Right)** Minimum distance between the input and the firing rate vector.

REFERENCES

[1] Bruno Cessac. In: *J Math Biol* 56.3 (2008). Pp. 311–345.