Interval Observers for Non Monotone Systems
Application to Bioprocess Models

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1. Introduction

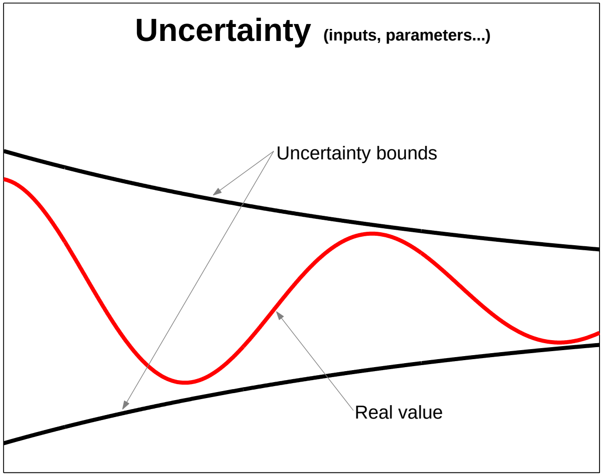
- Online monitoring of biotechnological processes:

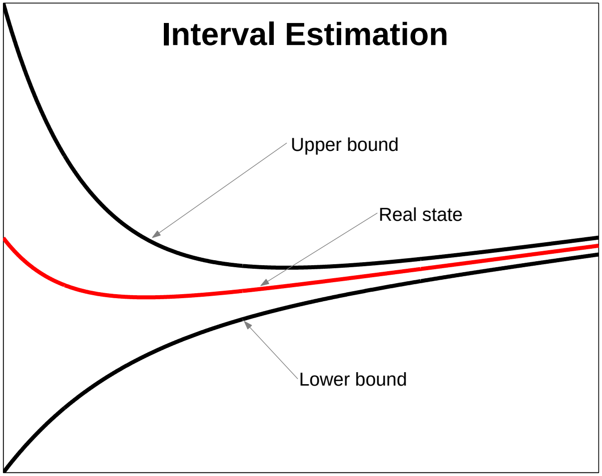
  $\longrightarrow$ Control and diagnosis tasks.
  $\longrightarrow$ Lack of hardware sensors.
  $\longrightarrow$ Development of observers.

- Mathematical models for bioprocesses:

  $\longrightarrow$ Available models are not accurate.
  $\longrightarrow$ Uncertainties.
  $\longrightarrow$ Interval observers.

2. Interval observers





- Suitable for estimation in uncertain systems.
- Known bounds of uncertainties are required.

Guaranteed (upper/lower) bounded estimation based on positivity

3. Class of system

$$(S_1) : \begin{cases} \dot{X}(t) = AX(t) + \psi(X, y); \quad X(0) = X_0 \\ y = CX(t) \end{cases} \tag{1}$$

where:

- $X \in \Omega \subset \mathbb{R}^n$: state vector.
- $A \in \mathcal{M}^{n \times n}$, $C \in \mathcal{M}^{1 \times n}$.
- $y \in \mathbb{R}$: system output.
- $\psi(X, y)$ is a $\mathcal{C}^1$ Lipschitz function

Example: Wastewater dynamical model

A two dimensional bioreactor model is considered:

$$(S_2) : \begin{cases} \dot{x} = r(X,t) - \alpha D x \\ \dot{s} = D(s_{in} - s) - k_1 r(X,t) \\ y = x \end{cases} \quad (2)$$

- Growing rate: Haldane model $\Rightarrow r(X,t) = x\mu_H(s)$, where $\mu_H(s) = \dfrac{\mu_0 s}{s + k_s + s^2/k_i}$
- Matrix A and function $\psi(X,y)$ are featured by: $A = \begin{pmatrix} -\alpha D & 0 \\ 0 & -D \end{pmatrix}$ and $\psi(s,x) = \begin{pmatrix} \mu(s)x \\ Ds_{in} - k\mu(s)x \end{pmatrix}$
- System main uncertainties: $s_{in} \in [s_{in}^-, s_{in}^+]$ and $\mu_0 \in [\mu_0^-, \mu_0^+]$.

4. Interval Observer: Closed loop approach

A property on non monotone functions:

$\psi(X,y) = f(X,y) - g(X,y),$ where f and g are increasing functions

Proof: Let us consider $f(X) = \gamma X$ and $g(X,y) = \gamma X - \psi(X,y)$, ($\gamma \geq 0$ is the Lipschitz constant).

$$\frac{\partial g}{\partial X_i} = \gamma - \frac{\partial \psi}{\partial X_i} \geq 0$$

Consequence:

$$X^- \leq X \leq X^+ \Rightarrow \bar{\psi}(X^+, X^-, y) \leq \psi(X,y) \leq \bar{\psi}(X^-, X^+, y) \qquad (3)$$

