Towards formal verification of imperative concurrent data structures

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- Imperative programs

$$P$$

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data structures
(heap)

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- Temporal property (safety, liveness)

$$P_1 \parallel \cdots \parallel P_n \models \varphi$$

$\underbrace{P_1 \parallel \cdots \parallel P_n}$ ↓ data structures (heap)

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$$\underbrace{P_1 \parallel \cdots \parallel P_n}_{\text{data structures (heap)}} \models \varphi \,\}\; \text{LTL } (\Box, \Diamond, \bigcirc, \mathcal{U})$$

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Regional Logic { data structures (heap)

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Regional Logic { data structures (heap)

Verification Diagram

Reasoning about the heap

Reasoning about the heap

- Separation Logic

  Hoare logic extension to reason about shared mutable data structure

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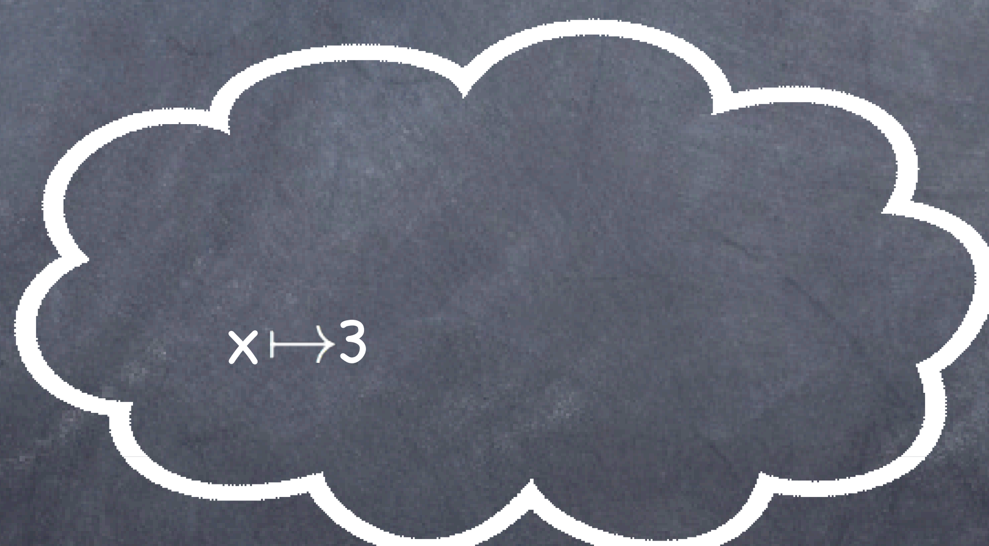

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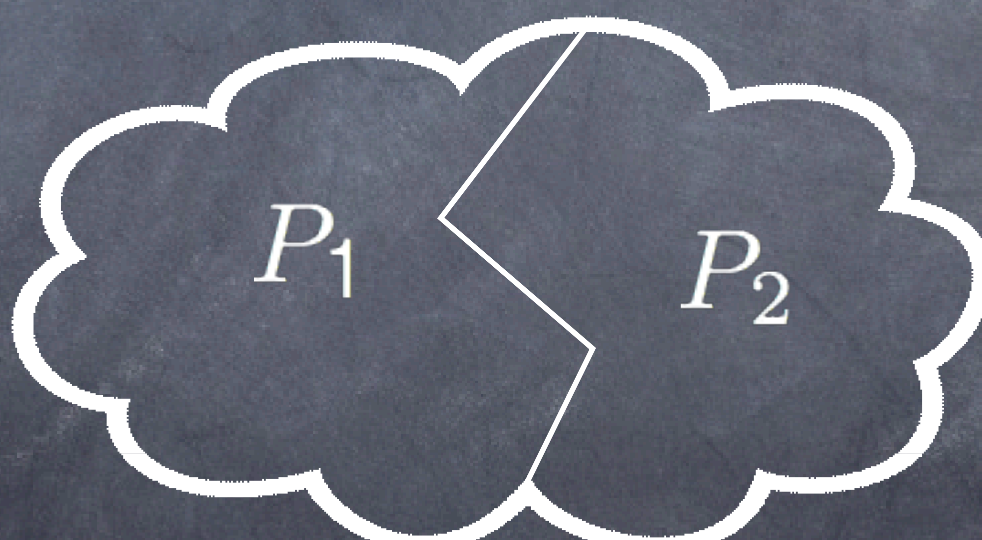

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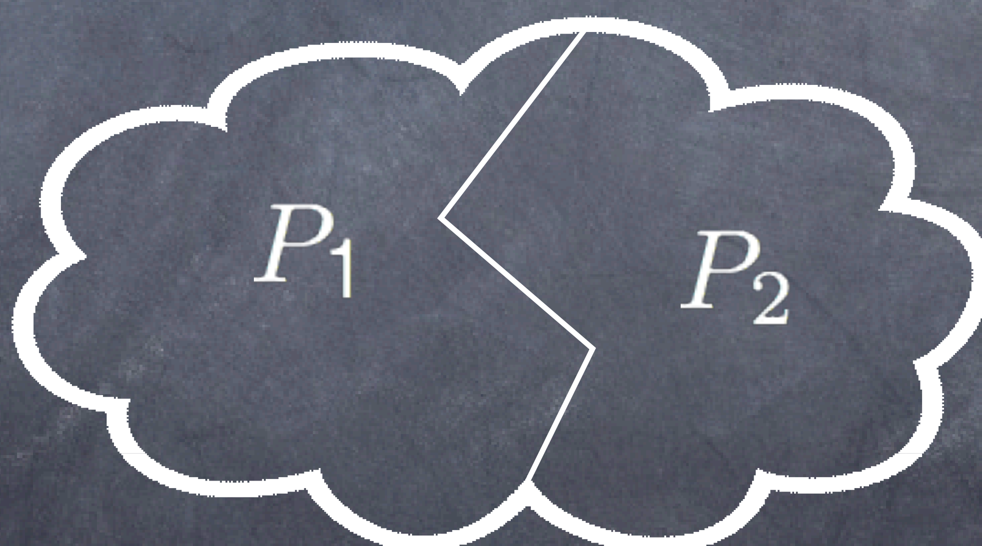

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    $\forall x : K \in R \mid P$

Verification Diagrams

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$$\begin{array}{c} P \models \varphi \\ \downarrow \\ \Psi \end{array}$$

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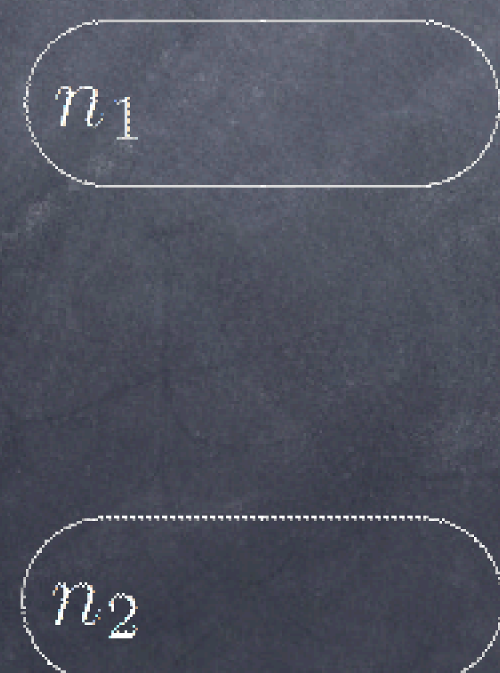

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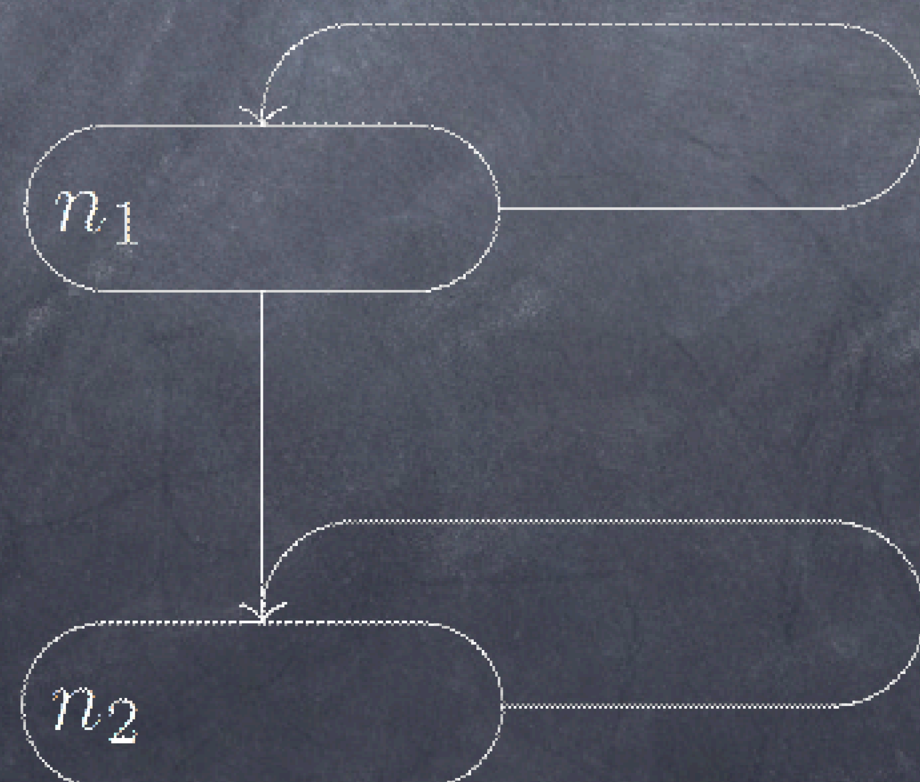

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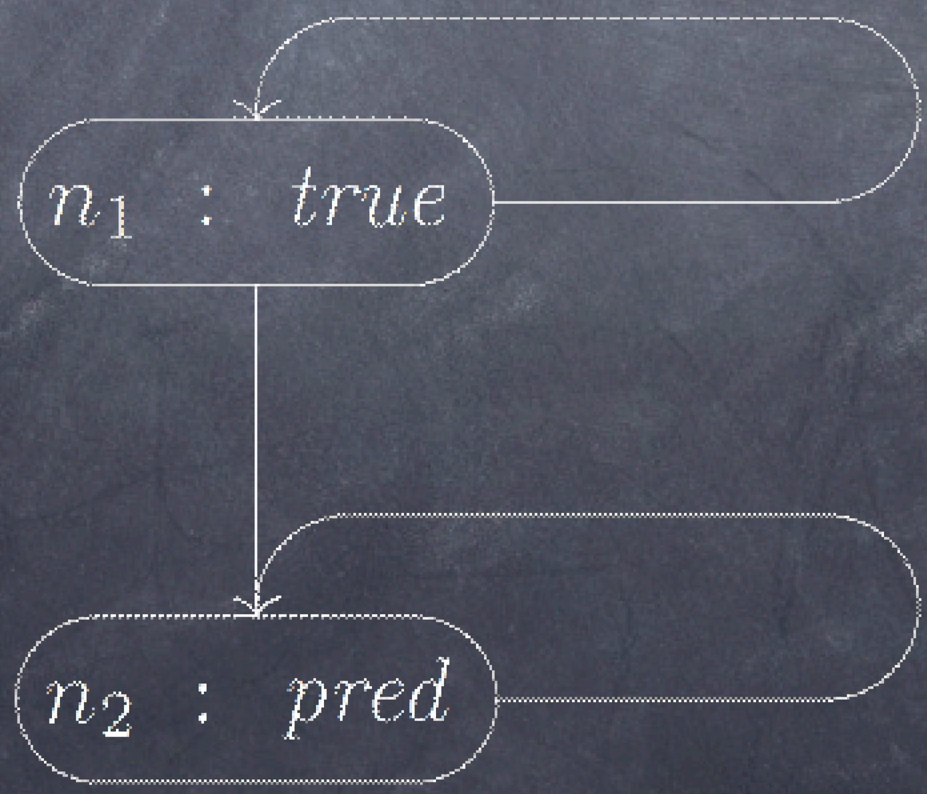

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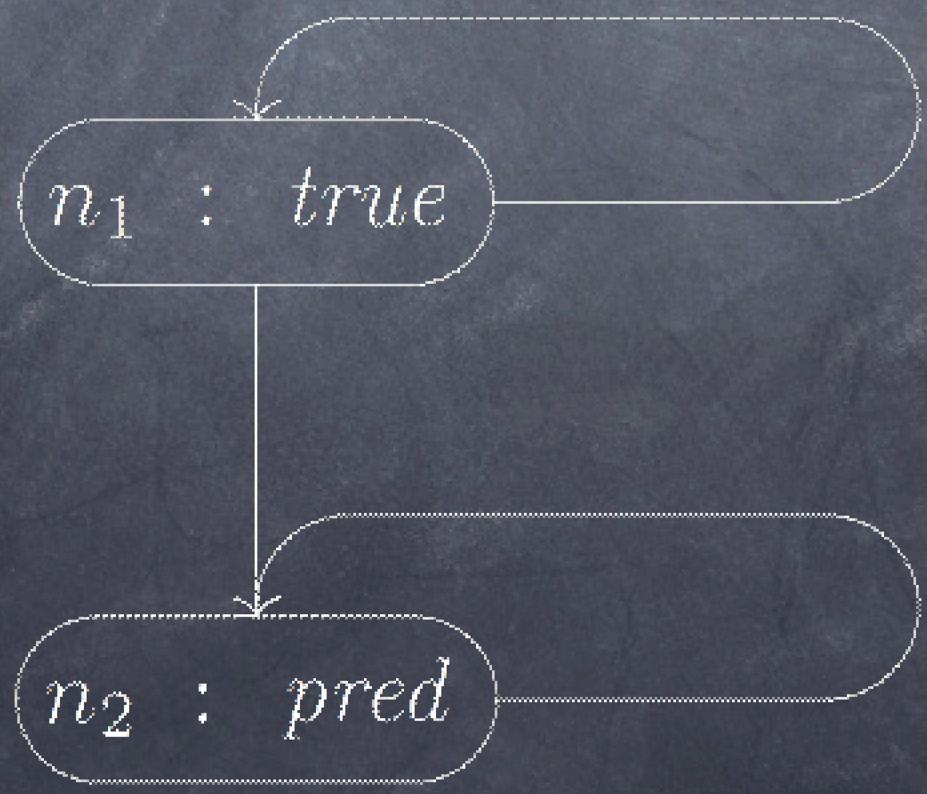

$\Diamond \Box pred$

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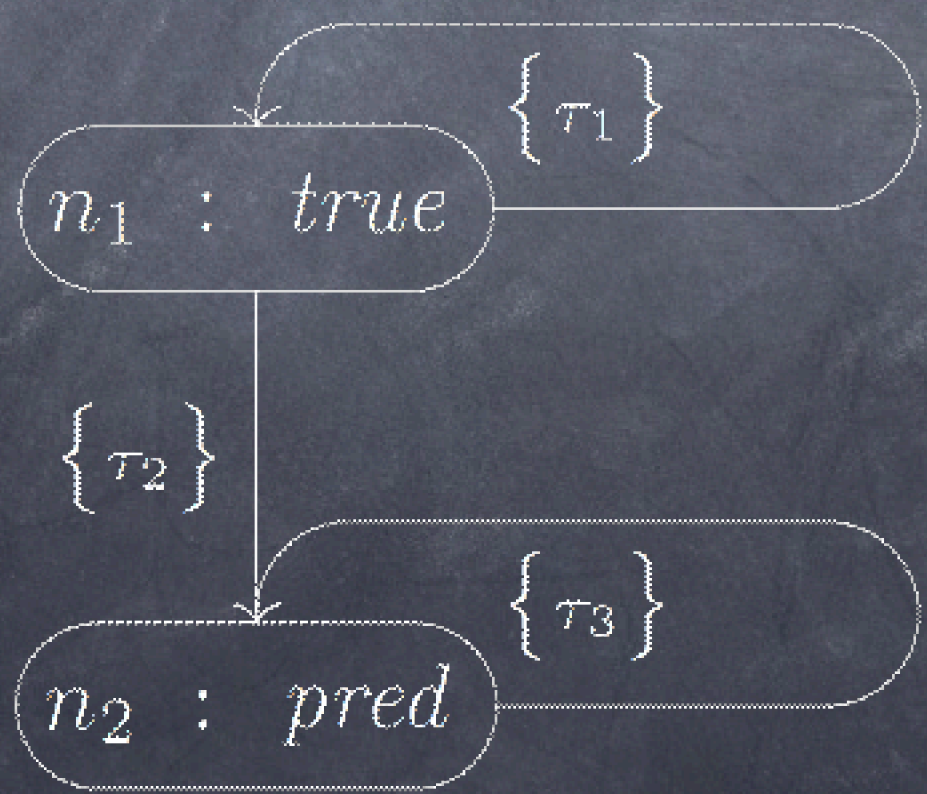

Main Idea

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Concurrent Data Structure

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φ

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Concurrent Data Structure

Most General Client [N] φ

(extended with GV)

Main Idea

Concurrent Data Structure

Most General Client [N] Ψ φ

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Main Idea

Concurrent Data Structure

Most General Client [N] Ψ φ

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- Verification conditions like: initialization, consecution, acceptance, fairness, satisfaction...

Main Idea

Concurrent Data Structure

↙

Most General Client [N] ⟷ Ψ ⟷ φ

(extended with GV)

- Verification conditions like: initialization, consecution, acceptance, fairness, satisfaction... ✓

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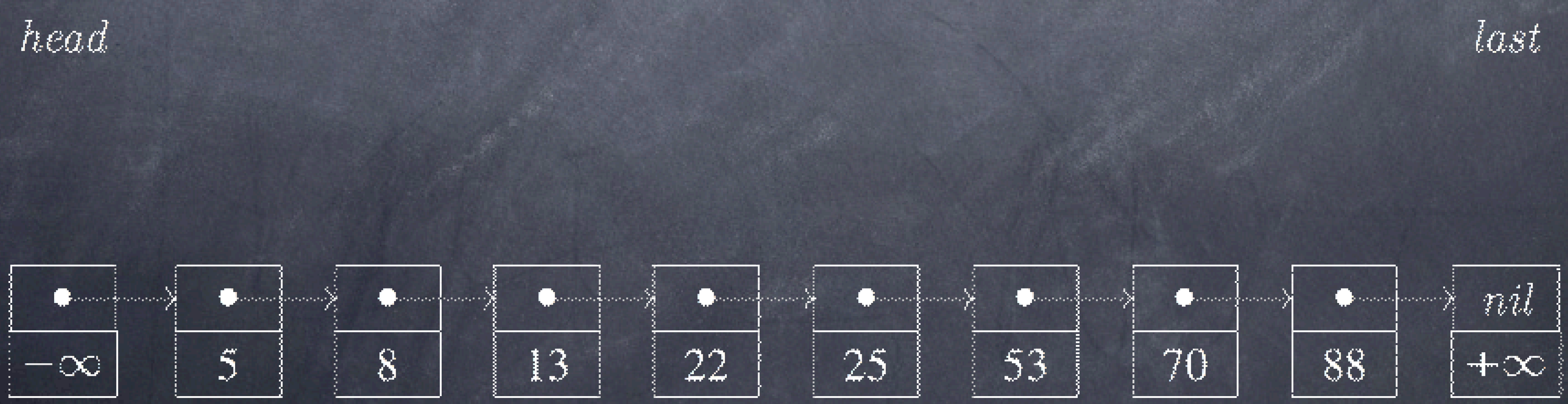
- Hierarchy of linked lists



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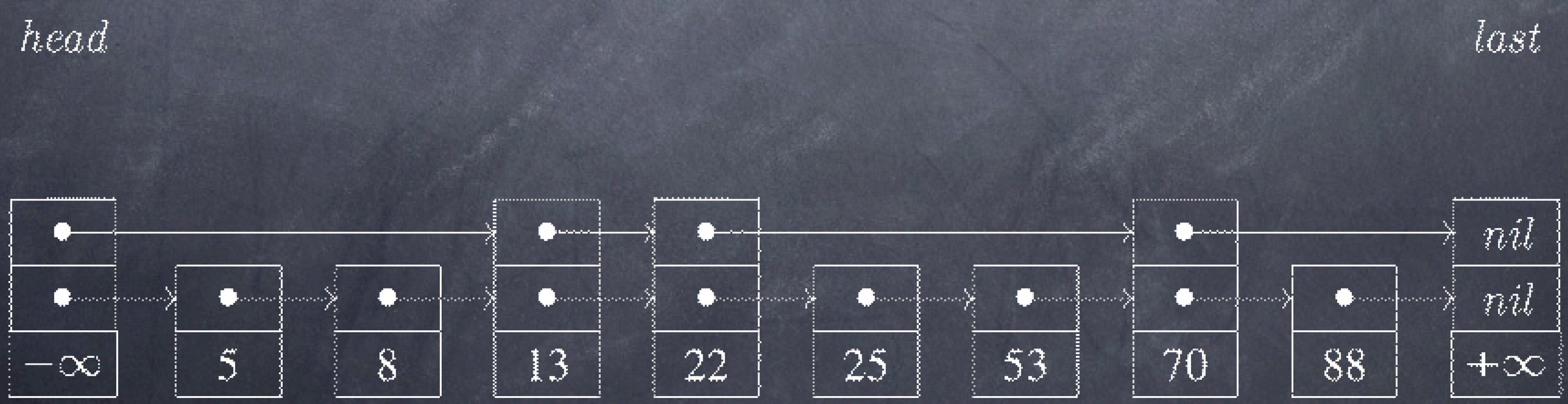

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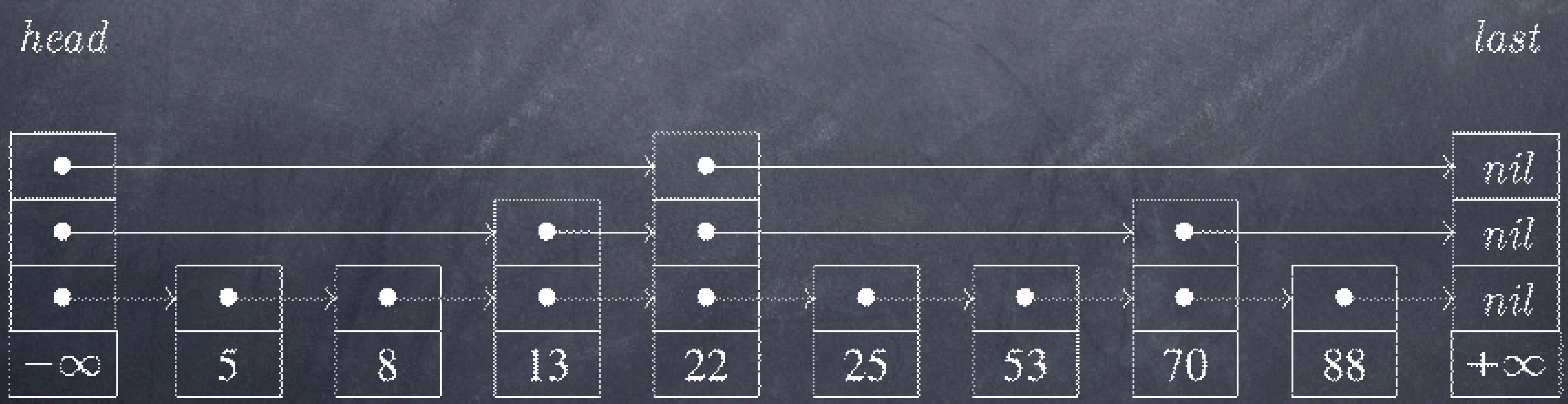