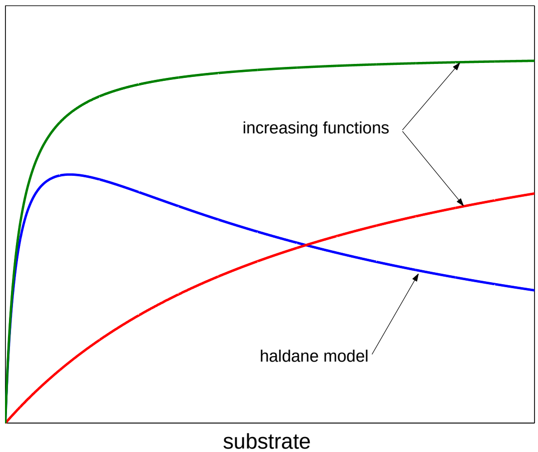
where $\bar{\psi}(X^1, X^2, y) = f(X^2, y) - g(X^1, y)$

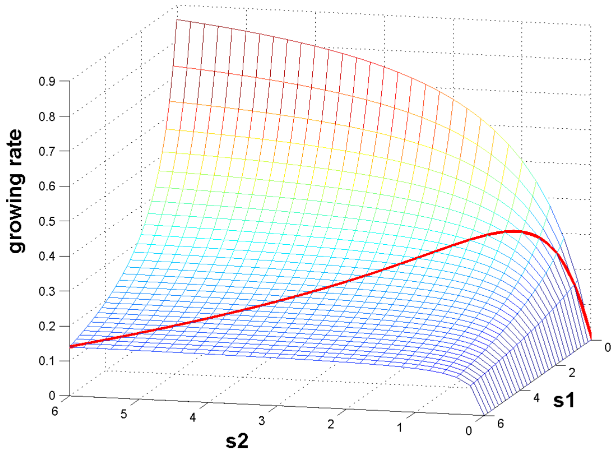
In our example:

$$s^- \leq s \leq s^+ \Rightarrow \mu_0 \bar{\rho}(s^-, s^+) \leq \mu(s) \leq \mu_0 \bar{\rho}(s^+, s^-)$$

where:

$$\bar{\rho}(s_2, s_1) = \frac{s_2}{s_2 + k_s + s_2 s_1 / k_i}$$





then we derive the bounds ψ^+ and ψ^-:

$$\psi(s,x) = \begin{pmatrix} \mu(s)x \\ Ds_{in} - k\mu(s)x \end{pmatrix} : \begin{cases} \psi^+(s^\pm, x) = \begin{pmatrix} \mu_0^+ \bar{\rho}(s^+, s^-)x \\ -\mu_0^- \bar{\rho}(s^-, s^+)x + Ds_{in}^+ \end{pmatrix} \\ \psi^-(s^\pm, x) = \begin{pmatrix} \mu_0^- \bar{\rho}(s^-, s^+)x \\ -\mu_0^+ \bar{\rho}(s^+, s^-)x + Ds_{in}^- \end{pmatrix} \end{cases} \quad (4)$$

Definition 1 *Cooperative Matrix.*

$$P \textit{ is cooperative} \Longleftrightarrow p_{ij} \geq 0 \quad \forall i, j \leq n, i \neq j$$

Considering $\zeta = (X^+, X^-)^t$: upper and lower estimates vector.

Lemma 1 (Closed Loop Interval Observer)

$$\frac{d\zeta}{dt} = \left(\begin{array}{c|c} A+\Theta_1 C & 0 \\ \hline 0 & A+\Theta_2 C \end{array}\right)\zeta - \begin{pmatrix}\Theta_1 \\ \Theta_2\end{pmatrix} y + \tilde{\psi}(\cdot) \qquad (5)$$

then $\forall X : X^-(t) \leq X(t) \leq X^+(t)$, provided that $X^-(0) \leq X(0) \leq X^+(0)$

where:

- $\Theta_i = (\theta_1^i, \ldots, \theta_n^i)^t$ gain vectors.
- $\tilde{\psi}(X^-, X^+, y) = \begin{pmatrix}\psi^+(X^+, X^-, y) \\ \psi^-(X^-, X^+, y)\end{pmatrix}$: bounds for ψ.

Proof:

- $\mathcal{P} = \left(\begin{array}{c|c} A+\Theta_1 C & 0 \\ \hline 0 & A+\Theta_2 C \end{array}\right)$, is a cooperative matrix.
- $\tilde{e} = (e^+, e^-)^t$: error vector where $e^+ = X^+ - X$ and $e^- = X - X^-$.