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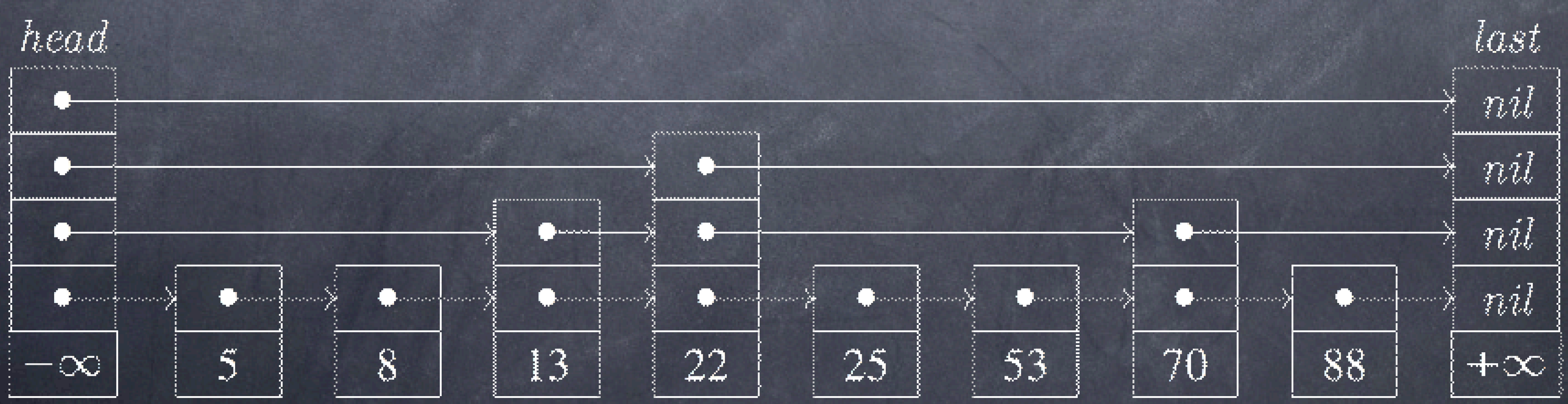

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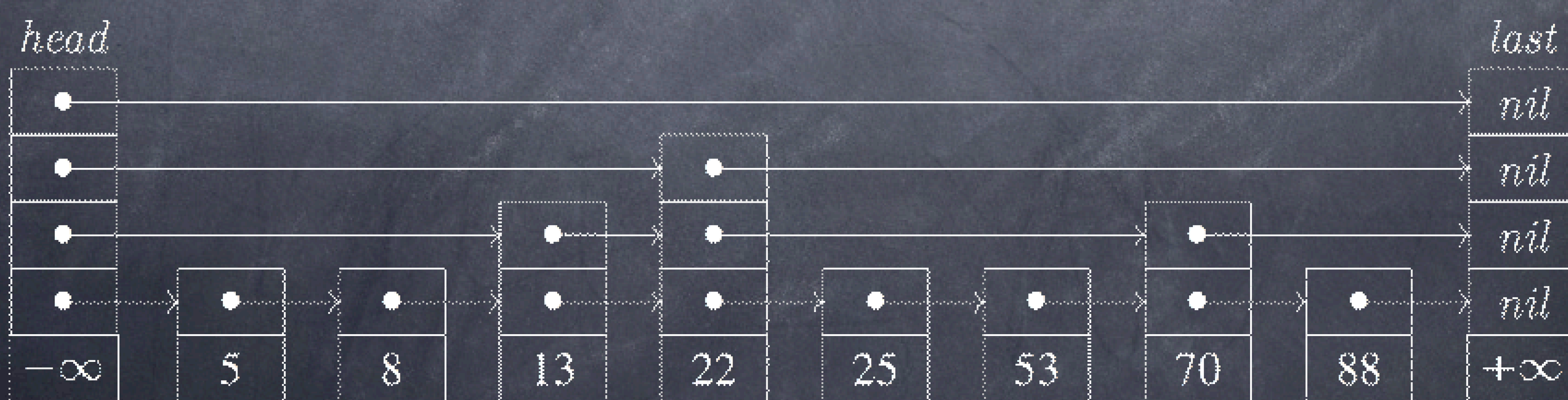

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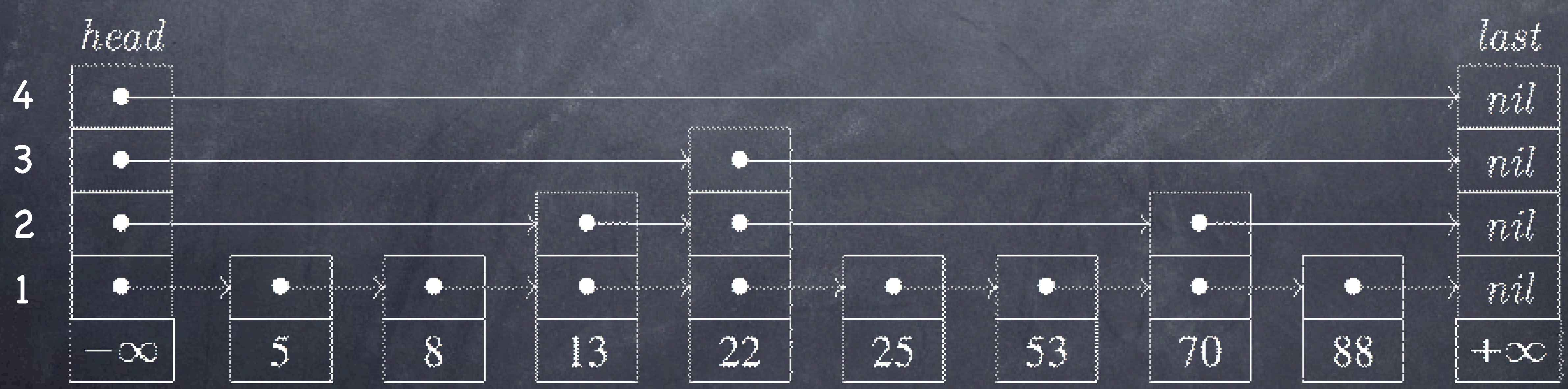

Fine-grained lock-coupling concurrent skiplists

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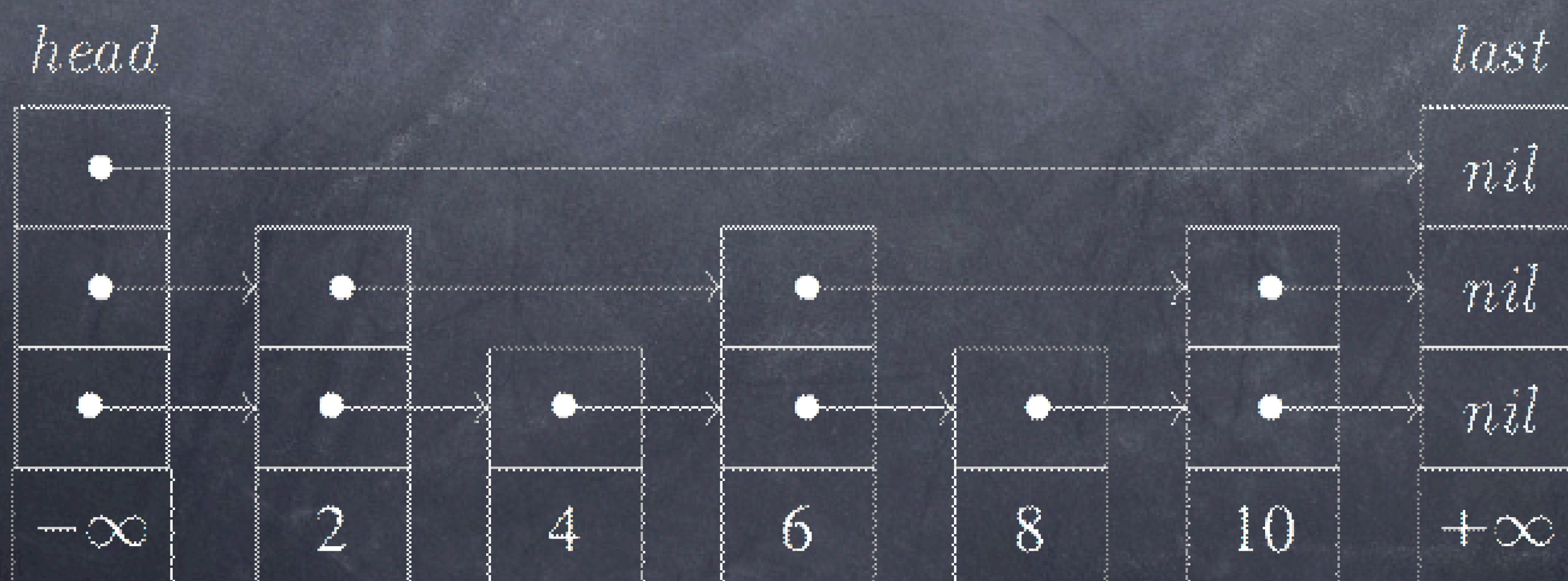

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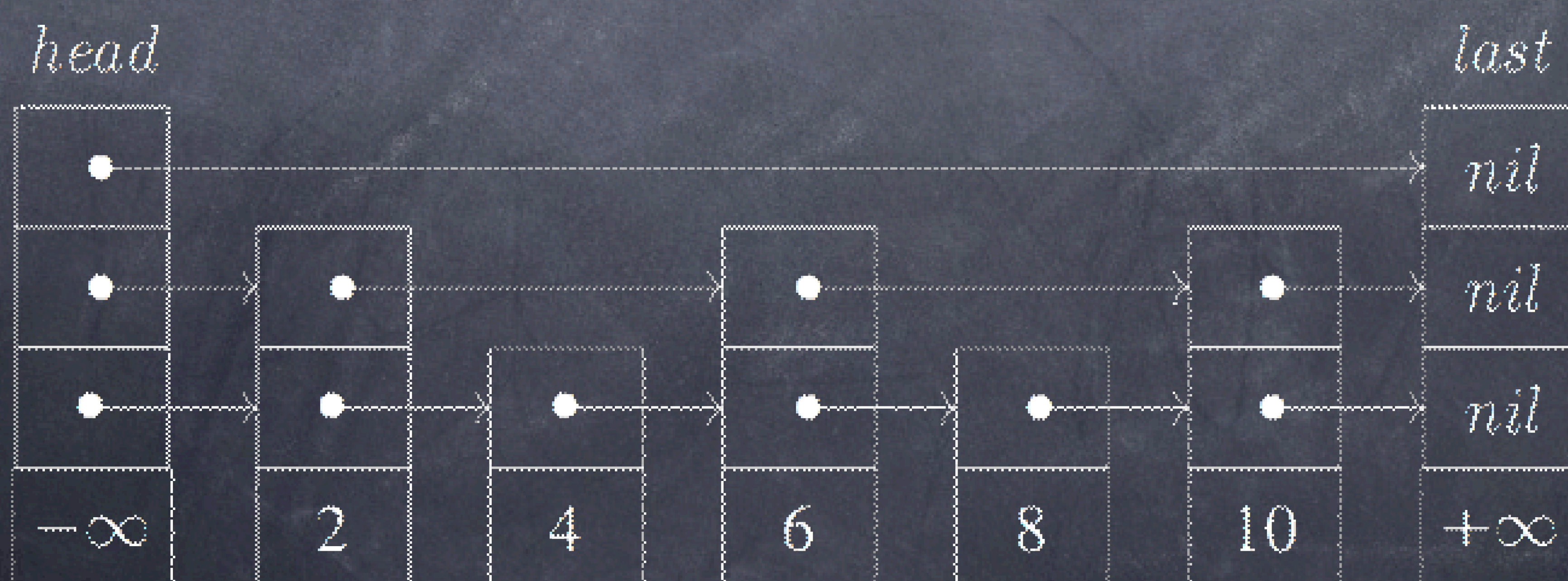
$insert^{[k]}(9)$



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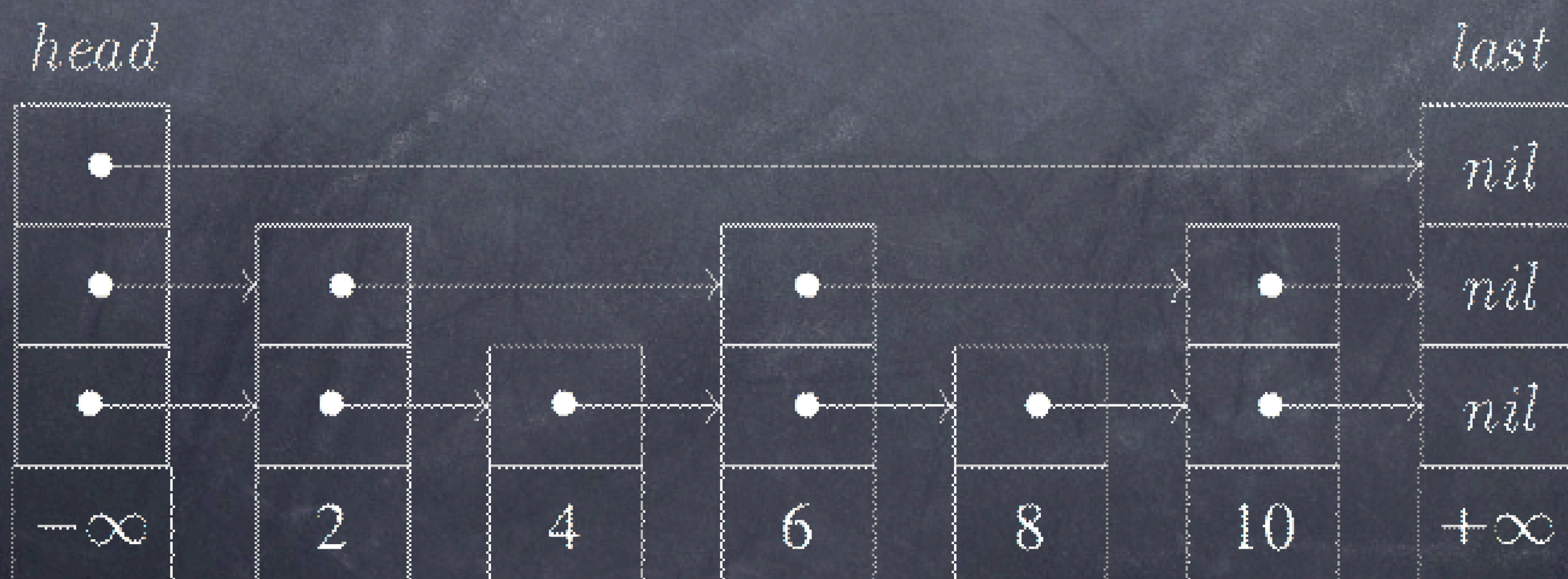

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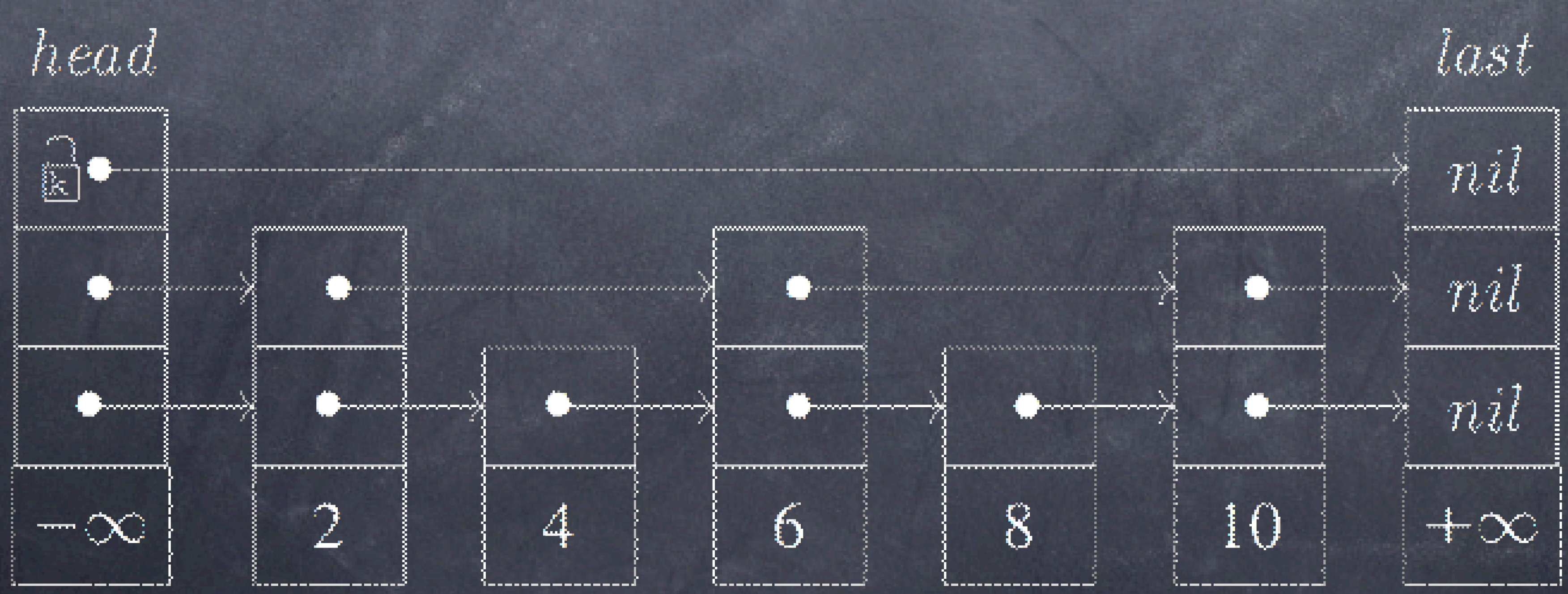

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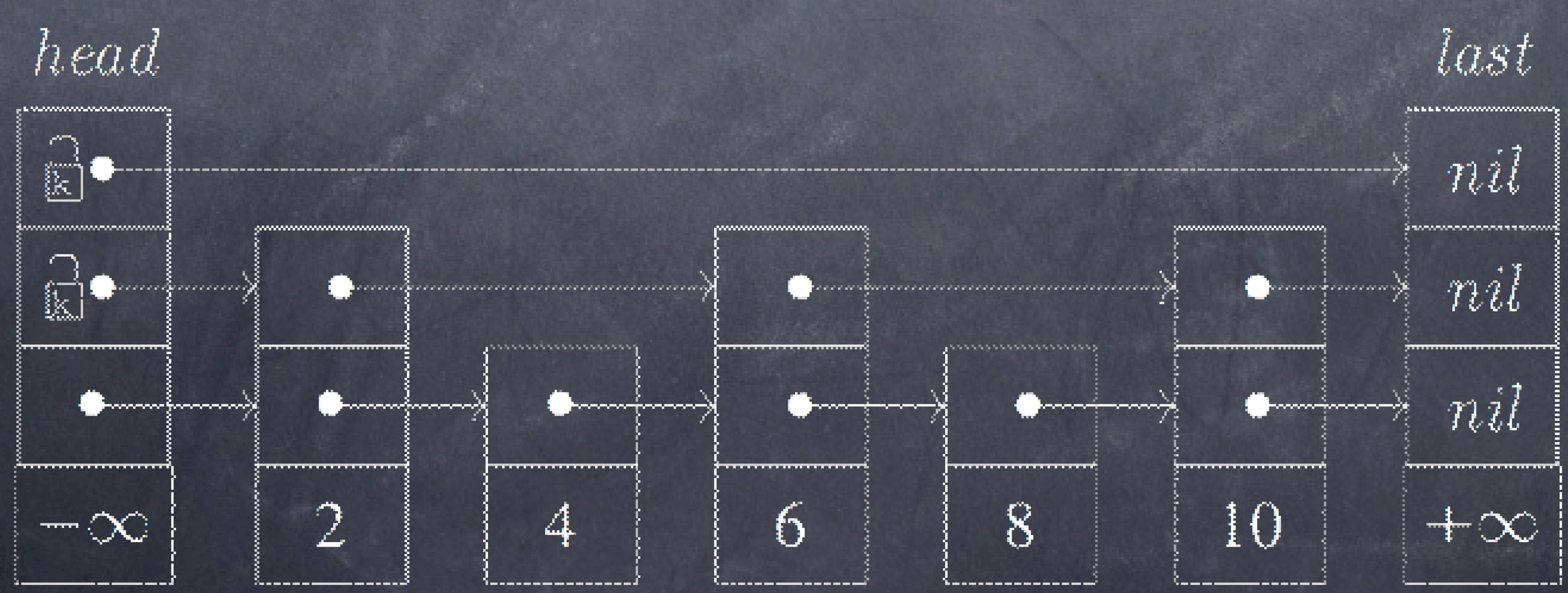

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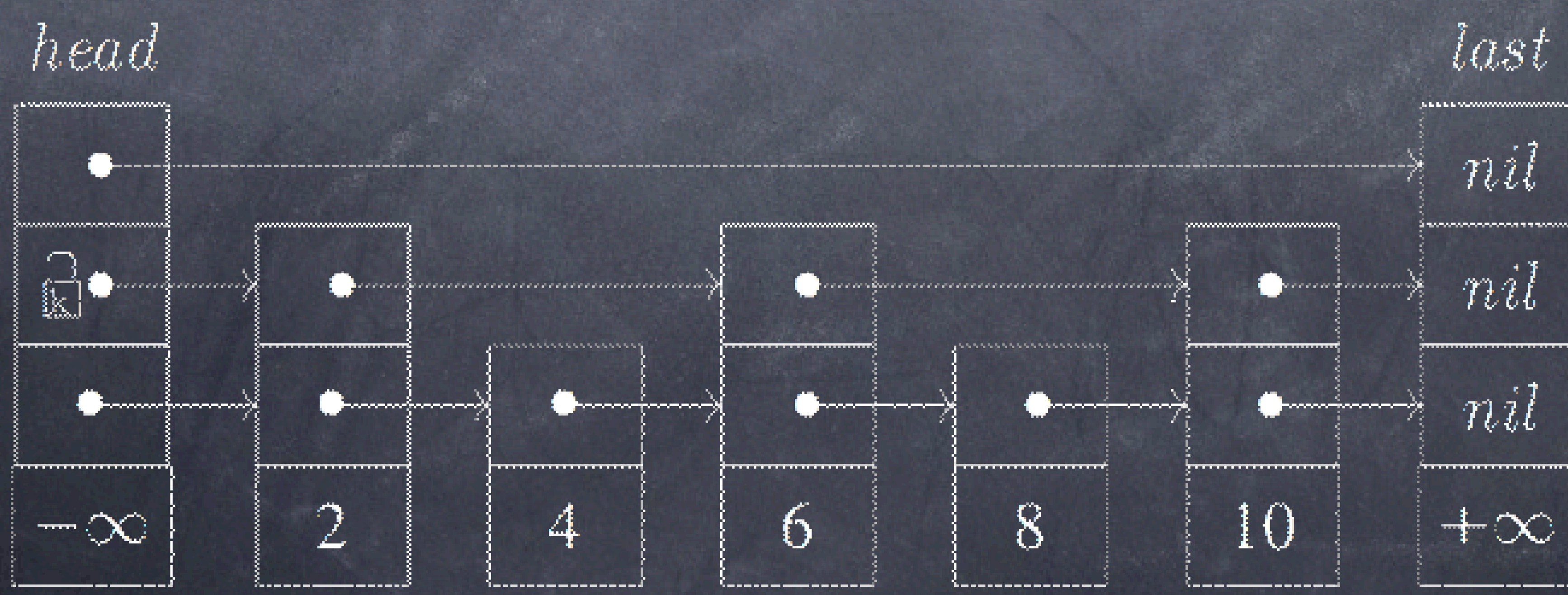