We have thus the following dynamics for the error:

$$\frac{d\tilde{e}}{dt} = \mathcal{P}\tilde{e} + \tilde{\phi}(X^+, X^-, y) \tag{6}$$

where $\tilde{\phi}(X^+, X^-, y)$ is defined as:

$$\tilde{\phi}(X^+, X^-, y) = \begin{pmatrix} \psi^+(X^+, X^-, y) - \psi(X, y) \\ \psi(X, y) - \psi^-(X^-, X^+, y) \end{pmatrix} \tag{7}$$

Consider the first time instant t_0 when one of the component of vector $\tilde{e}$ is equal to zero ($\mathcal{P}$=ρ_{ij}):

$$\frac{d\tilde{e}_k}{dt} = \rho_{kk}\tilde{e}_k + \sum_{i \neq k}^{2n} \rho_{ki}\tilde{e}_i + \tilde{\phi}_k(X^+, X^-, y)$$

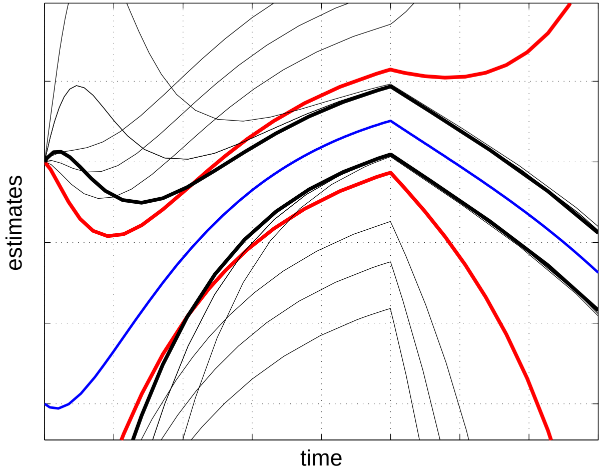
and thus at time instant t_0:

$$\left.\frac{d\tilde{e}_k}{dt}\right|_{t=t_0} = \sum_{i \neq k}^{2n} \rho_{ki}\tilde{e}_i + \tilde{\phi}_k(X^+, X^-, y) \geq 0$$

because $\tilde{e}_i(0) > 0$.

Remark on:
$\longrightarrow$ Cooperativity constraint.
$\longrightarrow$ Observer stability.

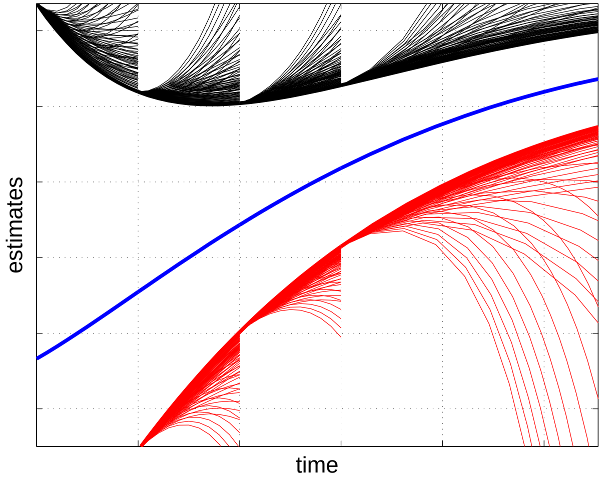
5. Bundle of Observers



$\longrightarrow$ Different observers are obtained for each value of Θ_i

Improvement of the estimate by varying the observer gains

6. Reinitialisation



- $[X_0^-(t_r), X_0^+(t_r)] = [\max\{X_{\Theta_i}^-\}(t_r), \min\{X_{\Theta_i}^+\}(t_r)]$, where t_r is the reinitialisation time.
- Regular reinitialisation let us improve the convergence rate of the observer.

7. Application to the example

- The matrix $A+\Theta C$ is expressed by: $A+\Theta C = \begin{pmatrix} -\alpha D + \theta_1 & 0 \\ \theta_2 & -D \end{pmatrix}$

Matrix $\mathcal{P}$ is cooperative $\Leftrightarrow \theta_2 \geq 0$

- The eigenvalues of matrix $A+\Theta C$ are: $\lambda_1 = -\alpha D + \theta_1$ and $\lambda_2 = -D$