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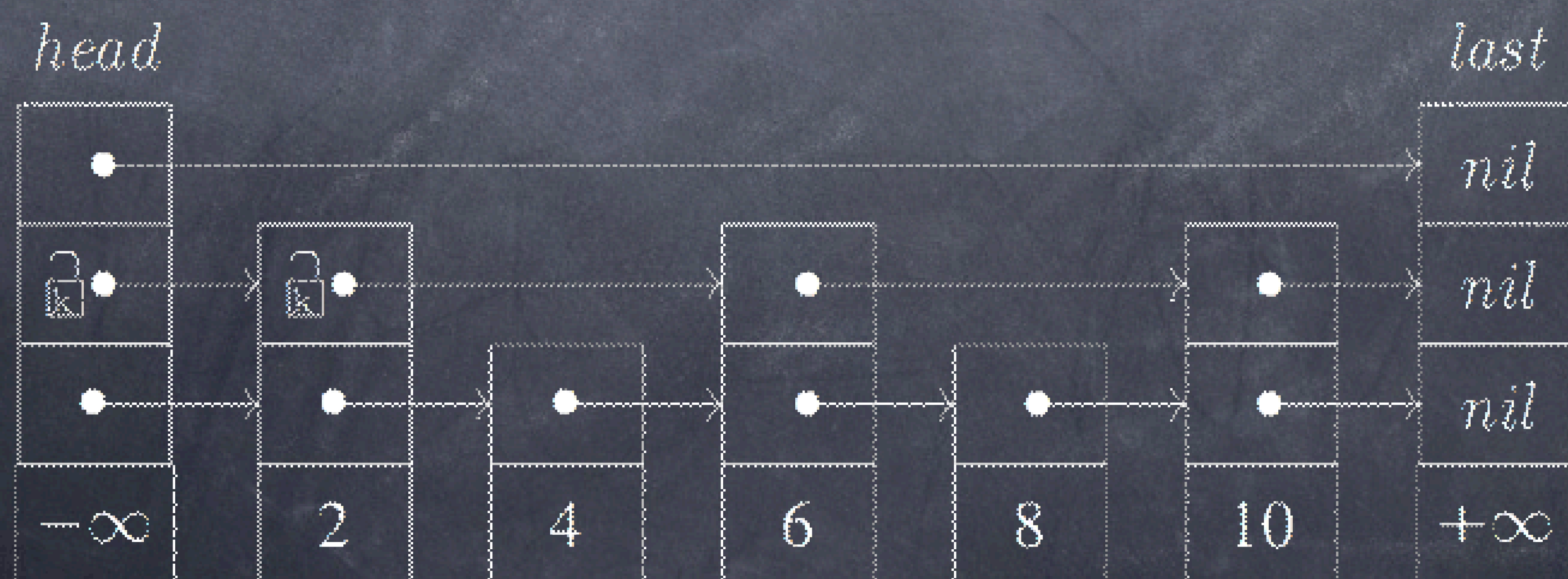

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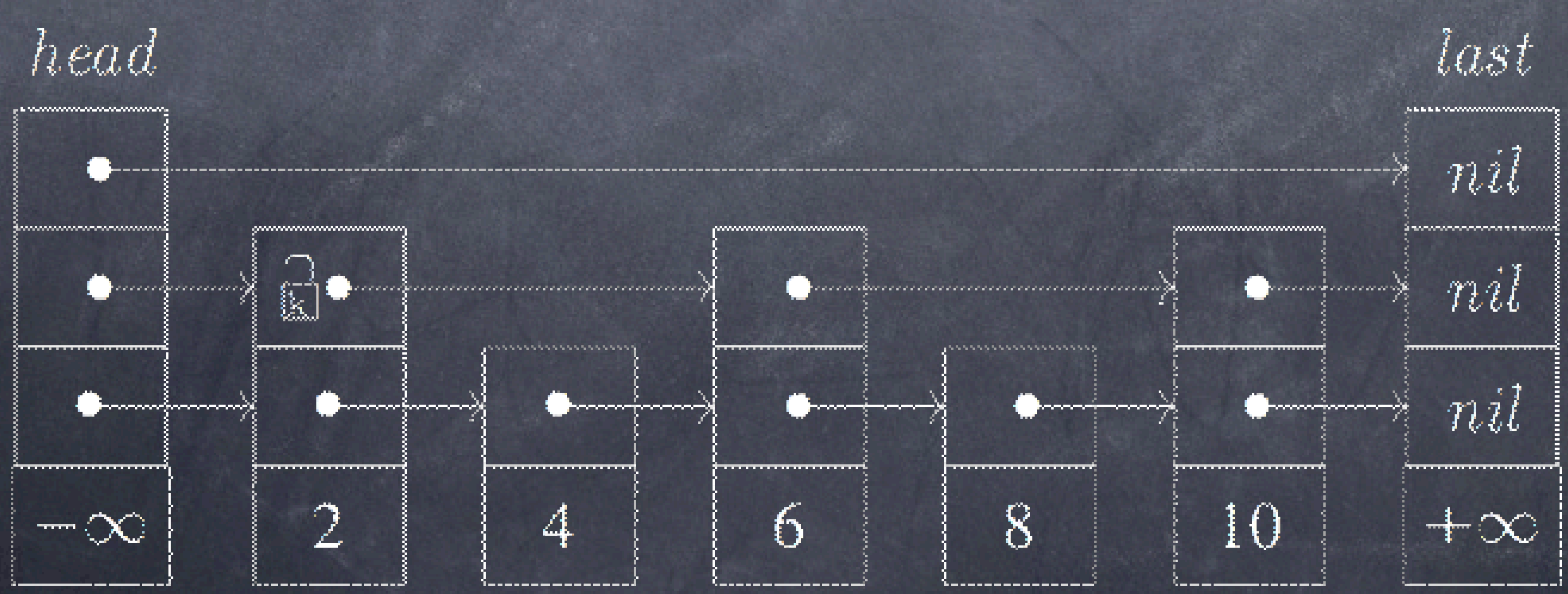

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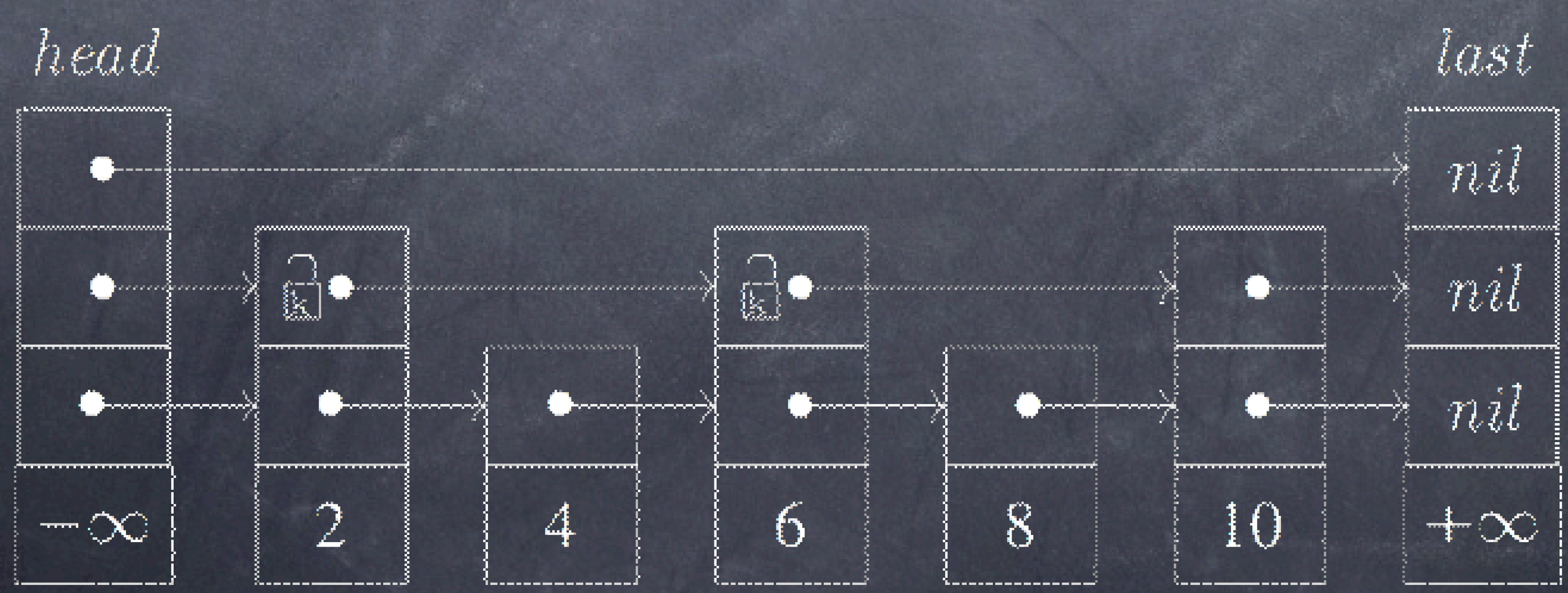

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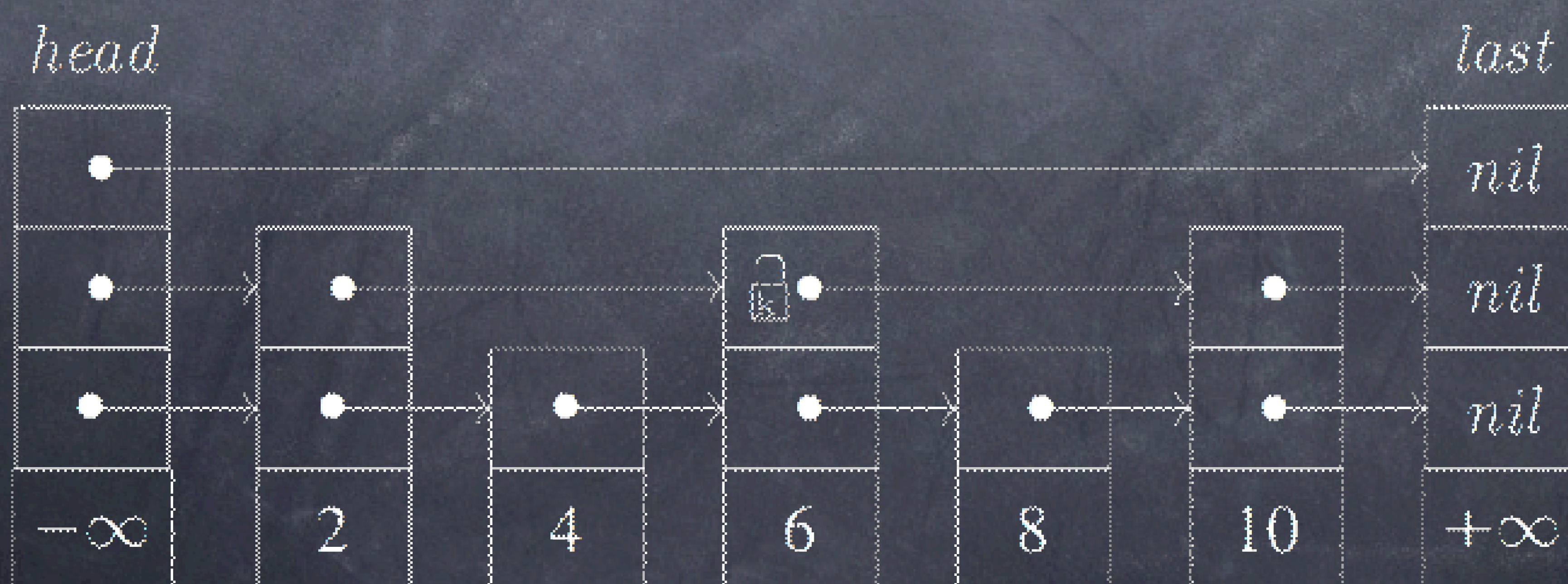

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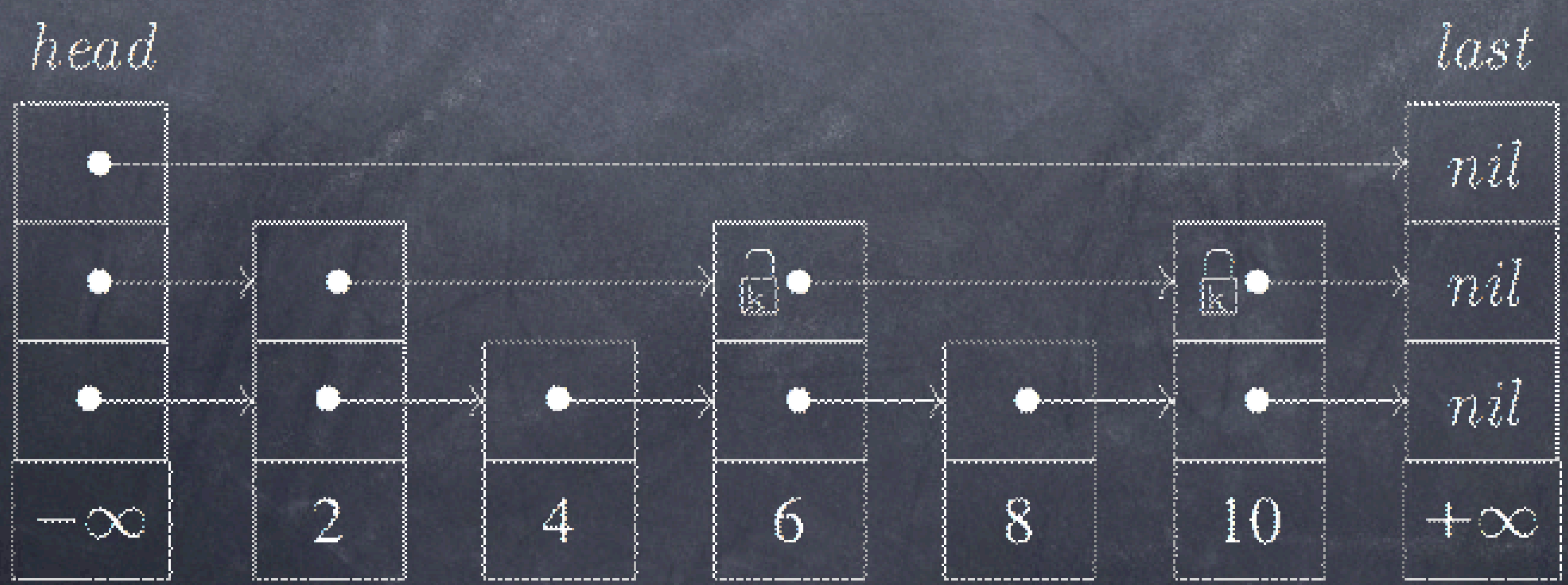

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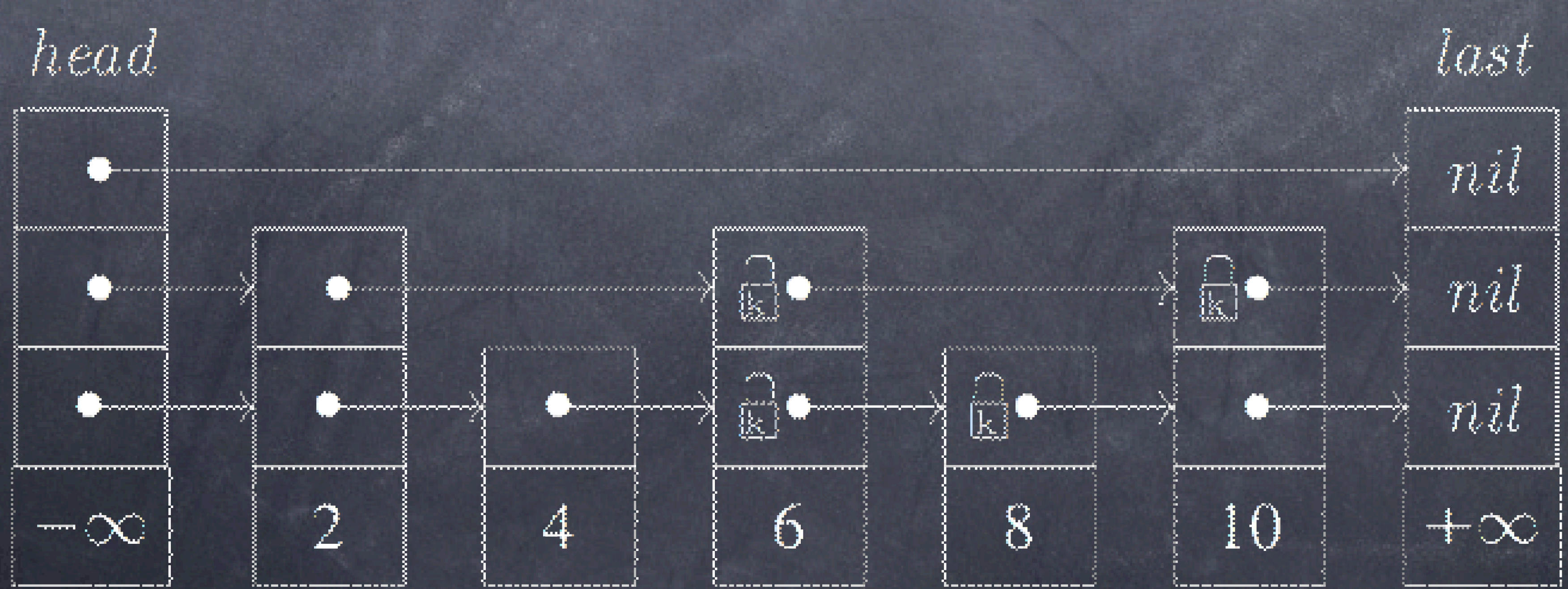

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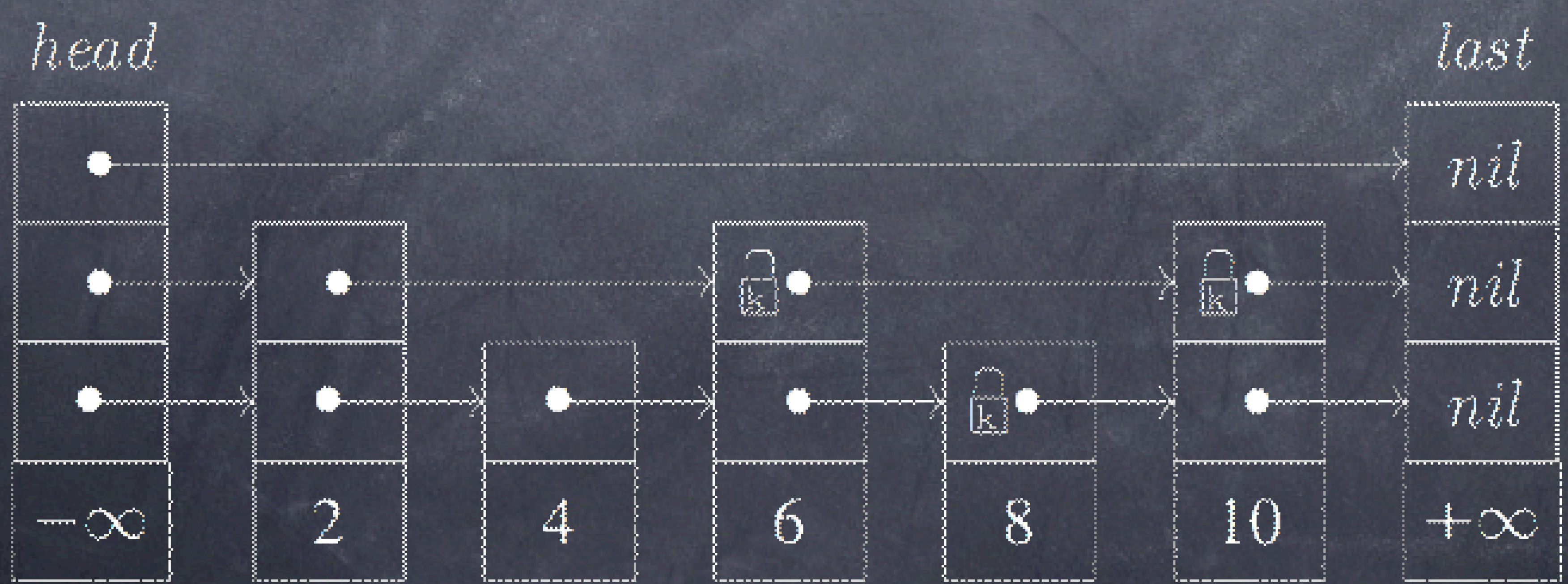

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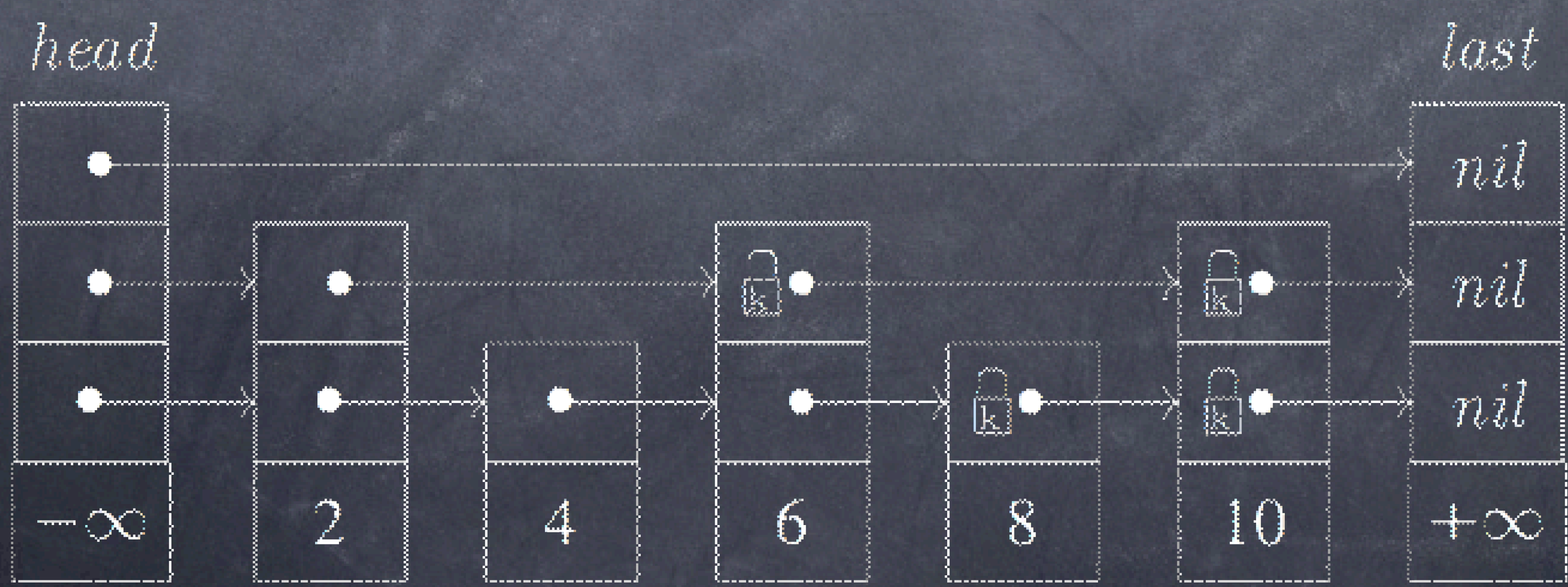