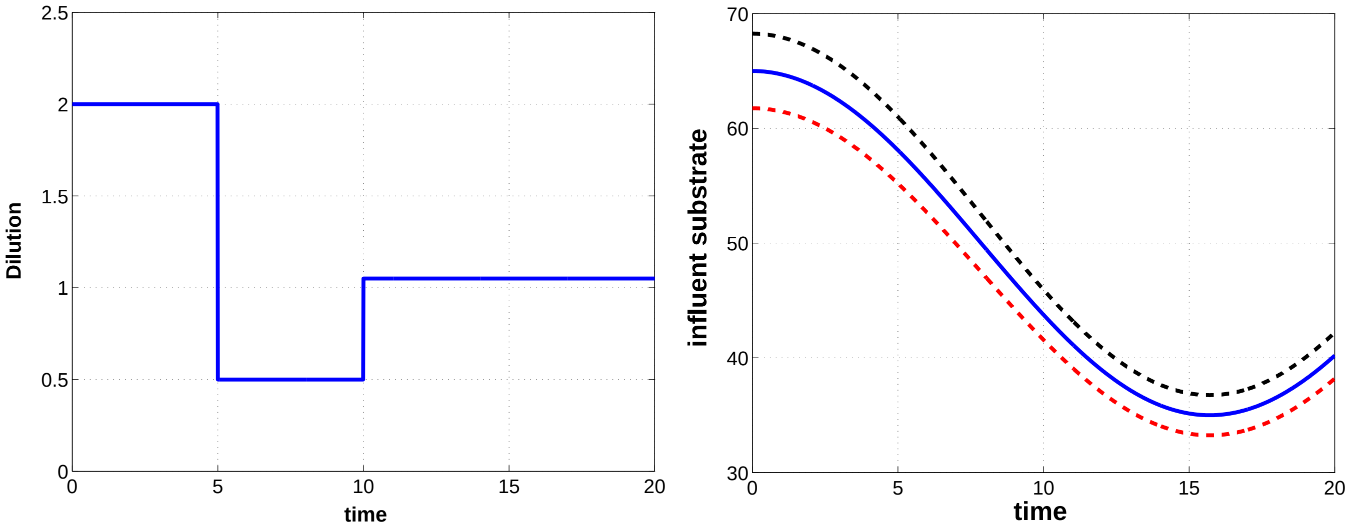
The boundness of the observer is guaranteed $\Leftrightarrow \theta_1 < \alpha D$

- Bundle of estimations: 77 observers working in parallel.
  With gains:

$$-10 \leq \theta_1 \leq 2 \text{ and } 0 \leq \theta_2 \leq 100.$$

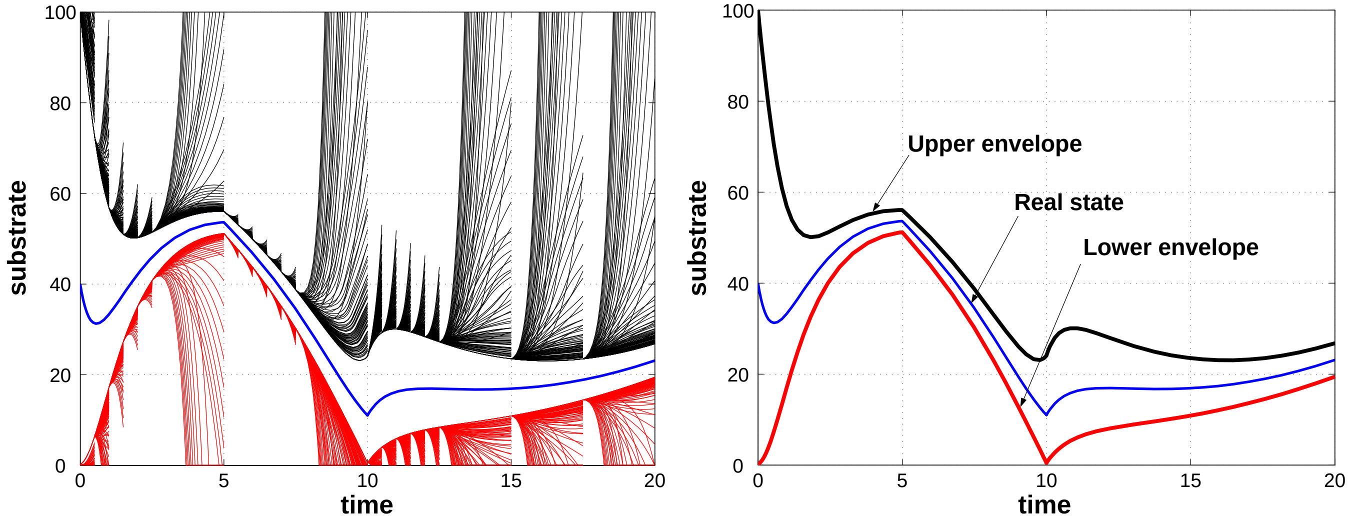
- Bounds for s_{in} and μ_0 have been fixed in a $\pm 5\%$ of their real values.

8. Simulation Results: Inputs



Dilution input and influent substrate

9. Simulation Results: Estimates



Observers bundle and final envelope

10. Conclusions

- Interval observer for non monotone systems:

n dimensional non monotone mapping	$\Rightarrow$	$2n$ dimensional monotone mapping

- The observer involves a strong coupling between the upper bound and the lower bound.
- The performance of the observer is improved with a regular reinitialisation maintaining both bounds in a reasonable range.
- Note that we could consider a bundle of observers issued from several observers based on various bounding functions ψ^+ and ψ^-.