Fine-grained lock-coupling CSL

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$insert(sl, v)$

Algorithm 4 Insertion on a lock-coupling concurrent skiplist

```
procedure INSERT(SkipList sl, Key k, Value newval)
    Vector < Node* > update[1..sl.maxLevel]    //@ mrgn m_r := emp
    lvl := randomLevel()
    Node * pred := sl.head
    pred.locks[lvl].lock()                     //@ m_r := m_r ∪ (pred.forward[lvl])
    Node * curr := pred.forward[lvl]
    curr.locks[lvl].lock()                     //@ m_r := m_r ∪ (curr.forward[lvl])
    for i := lvl downto 1 do
        if i < lvl then
            pred.locks[i].lock()               //@ m_r := m_r ∪ (pred.forward[i])
            curr := pred.forward[i]
            curr.locks[i].lock()               //@ m_r := m_r ∪ (curr.forward[i])
        end if
        while curr.key < k do
            pred.locks[i].unlock()             //@ m_r := m_r − (pred.forward[i])
            pred := curr
            curr := pred.forward[i]
            curr.locks[i].lock()               //@ m_r := m_r ∪ (curr.forward[i])
        end while
        update[i] := pred
    end for
    if curr.key = k then
        curr.val = newval
        for i := 1 to lvl do
            update[i].forward[i].locks[i].unlock()
                                 //@ m_r := m_r − (update[i].forward[i].forward[i])
            update[i].locks[i].unlock()      //@ m_r := m_r − (update[i].forward[i])
        end for
    else
        x := CreateNode(lvl, k, newval)
        for i := 1 to lvl do
            x.forward[i] := update[i].forward[i]
            update[i].forward[i] := x          //@ sl.r := sl.r ∪ (x)
            x.forward[i].locks[i].unlock() //@ m_r := m_r − (x.forward[i].forward[i])
            update[i].locks[i].unlock()      //@ m_r := m_r − (update[i].forward[i])
        end for
    end if
end procedure
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Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v)$

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Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v) \quad decide(sl)$

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T_i

Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v) \quad decide(sl)$

$$T_i \models \Box \varphi_{insert}(i)$$

$$\varphi_{insert}(i) \hat{=} at\_insert^{[i]}_{8..36} \rightarrow at\_insert^{[i]}_{8..36}\, \mathcal{U}\, at\_insert^{[i]}_{37}$$

Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v) \quad decide(sl)$

$$\|_{j \in T_{ID} - \{i\}} T_j \parallel T_i \models \Box \varphi_{insert}(i)$$

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$$insert(sl, v) \quad search(sl, v) \quad remove(sl, v) \quad decide(sl)$$

$$\|_{j \in T_{ID} - \{i\}} T_j \parallel T_i \models \Box \varphi_{insert}(i)$$

$$\Updownarrow$$

$$\Psi$$

$$\Updownarrow$$

$$\varphi_{insert}(i) \hat{=} at\_insert^{[i]}_{8..36} \to at\_insert^{[i]}_{8..36} \, \mathcal{U} \, at\_insert^{[i]}_{37}$$

Fine-grained lock-coupling CSL

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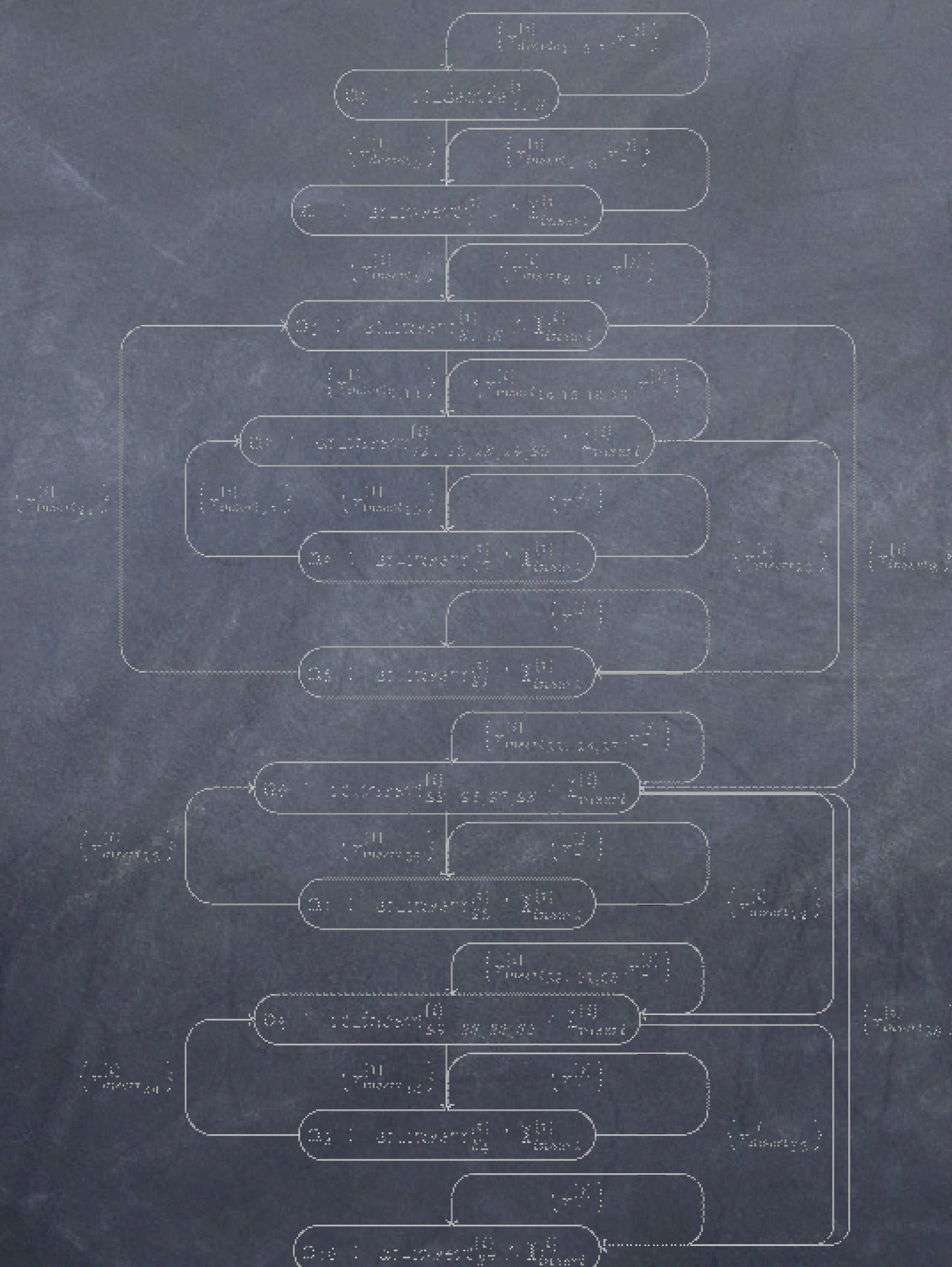

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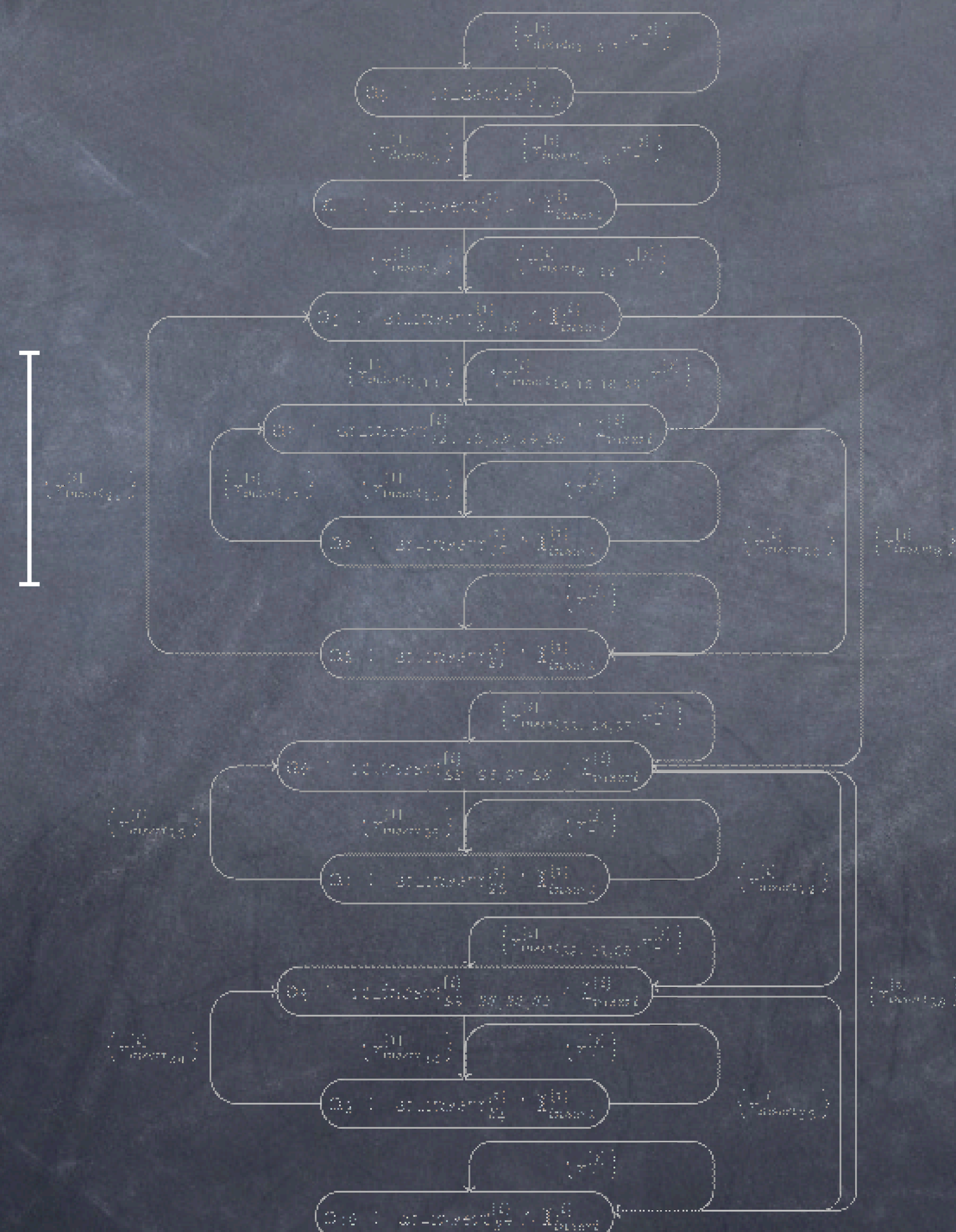

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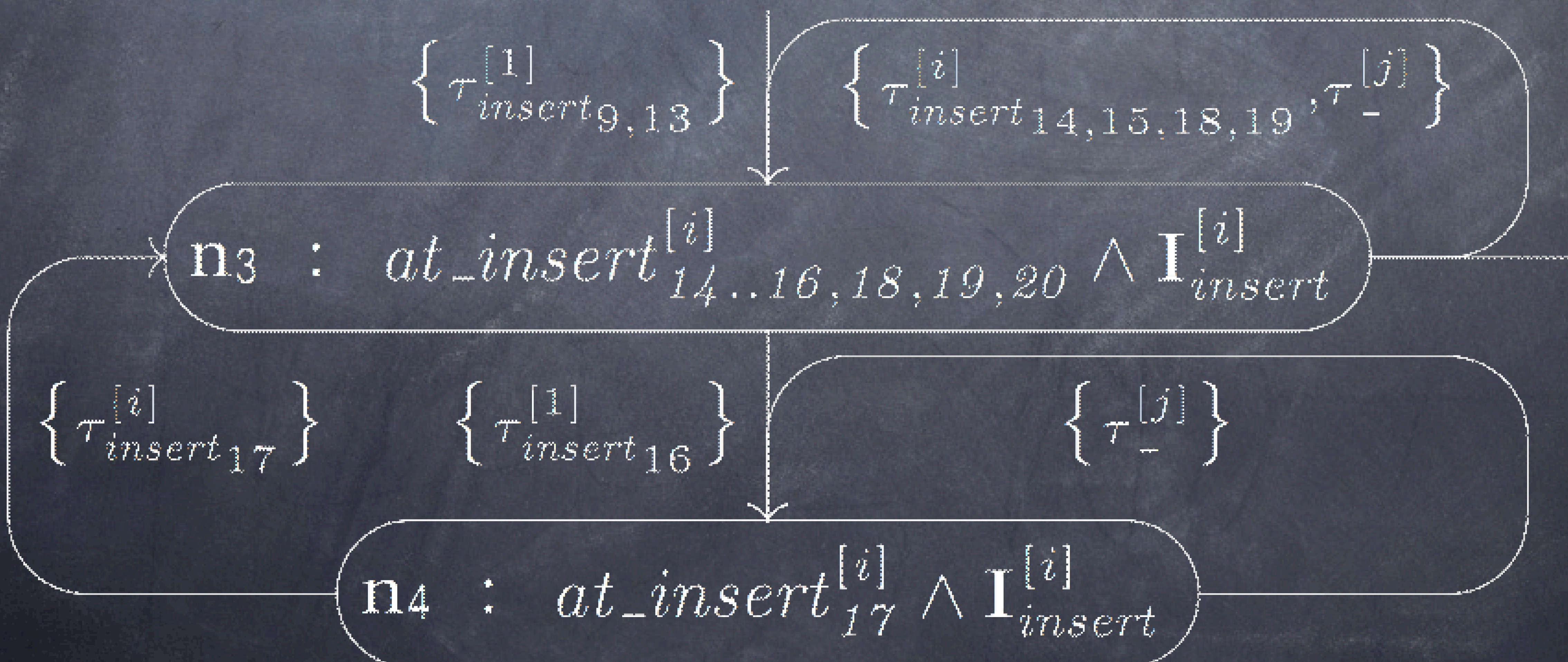
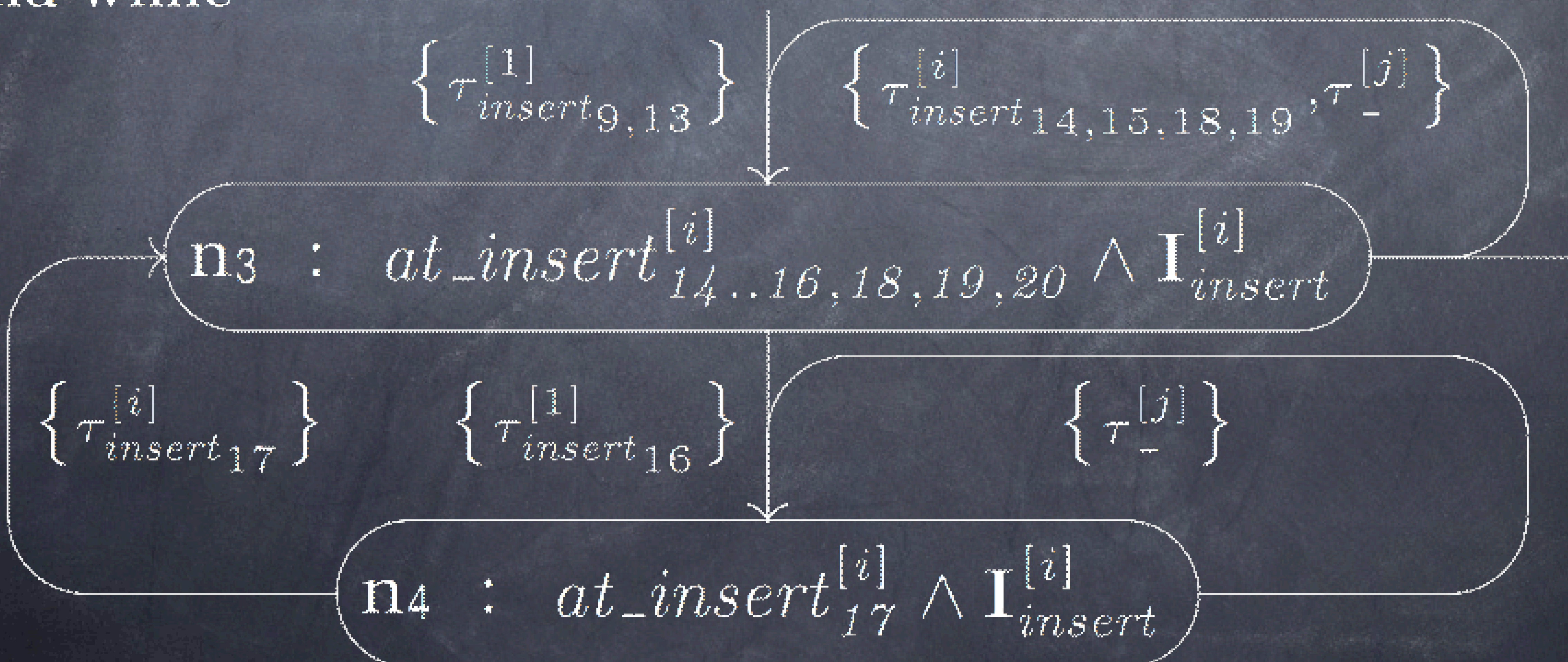


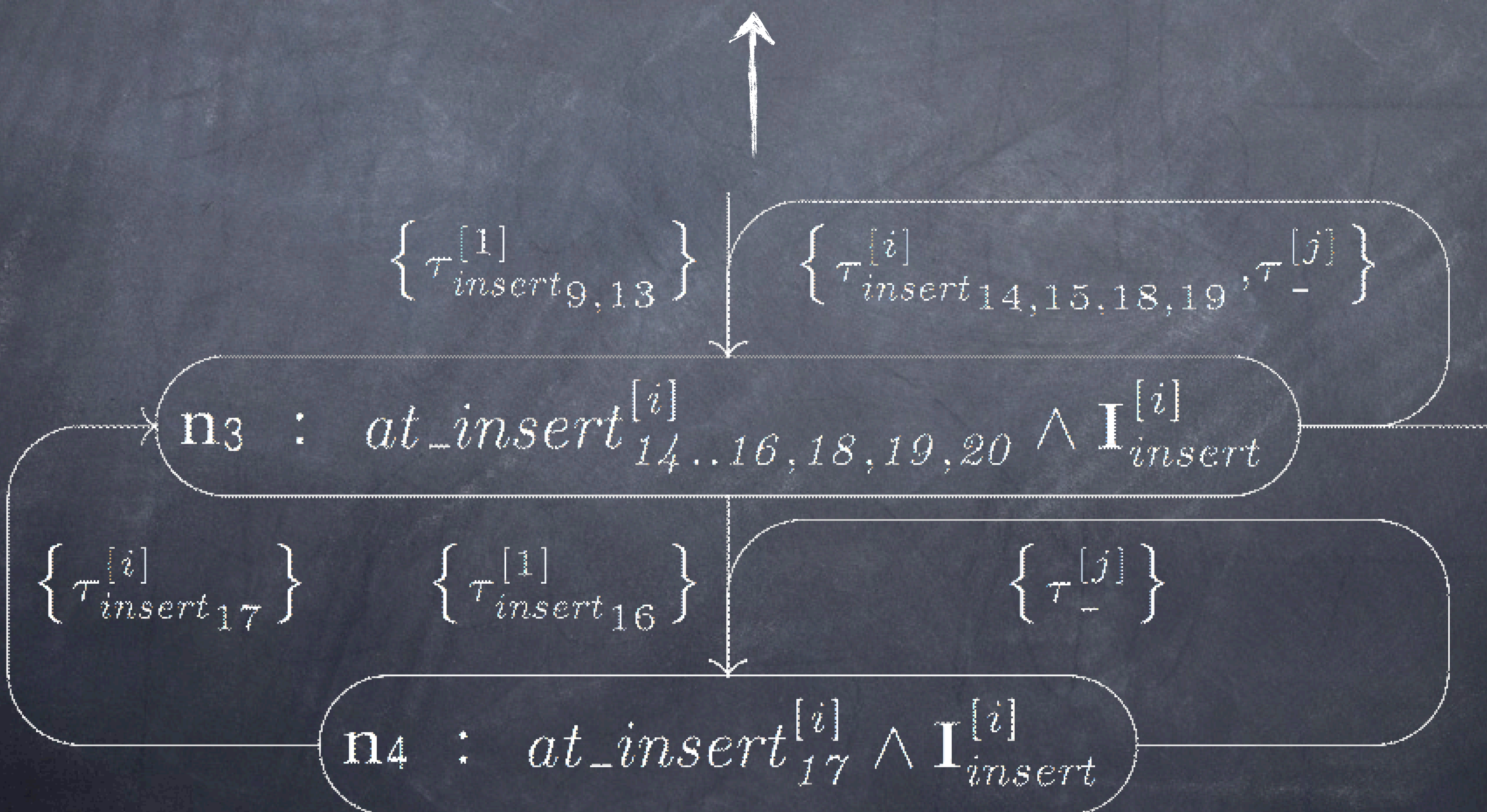
Fine-grained lock-coupling CSL

14: **while** $curr.key < k$ **do**
15: $\quad pred.locks[i].unlock()$ $\qquad$ $//@\ m_r := m_r - (pred, forward[i])$
16: $\quad pred := curr$
17: $\quad curr := pred.forward[i]$
18: $\quad curr.locks[i].lock()$ $\qquad$ $//@\ m_r := m_r \cup (curr, forward[i])$
19: **end while**



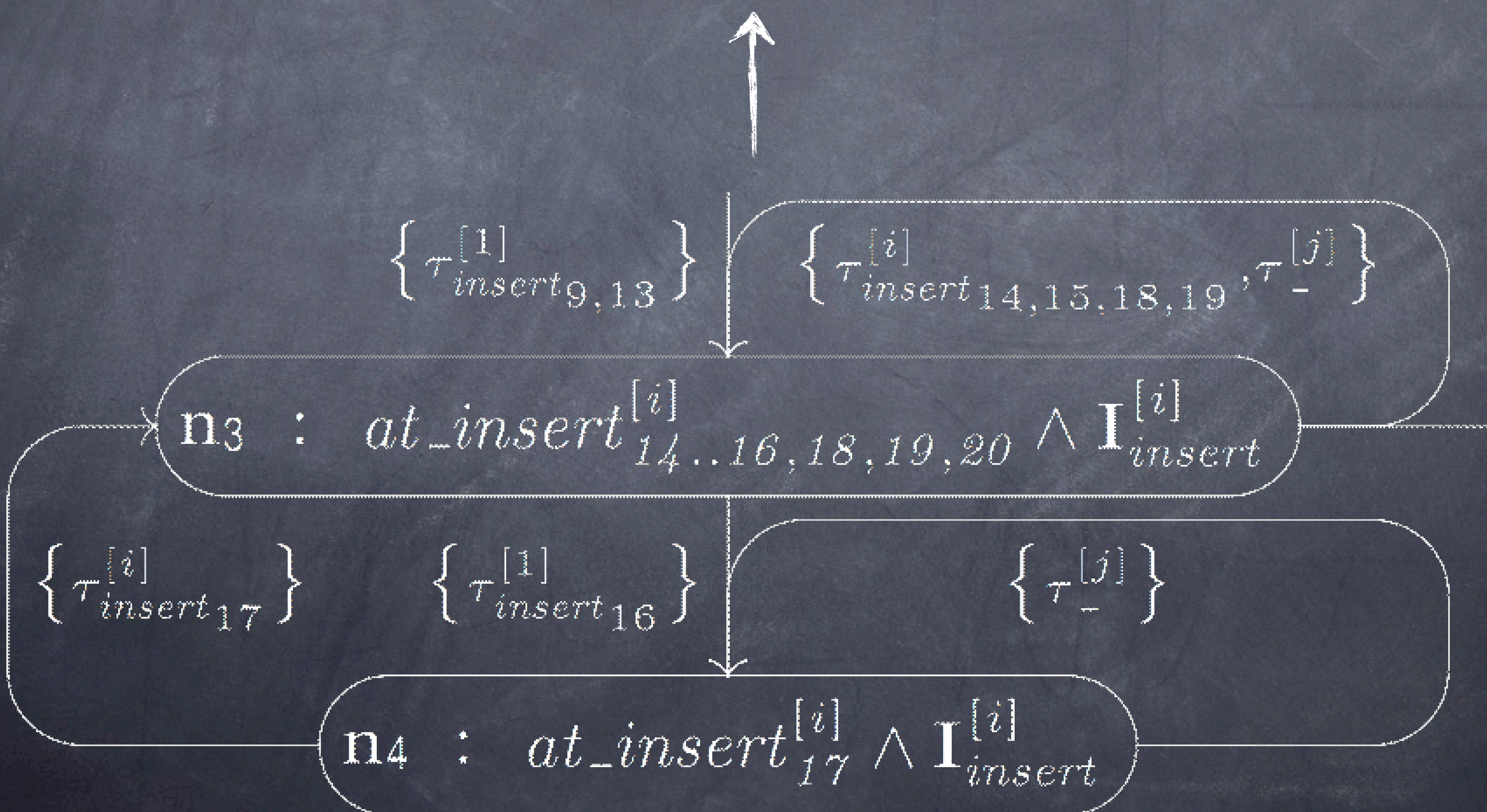
Fine-grained lock-coupling CSL

Verification conditions



Fine-grained lock-coupling CSL

Verification conditions ✓



Fine-grained lock-coupling CSL

Verification conditions ✓

$$\Psi$$

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Fine-grained lock-coupling CSL

Verification conditions ✓

$$\Psi$$

$$\downarrow$$

$$\|_{j \in T_{ID}-\{i\}} T_j \parallel T_i \models \varphi_{insert}(i)$$

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- Experience shows possibility of working with parameterized VD

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- This is just the beginning

Questions ?